THE FUZZY MATHEMATICS MODEL OF FORECASTING SAND LIQUEFACTION

Li ping

Department of Mathematics, Jilin province college of Education, 130022 Changchun, China.

Abstract: According to the ISODATA algorithm and the data of the earthquake taken place in Tangshan in 1976, a fuzzy mathematics model of forecasting sand liquefaction is constructed.

Keywords: Sand liguefaction, Fuzzy ISODATA algoritnm, Fuzzy mathematics model.

1. The fuzzy ISODATA algorithm

1) First the samples

$X_1, X_2 \cdots\cdots, X_n \quad (X_j=(X_{j1}, X_{j2}, \cdots\cdots X_{jm}), j=1,2\cdots\cdots,n)$ are dirided k groups, obtain primary matrix

$U=[u_{ij}]_{nxk}$

where u_{ij} satisfied

(1) $u_{ij} \in [0,1], i=1,2,\cdots\cdots,k, j=1,2,\cdots\cdots,n.$

(2) $\sum_{i=1}^{k} u_{ij}=1, \ j=1,2\cdots\cdots,n.$

(3) $\sum_{j=1}^{n} u_{ij}>0, \ i=1,2\cdots\cdots,k.$

2) The centers of clusters V_i:

$$V_i = \frac{\sum_{j=1}^{n} u_{ij}^2 \quad X_j}{\sum_{j=1}^{n} u_{ij}^2}$$

where $X_j = (X_{j1}, X_{j2}, \cdots\cdots, X_{jm})$

3)Fuzzy partition matrix

$$U_f^* = [u_{ij}^*]_{kxn}$$

$$u_{ij}^* = \frac{1}{\sum_{P=1}^{K} \left(\frac{\| X_j - V_i \|}{\| X_j - V_p \|} \right)^2}, \quad i=1,2\cdots\cdots,k, j=1,2\cdots\cdots,n.$$

where $\| X_j - V_i \| = \sum_{q=1}^{m} (X_{jq} - V_{iq})^2$

U_f^* satisfied

(1) $u_{ij}^* \in [0,1]$, $i=1,2\cdots\cdots k, j=1,2\cdots\cdots,n.$

(2) $\sum_{i=1}^{k} u_{ij}^* = 1$, $j=1,2\cdots\cdots,n.$

(3) $\sum_{j=1}^{n} u_{ij}^* > 0$, $i=1,2\cdots\cdots,k.$

(4) If $\max \{ |u_{ij}^* - u_{ij}| \} < \varepsilon$

where ε is a positive number be before hand, then

$$V = [v_{ij}]_{kxm}$$

and

$$U_f^* = [u_{ij}^*]_{kxn}$$

are resultsed. If not, repeat steps 2)—4).

2. The model of forecast

First we choose the samples that sand was Liquefied

$X_1, X_2, \cdots\cdots, X_n.$

where each X_i relied on m factors. Therefore X_i is characterized by a by a m—dimensional vector, that is

$X_i=(X_{i1},X_{i2}\cdots\cdots,X_{im}),\ i=1,2,\cdots\cdots,n.$

Then we apply the fuzzy ISODATA algorithm to X_1, X_2, ……, X_n. Assume that the clusters are obtained as follows

$A_1,A_2,\cdots\cdots,A_k.$

Extend $A_1,A_2,\cdots\cdots,A_k$ into fuzzy subsets in the universe of discourse R^m (R^m is a m—dimensional linear space)

$\underset{\sim}{A}_1,\underset{\sim}{A}_2,\cdots\cdots,\underset{\sim}{A}_K$

and their membership functions are

$\underset{\sim}{A}_1(x),\underset{\sim}{A}_2(x),\cdots\cdots,\underset{\sim}{A}_K(x)$

respectively. Set

$$\underset{\sim}{A}=\bigcup_{i=1}^{k}\underset{\sim}{A}_i$$

Thus

$$\underset{\sim}{A}(x)=\bigvee_{i=1}^{k}A_i(x)$$

Fix a level λ_0. Let X denote an arbitrary sample, compute the grade of membership of X in $\underset{\sim}{A}$:

$\underset{\sim}{A}(x)=\lambda$

If $\lambda\geqslant\lambda_0$ then we forecast that sand will be liquefied; if $\lambda<\lambda_0$ then we forecast that sand will be unliquefied.

we shall employ the example of earthquake appeared in Tangshan in 1976. Choose samples

$X_1,X_2\cdots\cdots,X_{40}$

and 7 factors for each X_i:

seismic intensity scale (Y_1), epicentral distance (Y_2), arerage grain diameter (Y_3), nonuniform coefficient (Y_3), ground water level (Y_5), embedment depth of sand stratum (Y_6), standard penetration ralue (Y_7).

Thus X_i mag be expressed as $X_i=(X_{i1},X_{i2},\cdots\cdots,X_{i7}),i=1,2,\cdots\cdots 40$

Then applying the fuzzy ISODATA algorithm to $X_1,X_2,\cdots\cdots,$ X40, we obtain

$A_1=\{X_1\},A_2=\{X_2,X_{34}\},A_3=\{X_3\},A_4=\{X_4,X_{12}\},$

$A_5=\{X_5\},A_6=\{X_{22},X_{24},X_{26}\},A_7=\{X_7,X_{16}\},A_8=\{X_8\},$

$A_9=\{X_6,X_9,X_{14},X_{15}\},A_{10}=\{X_{10}\},A_{11}=\{X_{11},X_{13}\}$

$A_{12}=\{X_{27},X_{28},X_{33}\},A_{13}=\{X_{17},X_{18},X_{19},X_{20}X_{21}\},$

$A_{14}=\{X_{23},X_{25}\},A_{15}=\{X_{29},X_{30},X_{31}\},$

$A_{16}=\{X_{32},X_{35},X_{36},X_{40}\},A_{17}=\{X_{37},X_{38},X_{39}\}.$

Extend $A_1,A_2,\cdots\cdots,A_{17}$ into fuzzy subsets in R^7

$\underset{\sim}{A}_1,\underset{\sim}{A}_2,\cdots\cdots,\underset{\sim}{A}_{17}$

The membership function of A_i is defined as

$$\underset{\sim}{A}_i(x)=\begin{cases}1-b_i\|X-V_i\|^2, & 1-b_i\|X-V_i\|^2\geqslant 0;\\ 0, & 1-b_i\|X-V_i\|^2<0\end{cases} \quad (1)$$

where b_i is a parameter in the relation to the great cluster radius of center V_i.

$b_1=10000,b_2=22.22,b_3=2000,b_3=57.14,b_5=2000,$

$b_6=17.83,b_7=17.83,b_8=2000,b_9=17.83,b_{10}=2000,$

$b_{11}=57.14,b_{12}=17.83,b_{13}=7.69,b_{13}=200,b_{15}=22.22$

$b_{16}=7.69,b_17=7.69$。

The centers V of clusters:

$$V=\begin{vmatrix}
0.55 & 0.37 & 0.15 & 0.85 & 0.06 & 0.12 & 0.01 \\
0.55 & 0.35 & 0.31 & 0.29 & 0.07 & 0.28 & 0.03 \\
0.55 & 0.56 & 0.18 & 0.30 & 0.08 & 0.17 & 0.07 \\
0.55 & 0.55 & 0.12 & 0.20 & 0.07 & 0.11 & 0.03 \\
0.55 & 0.51 & 0.17 & 0.17 & 0.03 & 0.12 & 0.02 \\
0.63 & 0.33 & 0.16 & 0.23 & 0.16 & 0.16 & 0.03 \\
0.55 & 0.53 & 0.09 & 0.38 & 0.05 & 0.26 & 0.03 \\
0.55 & 0.53 & 0.17 & 0.17 & 0.08 & 0.22 & 0.06 \\
0.55 & 0.53 & 0.19 & 0.19 & 0.09 & 0.16 & 0.02 \\
0.55 & 0.55 & 0.13 & 0.28 & 0.08 & 0.32 & 0.08 \\
0.55 & 0.53 & 0.16 & 0.19 & 0.08 & 0.29 & 0.06 \\
0.63 & 0.29 & 0.16 & 0.23 & 0.09 & 0.29 & 0.01 \\
0.63 & 0.78 & 0.15 & 0.22 & 0.19 & 0.37 & 0.07 \\
0.63 & 0.33 & 0.22 & 0.19 & 0.17 & 0.35 & 0.09 \\
0.63 & 0.29 & 0.15 & 0.25 & 0.07 & 0.36 & 0.05 \\
0.72 & 0.12 & 0.18 & 0.18 & 0.05 & 0.21 & 0.12 \\
0.72 & 0.10 & 0.11 & 0.23 & 0.07 & 0.39 & 0.09
\end{vmatrix}$$

Fix a level value $\lambda_0 = 0.6$. For example, for sample $X=(0.55, 0.47, 0.15, 0.86, 0.06, 0.12, 0.01)$, then (1) yields

$$\underset{\sim}{A}_1(x) = 1 - 10000 \times 0.000004 = 0.96$$

$$\underset{\sim}{A}_2(x) = \underset{\sim}{A}_3(x) = \cdots\cdots\cdots = \underset{\sim}{A}_{17}(x) = 0$$

Thus

$$\underset{\sim}{A}(x) = \bigvee_{i=1}^{17} \underset{\sim}{A}_i(x) = 0.96$$

Because $0.96 > 0.6$, we forecast that sand will be liquefied. It conforms with reality.