Further Discussions on Fuzzy Derivative

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In [1], the fuzzy derivative was defined by using the Caratheodory's derivative notion and a few basic properties of fuzzy derivative was proved. In this paper, we will continuous consider two new properties of fuzzy derivative. At the same time, we also study the high order fuzzy derivative and prove its two important properties, these properties again generalize the usually derivative theory.

Definition 1. Let E be a vector space over the field K of real or complex numbers, (E, T) be a fuzzy topological space, if the two mappings

(i) $\sigma : E\times E \to E, (x, y) \to x+y$

(ii) $\pi : K\times E \to E, (\alpha, x) \to \alpha x$

where K is the induced fuzzy topology of the usual nom, are fuzzy continuous. Then (E, T) is said to be a fuzzy topological vector space over the field K.

Defintion 2 (Caratheodory). Let f: $(a, b) \subseteq \mathrm{R} \to \mathrm{R}$, $c \in (a, b)$, the function f is said to be differentiable at the point $c \in (a, b)$ if there exists a function ϕ_c that is continuous at $x=c$ and satisfies the relation

$$f(x) - f(c) = \phi_c(x)(x-c)$$

for all $x \in (a, b)$.

Definition 3. Let R be the field of real numbers and (R, T) be a fuzzy topological vector space over the field R. f: R→R, $a \in R$, the function f is said to be fuzzy differentiable at the point a if there is a function ϕ that is fuzzy continuous at the point a, and have

$$f(x)-f(a)=\phi(x)(x-a)$$

for all $x \in R$. $\phi(a)$ is said to be the fuzzy derivative of f at a and denote $f'(a)=\phi(a)$.

Definition 4. Let R be the field of real numbers and (R, T) be a fuzzy topological vector space over the field R. f: $R \to R$, $a \in R$, if f is fuzzy differentiable at each point x and $f'(x)$ is also fuzzy differentiable at the point a, then f is said to be 2 –order fuzzy differentiable, and the 2 –order fuzzy derivative is denoted by $f''(a)$. Similar, we can define n –order fuzzy derivative ($n \geqslant 1$).

Definition 5. Let $z=f(x, y)$: $R^2 \to R$, (R, T) be a fuzzy topological vector space over the field R, if the function $z_b=f(x,b)$ is fuzzy differentiable at a, then the function $z=f(x, y)$ with respect to x is said to have fuzzy partial derivative at (a, b), and denote $z_x=f_x(a, b)$ is the partial derivative of f with respect to x at (a,b).

Our main results are as follows:

Theorem 1. If f is fuzzy continuous on $[a, b]$ and fuzzy differentiable on (a, b), then

$$f(b)-f(a)=\int_a^b f'(x)dx.$$

Theorem 2. If f is fuzzy continuous on $[a, b]$ and fuzzy differentiable on (a, b) except possibly at a finite number ,n, of points.

Then there are $n+1$ points $a<c_1<c_2<\cdots<c_{n+1}<b$ and $n+1$ positive numbers α_1, $\alpha_2,\cdots\alpha_{n+1}$ such that $\sum_{i=1}^{n+1}\alpha_i=1$, and

$$f(b)-f(a)=\sum_{i=1}^{n+1}\alpha_i\, f'(c_i)(b-a).$$

Theorem 3. If f is n-order fuzzy differentiable at $a\in R$, then for each $x\in R$,

$$f(x)=f(a)+f'(a)(x-a)+\cdots+f^{(n-1)}(a)\,(x-a)^{n-1}+f^{(n)}(\xi)(x-a)^n,$$

where $|\xi-a|<|x-a|$.

Theorem 4. For $z=f(x, y)$, if z_{xy} and z_{yx} is fuzzy continuous at (a, b), then $z_{xy}=z_{yx}$ at (a, b).

References

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