Granular Computing : A Preamble

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Abstract. *The study is concerned with the fundamentals of granular computing. Granular computing, as the name itself stipulates, deals with representing information in the form of some aggregates (embracing a number of individual entitites) and their ensuing processing. We elaborate on the rationale behind granular computing. Next, a number of formal frameworks of information granulation are discussed including several alternatives such as fuzzy sets, interval analysis, rough sets, and probability. The notion of granularity itself is defined and quantified. A design agenda of granular computing is formulated and the key design problems are raised. A number of granular architectures are also discussed with an objective of dealinating the fundamental algorithmic and conceptual challenges. It is shown that the use of information granules of different size (granularity) lends itself to general pyramid architectures of information processing. The role of encoding and decoding mechanisms visible in this setting is also discussed in detail along with some particular solutions. The intent of this paper is to elaborate on the fundamentals and put the entire area in a certain perspective while not moving into specific algorithmic details.*

Keywords: *information granules, information granulation, fuzzy sets, interval analysis, rough sets, shadowed sets, pyramid architectures, encoding and decoding*

1 Introduction

Last years saw a rapid growth of interest in so-called granular computing and computing with words as one among its realizations [13]. In a nutshell, granular computing is geared toward representing and processing basic chunks of information – information granules [5][12][14]. Information granules, as the name itself stipulates, are collections of entities, usually originating at the numeric level, that are arranged together due to their similarity, functional adjacency, indistinguishability or alike. The process of forming information granules is referred to as information granulation. No matter how this granulation proceeds and what fundamental technology becomes involved therein, there are several essential factors that drive all pursuits of information granulation. These factors include

- A need to split the problem into a sequence of more manageable and smaller subtasks. Here granulation serves as an efficient vehicle to modularize the problem. The primary intent is to reduce an overall computing effort
- A need to comprehend the problem and provide with a better insight into its essence rather than get buried in all unnecessary details. In this sense, granulation serves as an abstraction mechanism that reduces an entire

conceptual burden. As a matter of fact, by changing the "size" of the information granules, we can hide or reveal a certain amount of details one intends to deal with during a certain design phase.

The long-lasting tradition of computing using some specific information granules is a visible testimony that some specific versions of granular computing are omnipresent indeed. As a matter of fact, as we discuss in this study, digital – to- analog transformation leading to digital computing for analog world is just a highly representative (albeit quite specific) instance of granular computing. By tradition (and the associated technology dominant at that time), we have embarked on the digital world of computing. To interact with the continuous (analog) world, we use set-based granulation (more specifically, interval-valued granulation). This specific type of granulation comes under the name of analog-to-digital conversion.

Information granules may arise as a phenomenon of inherent nonuniqueness associated with the problem at hand. As a simple example, one can resort himself to any inverse problem; the type of characteristics involved (as the functions may be non-invertible) gives rise to relations and as a result, a collection of information granules rather than single numeric quantities. Dropping some input variable in a model may also lead to the same effect of granular information.

We may witness (maybe not always that clearly and profoundly) that the concept of granular computing tends to permeate a number of significant endeavors. The reason is quite straightforward. Granular computing as opposed to numeric computing is *knowledge-oriented.* Numeric computing is *data-oriented.* Undoubtedly, knowledge-inclined processing is a cornerstone of data mining, intelligent databases, hierarchical control, etc.

While the idea of granular computing has been advocated and spelled out in the realm of fuzzy sets (and seems to be a bit biased in this way), there are a number of fundamental formal frameworks that can be exploited as well. Several alternative paths to follow including interval analysis, rough sets and probabilistic environments, to name a few dominant and most visible options.

The diversity of the formal means used for information granulation and further processing of the resulting information granules has a common denominator. All of these environments share the same research agenda that attempts to address the fundamentals of granular computing.

A way of constructing information granules and describing them in an analytical fashion is a common problem no matter which path (probability or set-theoretic) we follow. The question as to the definition of the "size", "capacity" or "dimension" of the information granule is of primordial interest. How to measure granularity of the constructed information granules? How to relate this granularity with computational complexity? Those are open questions in the framework of granular computing that still await solid answers.

What are sound methodologies when operating on information granules? How to evaluate (validate and verify) granular constructs? What would be appropriate measures of relevance of granular models? These are fundamental issues posed in the case of numeric modeling and discussed in detail. The same suite of questions expressed in the case of granular architectures begs for further thorough investigations.

There is an intriguing question as to a way of navigating between constructs (models) developed at various levels of information granularity. Is the structure developed with the use of "large" information granules useful when more specific results are required? It is apparent that when forming information granules, the contributing elements loose their identity that is essentially a non-recoverable process. Now, how this could effect the results of computing involving bigger information granules? If we want to recover the details, how efficient could be our attempt? What are the limits of this reconstruction? These aspects boil down to the mechanisms of encoding and decoding granular information. When any datum enters a system operating at a certain level of information granularity, it becomes encoded. As a result it becomes "accepted" (tuned) to the level of information granularity present within this system. Once the system tends to communicate its results, these need to be decoded. In other words, encoding and decoding are interfaces between worlds (systems) operating at various levels of information granularity. We have already encountered this scheme in digital processing: encoding corresponds to the analog-to-digital (A/D) conversion whereas the decoding comes under the name of digital-to-analog (D/A) conversion.

Fuzzy modeling has emerged as an interesting, attractive, and powerful modeling environment applied to numerous system identification tasks. Granular computing forms a useful environment supporting all modeling pursuits and adding another dimension to the modeling itself. The key features being emphasized very often in this setting concern a way in which fuzzy sets enhance or supplement the existing identification schemes. It was Zadeh first [12] who has introduced the concept of fuzzy models and fuzzy modeling. The enhancements of system modeling conceived within this framework take place at the conceptual level as well as at the phase of detailed algorithms. In a nutshell, fuzzy models are concerned with the modeling pursuit that occurs at the level of linguistic granules (fuzzy sets or fuzzy relations) rather than the one that happens at a detailed and purely numeric level encountered in other modeling approaches. What fuzzy sets offer in system modeling is another more general and holistic view at the resulting model that gives rise to their augmented interpretation and better utilization. From a computational point of view, fuzzy sets are inherently nonlinear (viz. their membership functions are nonlinear mappings). As a consequence of such nonlinear character, one may anticipate that this feature augments the representation power of the fuzzy models. There have been a substantial number of various schemes of fuzzy modeling along with specific algorithmic variations that help eventually capture some specificity of the problem at hand and contribute to the efficiency of the overall identification schemes, cf. [4][6][8]. Quite often, in order to take advantage of numeric experimental data, the modeling algorithms resort themselves to a vast spectrum of neurofuzzy techniques, see e.g., [1][2][3][5][7][11].

Granulation is a necessary prerequisite that is required to take advantage of discrete models (where by "discrete" we mean granular) such as finite-state machines, Petri nets, and alike.

The objective of this study is to raise fundamental issues of granular computing as a new and unified paradigm of information processing, elaborate on a family of possible formal frameworks and formulate the key design problems associated with this form of computing.

2 Granular Computing: an Information Processing Pyramid

In granular computing we operate on information granules. Information granules exhibit different levels of granularity. Depending upon the problem at hand, we usually group granules of similar "size" (that is granularity) together in a single layer. If more detailed (and computationally intensive) processing is required, smaller information granules are sought. Then these granules are arranged in another layer. In total, the arrangement of this nature gives rise to the information pyramid. As portrayed schematically in Figure 1, in granular processing we encounter a number of conceptual and algorithmic layers indexed by the "size" of

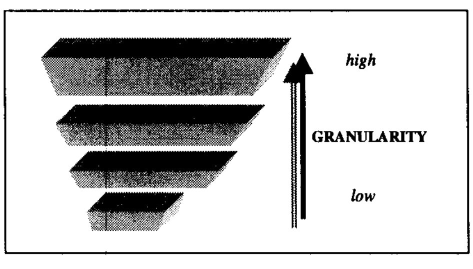

Figure 1. An information-processing pyramid (the respective layers are indexed by the corresponding level of information granularity)

information granules. Information granularity implies the usage of various techniques that are relevant for the specific level of granularity. Alluding to system modeling, we can refine Figure 1 by associating the layers of the information processing pyramid with the pertinent most commonly used classes of processing and resulting models

- at the lowest level we are concerned with numeric processing. This is a domain completely overwhelmed by numeric models such as differential equations, regression models, neural networks, etc.
- at the intermediate level we encounter larger information granules (viz. those embracing more individual elements)
- the highest level can be solely devoted to symbol-based processing and as such invokes well-known concepts of finite state machines, bond graphs, Petri nets, qualitative simulation, etc. Note that some of these classes emerge at the intermediate level of information granularity and at that level their conceptual and symbolic fabric is usually augmented with some numeric component.

The general characteristics of the principle of granular computing can be enumerated as shown in Table 1.

Allow for multiple abstraction levels (granularity levels)
Allow for several methods of traversing various levels of hierarchy (encoding – decoding mechanisms)
Allow for nonhomogeneous methods (differential or difference equations, Petri nets, finite state machines)

Table 1. The fundamental features of granular computing

3 Information Granulation

In this section, we look into the underlying rationale behind information granulation and discuss various means supporting the construction of information granules. The starting point is to look at any linguistic model as an association of information granules (linguistic terms) defined over some variables of the system. Quite descriptively, one may allude to such linguistic granules or linguistic landmarks as being focal point of all modeling activities. Linguistic granules are viewed as linked collections of objects (data points, in particular) drawn together by the criteria of indistinguishability, similarity or functionality. Such collections can be modeled in several formal environments including set theory, rough sets, random sets, shadowed sets or fuzzy sets.

Informally speaking, information granules [9][10][12][14] are viewed as linked collections of objects (data points, in particular) being drawn together by the criteria of indistinguishability, similarity or functionality. Information granules and the ensuing process of information granulation is a vehicle of abstraction leading to the emergence of concepts.

Granulation of information is an inherent and omnipresent activity of human beings carried out with intent of better understanding of the problem. In particular, granulation of information is aimed at splitting the problem into several manageable chunks. In this way, we partition the problem into a series of well-defined subproblems (modules) of a far lower computational complexity than the original one.

Granulation occurs everywhere; the examples are numerous and they originate from various areas

- we granulate information over time by forming information granules over predefined time intervals. For instance, one computes a moving average with its confidence intervals
- in any computer model we granulate memory resources by subscribing to the notion of pages of memory as its basic operational chunks (then we may consider various swapping techniques to facilitate an efficient access to individual data items)
- We granulate information available in the form of digital images – the individual pixels are arranged into larger entities and processed as such. This leads us to various issues of scene description and analysis
- In describing any problem, we tend to shy away from numbers. Instead, we tend using aggregates and building rules (*if-then statements*) that dwell on them.
- We live in an inherently analog world. Computers, by tradition and technology, perform processing in a digital world. Digitization of this nature (that dwells on set theory - interval analysis) is an example of information granulation
- All mechanisms of data compression are examples of information granulation that is carried in a certain sense

Overall, there is a profound diversity of the situations that call for information granulation. There is also panoply of possible formal vehicles to be used to capture the notion of granularity and provide with a suitable algorithmic framework in which all granular computing can be efficiently completed. In the ensuing section, we elaborate on those commonly encountered in the literature. Examples of such formal environments include set theory, rough sets, random sets, shadowed sets or fuzzy sets.

4 Fundamental Issues of Traversing Information Pyramid: Encoding and Decoding

Granular computing supports modeling activities carried out at various levels of information granularity, refer again to Figure 2.

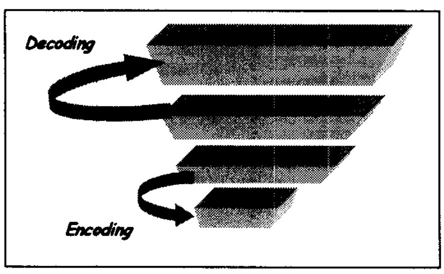

Figure 2. Decoding and encoding information granules as a vehicle of traversing the information pyramid

The ability to traverse through the layers characterized by different sizes of information granules is one of the dominant features of the modeling pursuits discussed in this framework. Each modeling layer indexed by the assumed level of granularity, comes with its own repertoire of modeling techniques. For instance, for the highest level of information granularity, viz. numeric data, we are dealing with differential equations and regression models as basic vehicles of system modeling. Commonly used neural networks fall under the same category.

When moving towards nonnumeric layer where some information granules of lower granularity are formed, we encounter a diversity of models such as Petri nets, finite state machines, bond graphs, constraint-based, etc. Depending on the specific form of granulation, we subsequently allude to fuzzy Petri nets, probabilistic Petri nets, etc.

The layers communicate between themselves. They receive data from other layers, complete computing (processing) and return the results to some other layers. These communication mechanisms are referred to as encoding and decoding, respectively. The role of the encoder is to transform the input information entering the given layer. The objective of the decoder is to convert the information granules produced by the given layer into the format acceptable by the destination layer. Depending on the problem at hand and the formalism of information granulation being used, a specific naming comes into play.

The general formulation of the encoding – decoding problem can be delineated as follows

- develop encoding (Enc) and associated decoding (Dec) algorithms such that the following relationship is satisfied

$$\mathrm{Dec}\,(\mathrm{Enc}\,(X)\,) = X$$

for all information granules (X) defined in a certain formal framework of information granulation and for a broad range of sizes of the information granules involved. In a limit case numeric granules are also included. Note, however, that the decoding-encoding scheme could be very demanding and one may not be able to meet the equality. More practically, we request that the design of these transformation should minimize the associated transformation error meaning that we are interested in minimizing the expression involving the distance $\|.\|$ between the original information granule and its transformation

$$\|\mathrm{Dec}\,(\mathrm{Enc}\,(X)\,) - X\| \rightarrow \mathrm{Min}$$

over a given range of granularity of X's involved there and for a fixed granulation environment.

The A/D and D/A conversions form an interesting illustration to the formulation of the problem given above, see Figure 3. We have

A/D: $\mathrm{Enc}(X) : X= \{x\} \in \mathbf{R} \rightarrow X \in \mathcal{P}(\mathbf{R})$ (the resulting granules are intervals in **R**; depending how the intervals are formed, one encounters either uniform quantization or a non-uniform one)

D/A: $\mathrm{Dec}(X) : X\in \mathcal{P}(\mathbf{R}) \rightarrow X= \{x'\} \in \mathbf{R}$ (usually a quantization error occurs so we never obtain the original numeric entity, $x \neq x'$).

The A/D and D/A conversions can be revisited and generalized in the framework of fuzzy sets, $\mathcal{F}(\mathbf{X})$. This leads to the following formulation of the problem

A/D: $\mathrm{Enc}(X) : X= \{x\} \in \mathbf{R} \rightarrow X \in \mathcal{F}(\mathbf{R})$ (the resulting granules are fuzzy sets in **R**; depending how they are formed, one encounters either uniform quantization or a non-uniform linguistic discretization of **X**)

D/A: $\mathrm{Dec}(X) : X\in \mathcal{P}(\mathbf{R}) \rightarrow X= \{x'\} \in \mathbf{R}$ (usually a quantization error it can be avoided by selecting a proper family of fuzzy sets. The zero error occurs for the triangular fuzzy sets with ½ overlap between successive membership functions).

In fuzzy controllers, the process of converting numeric data into the format accepted by the inference engine is called *fuzzification*. This is the name used for the encoding mechanism. The decoding is referred to as a *defuzzification* scheme.

One may also envision also a mixed form of information granules, namely they may originate from different formal environments of information granulation.

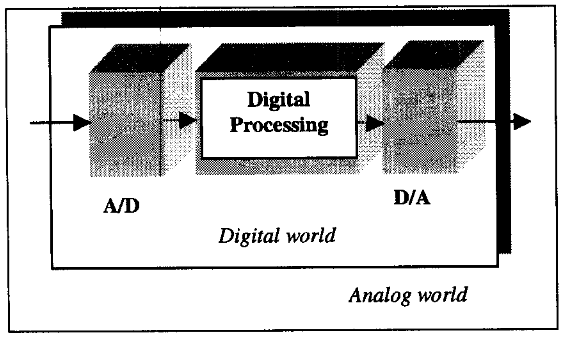

Figure 3. Digital processing as an example of commonly encountered granular computing; note a role of A/D and D/A converters utilized as the encoding and decoding modules

5 Conclusions

We have discussed the fundamentals of granular computing viewed as a new unified paradigm of processing information granules. Granular computing subsumes commonly encountered numeric processing as its special (limit) case.

The research agenda of granular computing includes a series of key and well-defined methodological and algorithmic issues

- Construction of information granules. This deals both with the selection of the formal framework of information granulation and detailed estimation procedure producing information granules. The latter dwells on the usage of the setting in which the granules are constructed
- Characterization of dimension (granularity) of information granules. This task is crucial as providing us with a better insight as to the essence of the granulation process and its implications both at the level of
- The development of the encoding and decoding mechanisms. These are essential to the functioning of any granular architecture. The encoding and decoding schemes are essential to the performance of granular computing. Interestingly enough, the essence of information compatibility expressed in terms of its granularity is inherently related with granular computing and nonexistent with

The study is an introduction to the rapidly growing research area. It concentrates on the methodology, attempts to identify the common features and help put the existing somewhat scattered approaches under the same conceptual umbrella.

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