General fuzzy least squares fitting of several fuzzy variables

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Abstract: A model for least-squares fitting of more than two fuzzy variables is described. Previous process obtained for two fuzzy variables by Ma Ming is generalized and the method is different from the previous.

. Keywords: fuzzy least squares fitting, partition, J − consistent

1 Prelimineries

Let E^1 denote a function space such that $u \in E^1$ if and only if $u: R \rightarrow [0,1]$ is a function which satisfies the following requirements

(1) normality: $u(x_0) = 1$, for some x_0, $-\infty < x_0 < +\infty$

(2) u is upper semicontinuous, i.e.

$$\limsup_{x \to t} u(x) = u(t) \quad -\infty < t < +\infty$$

(3) u is a convex fuzzy set, i.e.

$$u(\lambda x + (1 - \lambda)y) \geqslant \min\{u(x), u(y)\}, \quad x, y \in R, \ 0 \leqslant \lambda \leqslant 1$$

(4) $[u]^0 = \text{closure}\{t \mid t \in R, u(t) > 0\}$ is compact

The space E^1 is called a fuzzy number space and each $u \in E^1$ is called a fuzzy number. Especially, for arbitrary $r \in R$, we call r degenerated fuzzy number.

For $u \in E^1$ and r: $0 \leqslant r \leqslant 1$, we define

$$[u]^r = \begin{cases} \{t \mid u(t) \geqslant r\}, & 0 < r \leqslant 1 \\ \text{closure}\{t \mid u(t) > 0\}, & r = 0 \end{cases}$$

The requirements (1) − (4) imposed on the elements of E^1 imply that $[u]^r, 0 \leqslant r \leqslant 1$ are closed intervals.

Let $\underline{u}(r)$ and $\overline{u}(r)$ denote the left and right endpoints of the closed interval $[u]^r$, respectively ($\underline{u}(0)$ and $\overline{u}(0)$ are the endpoints of the closed interval $[u]^0$).

Definition 1.1 For arbitrary $u, v, w \in E^1$ if they satisfy

$$\underline{w}(r) = \underline{u}(r) + \underline{v}(r) \qquad \overline{w}(r) = \overline{u}(r) + \overline{v}(r)$$

Then w is said to be sum of u and v denoted by u + v

Definition 1.2 For arbitrary $u, w \in E^1$, $k \in R$ if they satisfy

$$\underline{w}(r)\begin{cases} k\underline{u}(r), & \text{for } k \geqslant 0 \\ k\overline{u}(r), & \text{for } k < 0 \end{cases} \qquad \underline{w}(r) = \begin{cases} k\overline{u}(r), & \text{for } k \geqslant 0 \\ k\underline{u}(r), & \text{for } k < 0 \end{cases}$$

Then w is said to be scalar multiplication of k and u denoted by ku.

It is easily seen that $(u + v)$ and (ku) are also on E^1 for $u, v \in E^1$ and $k \in R$. Thus E^1 is a convex cone.

Definition 1.3 Let $u, v \in E^1$, a metric D in E^1 is defined as follow

$$D^2(u,v) \triangleq \int_0^1 (\underline{u}(r) - \underline{v}(r))^2 dr + \int_0^1 (\overline{u}(r) - \overline{v}(r))^2 dr$$

Definition 1.4 Let V be a closed convex cone in E^1, $u \in E^1$ and v an arbitrary element in V. If a $v_0 \in V$ can be found such that

$$P(u, v_0, v) \triangleq \int_0^1 [(\underline{u} - \underline{v}_0)(\underline{v}_0 - \underline{v}) + (\overline{u} - \overline{v}_0)(\overline{v}_0 - \overline{v})] dr \geqslant 0, \quad v \in V$$

u is said to be v_0-orthogonal to V

Lemma 1 For arbitrary $u, v, w \in E^1$, the following relation holds

$$D^2(u,v) = 2D^2(v,w) + 2D^2(v,w) - 4D^2(w, \frac{1}{2}(u+v))$$

Lemma 2 Let V be a closed convex cone in E^1 and $u \in E^1$. Then
(a) if for some $v_0 \in V$, $D(u, v_0) \leqslant D(u, v)$ for all $v \in V$, then v_0 is unique.
(b) a necessary and sufficient condition for v_0 be a unique minimizing fuzzy number in V for $D(u, v)$, $v \in V$, is that u is v_0 − orthogonal to V.

Theorem 1 Let V be a closed convex cone in E^1 and u an arbitrary element in E^1. Then:
(a) a unique $v_0 \in V$ for which $D(u, v_0) \leqslant D(u, v)$ for all $v \in V$, exist;
(b) a necessary and suficient condition for $v_0 \in V$ to be a unique minimizing element of $D(u, v)$, $v \in V$ is that u is v_0 − orthogonal to V.

Corollary 1 Let N be a positive integer and V a closed convex cone in $E^1 \times E^1 \times \cdots \times E^1 = (E^1)^N$. Denote by D_N the metric on $(E^1)^N$ define by

$$D_N^2(u,v) = \sum_{i=1}^{N} D_2^2(u_i, v_i) \qquad u, v \in (E^1)^N$$

where u_i, v_i are the components of u, v. Then for any $u \in (E^1)^N$ there is a unique $v_0 \in V$ such that

$$D_N(u, v_0) \leqslant D_N(u, v)$$

for all $v \in V$

2 Least squares fitting

Let it be supposed that observational results consist of (n + 1) − tuples data

$$X_{1i}, X_{2i}, \cdots, X_{ni}, Y_i (i = 1, 2, \cdots, N)$$

where $X_{pi} \in E^1 (p = 1, 2, \cdots, N)$ $Y_i \in E^1 (i = 1, 2, \cdots, N)$

Let $X_1, X_2, \cdots, X_n$ are vectors on $(E^1)^N$. $X_{1i}, X_{2i}, \cdots X_{ni}$ $(i = 1, 2, \cdots, N)$ are components of $X_1, X_2, \cdots, X_n$, respectively. And let it be supposed that $X_1, X_2, \cdots, X_n$

are independent variables. Then $V \stackrel{\triangle}{=} \{\beta_0 + \beta_1 X_1 + \cdots + \beta_n X_n, \beta_0, \beta_1, \cdots, \beta_n \in R\}$ is a convex cone on $(E^1)^N$. Affine function from V to $(E^1)^N$ will be considered

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_n X_n, \qquad \beta_0, \beta_1, \cdots, \beta_n \in R$$

Definition 2.1 Let e denote the degenerated fuzzy number 1 and E a N − dimensional vector of components to be e.

Partition the set of integers $\{1, 2, \cdots, n\}$ into two exhaustive mutually exclusive subsets $J(+)$ and $J(-)$. One of which may be empty. Each partition like this associate a binary multi − index

$$J = (j_1, j_2, \cdots, j_n)$$

defined by

$$j_p = \begin{cases} 0, & \text{if } p \in J(+) \\ 1, & \text{if } p \in J(-) \end{cases}$$

especially $J_0 = (0, 0, \cdots, 0)$, $J_1 = (1, 1, \cdots, 1)$

Definition 2.2 $C(J) = \{\beta_0 E + \beta_1 X_1 + \cdots + \beta_n X_n : \beta_p \geqslant 0, \text{if } j_p = 0; \beta_p < 0, \text{if } j_p = 1\}$
Then J is said to be conal index and C(J) a cone decided by conal index J

For a given conal index J, consider the problem of minimizing

$$M(J): r(\beta_0(J), \beta(J)) = \sum_{i=1}^{N} d(\beta_0 + \beta_1 X_{1i} + \cdots + \beta_n X_{ni}, Y_i)^2 \qquad (※)$$

we expect to find a parametric solution $\beta_0(J)$, $\beta_1(J), \cdots, \beta_n(J)$ of equation (※)
Definition 2.3 Denote by S(J) the system of n + 1 equations

$$\partial r(\beta_0(J), \beta(J))/\partial \beta_p = 0 \qquad (p = 0, 1, 2, \cdots, n)$$

Suppose that S(J) has a solution $\beta_0(J)$, $\beta_1(J), \cdots, \beta_n(J)$ such that $\beta_p(J) \geqslant 0$ if $j_p = 0$ and $\beta_p(J) < 0$ if $j_p = 1$. Then say that the model (※) is J − consistent with the data.

A model is thus J − consistent if the formal equation S(J) for unconstrained minimisation are compatible with $\beta_0 E + \beta_1 X_1 + \cdots + \beta_n X_n$ lying in C(J).

We can conclude the theorem as follow by corollary 1 directly.
Theorem 2 Let the data set $Y_i, X_{1i}, X_{2i}, \cdots, X_{ni}$ $i = 1, 2, \cdots, N$ be given for the model (※). For all conal index J, the system S(J) has a unique solution $\beta_0(J)$, $\beta_1(J), \cdots, \beta_n(J)$.

References

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