The Stability of Fuzzy Large Dynamic System

Cheng Gang
(Liao Ning Technical University, Fuxin, Liao Ning, 123000, P. R. China)

Abstract

In this paper, the sufficient condition of stability about large fuzzy dynamic system is given.

Key words: Fuzzy number; Fuzzy dynamic; Fuzzy number matrix.

1. Sing, definition and elementary lemma

Def1: suppose R is a set of real number, $\tilde{R} = \{\tilde{u}|\tilde{u}:R \to [0,1]$, satisfies the following condition (Ⅰ)-(Ⅳ).

(Ⅰ) $\exists x_0 \in R$, such that $\tilde{u}(x_0) = 1$.

(Ⅱ) "$\tilde{u}$" is a convex fuzzy set.

(Ⅲ) "$\tilde{u}$" is a upper semi-cotinuous function.

(Ⅳ) "$\tilde{u}_0 = \{x \in R | \tilde{u}(x) > 0\}$" is compact set. ie, the closure of supp$\tilde{u}$ is compact set, then $\tilde{u} \in \tilde{R}$ is a fuzzy number, "$\tilde{R}$" is fuzzy number space. Obvioully, $\tilde{u} \in \tilde{R}$, $\lambda \in [0, 1]$, $\tilde{u}_\lambda$ are close intervals of non-empty and bounded. When $\lambda \in (0, 1]$, $\tilde{u}_\lambda = \{x | \tilde{u}(x) > \lambda\}$; when $\lambda = 0$, $\tilde{u}_0$ is (Ⅳ) of Def1.

For fuzzy number, there is the following theorem:

Theorem 1: If $\tilde{u} \in \tilde{R}$, then

(Ⅰ) $\lambda \in [0, 1]$, $\tilde{u}_\lambda$ is nonempty bounded close interval.

(Ⅱ) If $0 < \lambda_0 \leq \lambda_1 \leq 1$, then $\tilde{u}_{\lambda_2} \subseteq \tilde{u}_{\lambda_1}$.

(Ⅲ) If λ_n is a decreasing sequence, $\lambda_n > 0. \lambda_n \to \lambda \in (0, 1]$, then

$$\bigcap_{n=1}^{\infty} \tilde{u}_{\lambda_n} = u_\lambda$$

Inversely, if $\lambda \in [0, 1]$, $\exists \tilde{A}_\lambda \subset R$ and satifies (Ⅰ)-(Ⅲ), then there is a unique $\tilde{u} \in \tilde{R}$, such that $u_\lambda = A_\lambda$, $\lambda \in (0, 1]$ and $u_0 = \overline{\bigcup_{\lambda \in (0,1]} u_\lambda} \subset A$.

The proof of the theorem see [2].

Def2: $\tilde{u}, \tilde{v} \in \tilde{R}, k \in R$, then using extension principle, we give the following definition:

$$(\tilde{u} + \tilde{v})(x) = \sup_{s+t=x} \min(u(s), v(t))$$

$$(k\tilde{u})(x) = \tilde{u}(x / k) \qquad k \neq 0$$

$$(0\tilde{u}) = \hat{O}$$

$$(\tilde{u}\tilde{v})(x) \sup_{u-x} \min(u(s),v(t))$$

In which $\hat{O}(x)=\begin{cases}0 & x\neq 0\\ 1 & x=0\end{cases}$

Now, we consider the following dynamic system:

$$\begin{cases}\dfrac{dx_i}{dt}=\sum_{j=1}^{n}\tilde{a}_{ij}x_j\\ x_i(t_0)=x_{i_0}\end{cases} \qquad (1)$$

In which $\tilde{a}_{ij}\in\tilde{R}$ is fuzzy number, (i, j= 1、2、……n) $\tilde{A}$ is called a fuzzy number matrix.

Def3: Fuzzy number matrix $\tilde{A}$ is called stable, if for any $\lambda\in(0,1]$, $A\in\tilde{A}_\lambda$, A is stable,i.e,

$dx/dt=Ax$ is asymptotic stable too.In which

$$\tilde{A}_\lambda \overset{\Delta}{=} ([(\tilde{a}_{ij})_\lambda^-,(\tilde{a}_{ij})_\lambda^+])_{n\times n}$$

$$a_{ij}\in[(\tilde{a}_{ij})_\lambda^-,(\tilde{a}_{ij})_\lambda^+], A\overset{\Delta}{=}(a_{ij})_{n\times n}$$

For the sake of convenience , we denote

$$\tilde{A}_0=([(\tilde{a}_{ij})_\lambda^-,(\tilde{a}_{ij})_\lambda^+])\overset{\Delta}{=}([a_{ij}^-,a_{ij}^+])_{n\times n}$$

Def4: If $\lambda\in[0,1]$, $A\in\tilde{A}_\lambda$, A is unstable, ie $dx/dt=Ax$ is not asymptotic stable, Then $\tilde{A}$ is unstable, thereby (2) is not asymptotic stable, Denote $\tilde{A}\in US$.

Def5: If $\tilde{A}$ is neither stable nor unstable, then $\tilde{A}$ is mixed type. i.e, $\exists\ \lambda\in[0,1]$, such that $B_1\in A_\lambda$, B_1 is stable, $B_2\in\tilde{A}_\lambda$, B_2 is unstable; or $\exists\ \lambda_1\in[0,1]$, $B_1\in\tilde{A}_{\lambda_1}$, B_1 is stable; $\exists\ \lambda_2\in[0,1]$, such that $B_2\in\tilde{A}_{\lambda_2}$, B_2 is unstable, then $\tilde{A}$ is mixed type, denote $\tilde{A}\in$ CT. At the time, system (2) is called mixed type.

It is difficult for using the definition to discriminate the stability of fuzzy number matrix. Following give the necessary and sufficent condition to discriminate stability of the fuzzy number matrix.

Theorem 2:1) Fuzzy number matrix $\tilde{A}$ is stable if and only if $A\in\tilde{A}_0$, A is stable, in which $A=(a_{ij})_{n\times n}$, $a_{ij}\in[a_{ij}^-,a_{ij}^+]$, $(\tilde{a}_{ij})_0)\overset{\Delta}{=}[a_{ij}^-,a_{ij}^+]$ (i,j= 1, 2, ……, n)

2) Fuzzy number matrix $\tilde{A}$ is unstable if and only if $A\in\tilde{A}_0$, A is unstable.

3) Fuzzy number matrix $\tilde{A}$ is mixed type if and only if $\exists\ B_1\in\tilde{A}_0$, B_1 is stable; $\exists\ B_2\in\tilde{A}_0$, B_2 is unstable.

Proof: Necessary is obvious.

Sufficient: $\lambda\in(0,1]$, $A\in\tilde{A}_\lambda=((\tilde{a}_{ij})_\lambda^-)_{n\times n}$, Because of $\tilde{A}_0\supseteq\tilde{A}_\lambda$, then $A\in\tilde{A}'_0$, for A is stable, $\lambda\in(0,1]$ is arbitrary and $A\in\tilde{A}_\lambda$ is arbitray, then $\tilde{A}$ is stable.

The proof of (2)–(3) is similar.

2) The stability of fuzzy large system

Using theorem 2 to discriminate stability of fuzzy system is difficult too. Especially, to study the stability of fuzzy large system is more difficult. In this chapter, we use the idea of decomposition, reduce dimension, iteration to analysis the stability of fuzzy large system. In which, if the dimension of fuzzy number matrix is high, and the construct of fuzzy number matrix have not regular, then $\tilde{A}$ is called fuzzy large matrix, and system (2) is called fuzzy large system.

For studing the stability of system (2), it is very difficulty to directly compute. We use the following method to study system (2).

If $\tilde{A}$ is a fuzzy large system, we denote:

$$\tilde{A} = diag(\tilde{A}_{11},\cdots,\tilde{A}_{rr}) + (1-\delta_{ij})\tilde{A}_{ij}$$

$$\tilde{A}_0^- = diag(\tilde{A}_{11}^-,\cdots,\tilde{A}_{rr}^-) + (1-\delta_{ij})\tilde{A}_{ij}^-$$

$$\tilde{A}_0^+ = diag(\tilde{A}_{11}^+,\cdots,\tilde{A}_{rr}^+) + (1-\delta_{ij})\tilde{A}_{ij}^+$$

$$A = diag(A_{11},\cdots,A_{rr}) + (1-\delta_{ij})A_{ij}$$

In which $A\in \tilde{A}_0$, $A_{ij}\in(\tilde{A}_{ij})_0$, $\tilde{A}_{ij}$ is $n_i\times n_j$ fuzzy number matrix, $\tilde{A}_{ij}^-$, $\tilde{A}_{ij}^+$, $\tilde{A}_{ij}$ is $n_i\times n_j$ common matrix, $\sum_{i=1}^{r} n_i = n$, δ_{ij} is Kronecker sign.

$$m_{ij} \overset{\Delta}{=} \max_{A_{ij}\in(\tilde{A}_{ij})_0} \|A_{ij}\|$$

$A\in \tilde{A}_0$, we have dynamic system

$$\begin{cases} \dfrac{d\overline{X}}{dt} = A\overline{X} \\ \overline{X}(t_0) = \overline{X}_0 \end{cases}$$

Therefor,

$$\begin{cases} \dfrac{d\overline{X}}{dt} = (diag(A_{11},\cdots,A_{rr}) + (1-\delta_{ij})A_{ij})\overline{X} \\ \overline{X}(t_0) = \overline{X}_0 \end{cases}$$

Or equivalent:

$$\begin{cases} \dfrac{dX_i}{dt} = A_{ii}X_i + \sum\limits_{j=1,j\neq i}^{r} A_{ij}X_j \qquad (i=1,2,\cdots\cdots,r) \\ \overline{X}_i(t_0) = \overline{X}_{i_0} \end{cases} \tag{4}$$

$$\overline{X}_i = (x_1^i,\cdots\cdots,x_{n_i}^i), \qquad \sum_{i=1}^{r} n_i = n$$

The isolated sub-system of (4) is as the following:

$$d\overline{X}_i/dt = A_{ii}\overline{X}_i \qquad (i=1,2,\cdots\cdots,r) \tag{5}$$

Theorem 3: If the following condition are satisfied.

1) For any $A_{ii} \in (\tilde{A}_{ii})_0$, $\exists$ constant M_i, $\alpha_i > 0$, such that:

$$\|exp(A_{ii}(t-t_0))\| \leq M_i exp(-\alpha_i(t-t_0)) \qquad (i=1,2,\cdots,r)$$

2) Let $b_{ij} \stackrel{\Delta}{=} \frac{M_i}{\alpha_i}(1-\delta_{ij})m_{ij}$,

$B \stackrel{\Delta}{=} (b_{ij})_{r\times r}$, spectral radius $\rho(B) < 1$ (especially $\|B\| < 1$)

then $\tilde{A}$ is stable.

Proof: For any $A \in \tilde{A}_0$, obtain system (3) and (4), the solution of

(3) $\bar{X}(t,t_0,x_0) \stackrel{\Delta}{=} \bar{X}(t)$ is:

$$\bar{X}_i(t) = exp(A_{ii}(t-t_0)\bar{X}_i(t_0)) + \int_{t_0}^{t} exp(A_{ii}(t-t_1))\sum_{j=1}^{r}(1-\delta_{ij})A_{ij})\bar{X}_i(t_1)dt_1 \qquad (6)$$

Let $\bar{b}_{ij} = \frac{M_i}{\alpha_i - \varepsilon}(1-\delta_{ij})m_{ij}$, $\bar{B} \stackrel{\Delta}{=} (\bar{b}_{ij})$

For the eigenvalve of the matrix is continuous for its element, then for $0 < \varepsilon << 1$,

$$\rho(B) < 1(\|B\| < 1) \Rightarrow \rho(\bar{B}) < 1(\|\bar{B}\| < 1)$$

For formular (6), using iterative method:

$$\begin{cases} \bar{X}_i^{(m)}(t) = exp(A_{ii}(t-t_0))X_{i_0} + \int_{t_0}^{t} exp(A_{ii}(t-t_1))\sum_{j=1}^{r}(1-\delta_{ij})A_{ij}\bar{X}_i^{(m-1)}(t_1)dt_1 \\ X_i^0(t) = exp(A_{ii}(t-t_0))X_{i_0} \qquad i=1,2,\cdots,r \qquad m=1,2,\cdots. \end{cases}$$

When $t > t_0$, $0 < \varepsilon << \min_{1\leq i\leq r} \alpha_i$, we have

$$\|x_i^{(0)}(t)\| \leq M_i\|X_{i0}\|exp(-\alpha_i(t-t_0)) \leq M_i\|X_{10}\|exp(-\varepsilon(t-t_0)) \qquad (8)$$

$$(\|X_i^0(t)\|,\cdots,\|X_r^{(0)}(t)\|)^T \leq E(M_1\|X_{10}\|,\cdots,M_r\|X_{r_0}\|)^T exp(-\varepsilon(t-t_0))$$

$$\|X_i^{(1)}(t)\| \leq M_i\|X_{i0}\|exp(-\varepsilon(t-t_0)) + (\int_{t_0}^{t} exp(-(\alpha_i-\varepsilon))(t-t_1)$$

$$\cdot \sum_{j=1}^{r}(1-\delta_{ij})\|A_{ij}\|\|M_j\|\|X_{j_0}\|dt_1 exp(-\varepsilon(t-t_0))$$

$$\leq M_i\|X_{10}\|exp(-\varepsilon(t-t_0)) + \sum_{j=1}^{r}\frac{(1-\delta_{ij})m_{ij}M_j\|X_{j0}\|}{\alpha_i-\varepsilon}exp(-\varepsilon(t-t_0))$$

$$(i=1,2,\cdots,r) \qquad (9)$$

$$\|X_i^{(1)}(t) - X_i^{(0)}(t)\| \leq \sum_{j=1}^{r}\frac{(1-\delta_{ij})m_{ij}M_{ij}}{\alpha_i-\varepsilon}\|X_{j0}\|exp(-\varepsilon(t-t_0))(i=1,2,\cdots,r) \qquad (10)$$

i.e:

$$(\|X_1^{(1)}(t)\|,\cdots,\|X_r^{(1)}(t)\|)^T \leqslant (E+\bar{B})(M_1\|X_{10}\|,\cdots,M_r\|X_{r_0}\|)^T exp(-\varepsilon(t-t_0)) \quad (11)$$

Because of $\rho(\bar{B})<1$, then $\sum_{m=0}^{+\infty}\bar{B}^m=(E-B)^{-1}$ is convergent.

Let $\bar{B}^m \overset{\Delta}{=} (\bar{b}_{ij}^m)$

$$(\|X_1^{(m)}(t)\|,\cdots,\|X_r^{(m)}(t)\|)^T$$

$$\leqslant (E+\bar{B}+\cdots+\bar{B}^m)(M_1\|X_{10}\|,\cdots,M_r\|X_{r_0}\|)^T exp(-\varepsilon(t-t_0))$$

$$\leqslant (E-\bar{B})^{-1}(M_1\|X_{10}\|,\cdots,M_r\|X_{r_0}\|)^T exp(-\varepsilon(t-t_0)) \quad (12)$$

$$(\|X_1^{(m)}(t)-X_1^{(m-1)}(t)\|,\cdots,\|X_r^{(m)}(t)-X_r^{(m-1)}(t)\|)^T$$

$$\leqslant \bar{B}^m(M_1\|X_{10}\|,\cdots,M_r\|X_{r_0}\|)^T exp(-\varepsilon(t-t_0)) \quad (13)$$

Uusing mathematical induction, for every integer m, we can obtain that (12), (13) is valid.

Because of $\rho(\bar{B})<1$ and uniform convergence of

$$\sum_{m=0}^{+\infty}\bar{B}^m(M_1\|X_{10}\|,\cdots,M_r\|X_{r_0}\|)^T exp(-\varepsilon(t-t_0))$$

in (t_0, T), then.

$$(X_1^{(m)}(t),\cdots,X_r^{(m)}(t))\to(X_1(t),\cdots,X_r(t))$$

Hence:

$$(\|X_1(t)\|,\cdots,\|X_r(t)\|)^T$$

$$\leqslant (E-\bar{B})^{-1}(M_1\|X_{10}\|,\cdots,M_r\|X_{r_0}\|)exp(-\varepsilon(t-t_0))\to 0 \qquad (t\to\infty)$$

As a result, A is stable. Because A∈ $\tilde{A}_0$ is arbitrarity, According to theorem 2, $\tilde{A}$∈ S.

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