Brief Notes on Fuzzy Cooperation

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The theory of games is one of mathematical tools used for the modelling and investigation of economic phenomena. One of them is cooperation and correlation of behaviour modelled also by the concepts of theory of coalition games. The classical form of this theory is strictly deterministic - the negotiation, even if it takes its place before the realizations of the game, is based on deterministic knowledge of the pay-offs expected by coalitions and on the deterministic patterns of re-distribution of the pay-offs among the players in coalitions. The reality is, at least in most cases, rather different. The negotiating players have usually only uncertain idea about the expected outcomes of the game. Guaranteed or almost guaranteed values of the profit are accompanied by other values which are only possible or "not excluded" but which have to be taken into consideration, as well. Fuzzy set and fuzzy quantity theory are a good tool for mathematical representation of such uncertain knowledge.

Some research in this field was recently realized in the Institute of Information Theory and Automation of the Academy of Sciences of the Czech Republic in Prague. It was supported in 1996 - 98 by grant no. 402/96/0414 of the Grant Agency of the Czech Republic and its results were summarized in the publication referred below.

The main interest was oriented to the relations between the fuzziness of the "input" information about the expected pay-offs and the fuzziness of the "output" concepts like superadditivity; core realized coalition structures and other similar components of the game. It is presumed that vague knowledge of the possible pay-offs taking place in the bargaining period of the game leads to vague validity of the results of bargaining and other properties of the game.

The deterministic coalition game theory deals with various types of games. Two of them are especially significant with respect to their generality, namely, the coalition games with side-payments and those without side-payments. The latter type of games is even more general and it includes, in certain sense, rather modified games with side-payments as its special case.

The ***coalition games with side-payments*** are described by a pair (I, v), where I is a non-empty and finite set of ***players*** and v, called ***characteristic function*** of the game, is a mapping of 2^I into the set of real numbers R. It means that for every ***coalition*** $K \subset I$ the number $v(K) \in R$ represents its total income which is re-distributed among its members. In this model, the analysis of the game means processing of real numbers and investigation of the relations among them. The result of bargaining is represented by a ***coalition structure*** (i.e., a partition of the set players into realized coalitions) and by the set of acceptable ***imputations*** which are I-dimensional real-valued vectors determining the distribution of the expected total profit among the players. The set of such admissible imputations is called a ***core*** of the considered game. The uncertainty entering the model is assumed to be connected, primary, with the expected incomes of coalitions $v(K)$. Instead of crisp real number, they are considered to be fuzzy numbers (or fuzzy quantities) and we denote them by $w(K)$. Each fuzzy number $w(K)$ is defined as an extension of the crisp number $v(K)$ in the sense that $v(K)$ is its modal value. The fuzziness of the core or the possibility of realization of particular coalitions and the vagueness of other concepts in such fuzzified game are derived from the fuzziness of incomes $w(K)$ by means of their processing. It was shown that, if the quantities $w(K)$ are fuzzy extensions of crisp modal values $v(K)$ then also other fuzzified concepts extend their deterministic counterpart. For example, imputation $\mathbf{x}$ belonging to the crisp core of the deterministic, game (I,v) belongs to the fuzzy core with possibility equal to 1.

Also the ***coalition game without side-payments*** can be fuzzified. This procedure seems to be easier than the one used in games with side-payments. A coalition game without side-payments is defined as a pair (I,V), where for any $K \subset I$, $V(K)$ is a subset of R^I. The input information about the expected pay-offs of coalitions is here represented by sets $V(K) \subset R^I$ that can be extended into fuzzy subset. We denote them by $W(K)$, $K \subset I$, and suppose that any imputation $\mathbf{x} \in V(K)$

certainly belongs to the fuzzy set $W(K)$, i.e. its value of membership function is equal to 1. Anyhow, further processing of this fuzzy extension of coalition game without side-payments is not as simple as it could be expected. Namely, the concept of the domination relation between imputations, which is the crucial one for the definition of fuzzy core, can be treated in different ways having specific advantages and disadvantages. The results derived during the referred research show that the relation between fuzzy core and its deterministic pattern is rather more complicated than in the previous case but even here exist some interesting general conclusions. Namely, if an imputation $\mathbf{x}$ belongs to the crisp core of the deterministic game then it belongs to the fuzzy core of the fuzzified extension of (I,V) with positive possibility. On the other hand, the possibility that an imputation that does not belong to the deterministic core will be an element of the fuzzy one is never equal to 1.

There exists a special class of coalition games without side-payments (in the general terminology) which are derived from the games with side-payments and which can serve for comparison of both approaches to the fuzzy coalition game theory. If (I,v) is a game with side-payments then it is easy to define a game (I,V), formally without them, by

$$V(K)=\left\{\mathbf{x}\in R^I:\sum_K x_i \le v(K)\right\},\quad K\subset I.$$

In the deterministic case, the properties of both games (I,v) and (I,V), are completely equivalent. But different approach to their fuzzification, which is based on the processing of fuzzy numbers $w(K)$ in the case of (I,v) and on the processing of fuzzy subsets $W(K)$ of R^I in the case of (I,V), leads to rather different results for (I,w) and (I,W). These differences regard the fuzzy cores as well as the concepts of superadditivity and subadditivity and also the effectivity (i.e. accessibility) of particular coalitions. This part of the research continues, and the results will be prepared for publication in the near future.

The practical processing of the described models and solving numerical examples illustrate again a well-known disadvantage of arithmetical operation of addition defined for fuzzy numbers by means of the extension principle. Namely, frequent repetition of the operation (which is quite usual in the fuzzy games with side-payments) enormously increases the extent of uncertainty (support sets of membership functions) of the results. This fact motivated the research of an alternative approach to fuzzy quantities based on consequent separation of their quantitative and qualitative component. The quantitative one is described by a numerical value. The qualitative component, representing usually the verbal description of uncertainty, is described by a normalized form of membership function. Their composition characterizes a verbally formulated fuzzy quantity. Separate processing of quantitative components by classical algebraic methods and qualitative component by fuzzy logical and fuzzy semantic methods allowes to keep the formal uncertainty of the result in acceptable limits. The fuzzy quantities decomposed into quantitative and qualitative part are called ***generated fuzzy quantities***, and some of their properties are also summarized in several of the referred papers.

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