On the Definition of Intuitionistic Fuzzy Topology

Liu Huawen
Department of Mathematics and Physics, Shandong University of Technology, Jinan, 250061, China
Song Wenqing
Shandong Hydraulic Engineering College, Taian, 271000, China

Abstract

The definition of intuitionistic fuzzy topology was given in literature [1]. In this paper, we introduce the concept of gradation of openness of intuitionistic fuzzy set and give a new definition of intuitionistic fuzzy topology. Furthermore, we discuss the relation between the two definitions of intuitionistic topology.

Keywords: Intuitionistic fuzzy sets, intuitionistic fuzzy topology, intuitionistic fuzzy topological space, gradation of openness.

1. Introduction

After the introduction of the concept of fuzzy sets by Zadeh several researches were conducted on the generalizations of the notion of fuzzy set. The idea of "intuitionistic fuzzy set" was first published by Atanassov [2] and many works by the same author and his colleagues appeared in the literature [3-5]. The concept of intuitionistic fuzzy topology was first introduced by Doğan Çoker [1]. In this paper, we first introduce the concept of gradation of openness on intuitionistic fuzzy set, and then, give a new definition of intuitionistic fuzzy topology by means of the concept of gradation of openness. Furthermore, the relations of two definitions of intuitionistic fuzzy topology above are discussed.

2. Preliminaries

First we shall present the fundamental definitions given by Atanassov.

Definition 2.1.([2]) Let X be a nonempty fixed set. An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A=\{<x, \mu_A(x), \nu_A(x)>: x\in X\}$$

Where the functions $\mu_A: X\to I$ and $\nu_A: X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\nu_A(x)$) of each element $x\in X$ to the set A, respectively, and $0\le\mu_A(x)+\nu_A(x)\le 1$ for each element $x\in X$. IFS(X) denotes the all IFSs on X. Where I denotes unit interval [0,1].

Definition 2.2.([2]) Let $A,B\in IFS(X)$, we define the operations as follows:

(1) $A\subset B$ iff $\mu_A(x)\le\mu_B(x)$ and $\nu_A(x)\ge\nu_B(x)$ for all $x\in X$;

(2) $A=B$ iff $A\subset B$ and $B\subset A$;
(3) $A'=\{<x,\nu_A(x),\mu_A(x)>:x\in X\}$;
(4) $A\cap B=\{<x,\mu_A(x)\wedge\mu_B(x),\ \nu_A(x)\vee\nu_B(x)>:x\in X\}$;
(5) $A\cup B=\{<x,\mu_A(x)\vee\mu_B(x),\nu_A(x)\wedge\nu_B(x)>:x\in X\}$.

Definition 2.3. Let $A_t\in IFS(X),t\in T$.We define
(1) $\cap A_t=\{<x,\wedge\mu_{A_t}(x),\vee\nu_{A_t}(x)>:x\in X\}$;
(2) $\cup A_t=\{x,\vee\mu_{A_t}(x),\ \wedge\nu_{A_t}(x)>:x\in X\}$.

Since our main purpose is to construct the tools for developing intuitionistic fuzzy topological space,we must introduce the IFSs $0_\sim$and $1_\sim$ in X as follows:

Definition 2.4. $0_\sim=\{<x,0,1>:x\in X\}$ and $1_\sim=\{<x,1,0>:x\in X\}$.

3.Intuitionistic fuzzy topology

Definition 3.1.([1]) An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family τ of IFSs in X satisfying the following axioms:
(T1) $0_\sim,1_\sim\in\tau$;
(T2) $G_1\cap G_2\in\tau$ for any $G_1,G_2\in\tau$;
(T3) $\cup G_i\in\tau$ for arbitrary family $\{G_i:i\in J\}\subset\tau$.
In this case the pair $(IFS(X),\tau)$ is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS for short) in X.The complement A′ of an IFOS A in $(IFS(X),\tau)$ is called an intuitionistic fuzzy closed set (IFCS for short) in X.

In the following,we write $L=\{<a,b>:a+b\leq 1,a,b\in I\}$,and define operations in L as follows:

$<a,b>=<c,d>$ iff $a=c$ and $b=d$;
$<a,b>\leq<c,d>$ iff $a\leq c$ and $b\geq d$;
$<a,b><<c,d>$ iff $<a,b>\leq<c,d>$ and $<a,b>\neq<c,d>$;
$<a,b>'=<b,a>$.

It is easy to prove that Lis a complete lattice having the order-reversing involution "′",and it has maximal element $1=<1,0>$ and minimal element $0=<0,1>$.

Definition 3.2. Let X be a nonempty set and $\delta:IFS(X)\to L$ be a mapping satisfying the following properties:
(O1) $\delta(0_\sim)=\delta(1_\sim)=1$;
(O2) $\delta(A)>0$ and $\delta(B)>0$ implies $\delta(A\cap B)>0$ for $A,B\in IFS(X)$;
(O3) $\delta(A_t)>0,t\in T$ implies $\delta(\cup A_t)>0$ for $A_t\in IFS(X)$.
Then we call δ a gradation of openness on IFS(X) or an intuitionistic fuzzy topology (IFT for short).The pair $(IFS(X),\delta)$ is called an intuitionistic fuzzy topological space (IFTS for short).
If we rewrite properties (O2) and (O3) as follows:

(O2)′ $\delta(A)>0$ and $\delta(B)>0$ implies $\delta(A\cap B)\geq 0$ for $A,B\in IFS(X)$;

(O3)′ $\delta(A_t)>0, t\in T$ implies $\delta(\cup A_t)\geq 0$ for $A_t\in IFS(X)$.

Then we call δ a weak gradation of openness on IFS(X) or a weak intuitionistic fuzzy topology.The pair (IFS(X),δ) is called a weak intuitionistic fuzzy topological space.

Definition 3.3. Let X be a nonempty set and F:IFS(X)→L be a mapping satisfying the following properties:

(C1) $F(0_\sim)=F(1_\sim)=1$;

(C2) $F(A)>0$ and $F(B)>0$ implies $F(A\cup B)>0$ for $A,B\in IFS(X)$;

(C3) $F(A_t)>0, t\in T$ implies $F(\cap A_t)>0$ for $A_t\in IFS(X)$.

Then we call F a gradation of closedness on IFS(X).

Theorem 3.1. Let δ be a gradation of openness on IFS(X) and F be a gradation of closedness on IFS(X).We define the mapping:

$$F_\delta: IFS(X)\to L, A\to\delta(A') \text{ and } \delta_F: IFS(X)\to L, A\to F(A')$$

Then δ_F is a gradation of openness on IFS(X),F_δ is a gradation of closedness on IFS(X).

The proof is straightforward.

Definition 3.4. Let (IFS(X),δ) be an IFTS and $A\in IFS(X)$.Then the fuzzy interior and fuzzy closure of A are defined by

$$int(A)=\cup\{G\in IFS(X):\delta(G)>0, G\subset A\}$$
$$cl(A)=\cap\{K\in IFS(X):F_\delta(K)>0, K\supset A\}$$

It is easy to prove the following properties:

Theorem 3.2. Let (IFS(X),δ) be an IFTS,$A,B\in IFS(X)$,then

(1) $int(1_\sim)=1_\sim$; (2) $int(A)\subset A$; (3) $int(A\cap B)=int(A)\cap int(B)$;

(4) $int(int(A))=int(A)$; (5) $cl(0_\sim)=0_\sim$; (6) $cl(A)\supset A$;

(7) $cl(A\cup B)=cl(A)\cup cl(B)$; (8) $cl(cl(A))=cl(A)$.

Theorem 3.3. Let (IFS(X),δ) be an IFTS,$A\in IFS(X)$,then

(1) $\delta(A)>0$ iff $int(A)=A$; (2) $F_\delta(A)>0$ iff $cl(A)=A$.

In the following,we discuss the relation of definition 3.1 and definition 3.2.

Theorem 3.4. Let (IFS(X),δ) be an IFTS.We define $supp\delta=\{A\in IFS(X):\delta(A)>0\}$. Then $supp\delta$ is an IFT in literature [1].

Proof.(1) As δ is a gradation of openness on IFS(X),therefore $\delta(0_\sim)=\delta(1_\sim)=1>0$,i.e. $0_\sim,1_\sim\in supp\delta$;

(2) Let $A,B\in supp\delta$.Then $\delta(A)>0,\delta(B)>0$.we get $\delta(A\cap B)>0$ from property (O2),i.e. $A\cap B\in supp\delta$;

(3) Let $A_t\in supp\delta, t\in T$.Then $\delta(A_t)>0$ for any $t\in T$.We get $\delta(\cup A_t)>0$ from property (O3).

From above,we get that $supp\delta$ is an IFT in literature [1].

Definition 3.5. Let (IFS(X),τ) be an IFTS in [1].We call δ and τ is compatible if supp$\delta=\tau$.

Theorem 3.5. Let (IFS(X),τ) be an IFTS in [1].Then exist a gradation of openness δ_a on IFS(X) for any $a\in L$ and $a\neq 0$,such that δ_a and τ is compatible,i.e. supp$\delta_a=\tau$.

Proof.We define δ_a:IFS(X)$\rightarrow$L for $a\in L$ and $a\neq 0$ as follows:

$$\delta_a(A)=\begin{cases} 1, A=0_\sim,1_\sim \\ a, A\neq 0_\sim,1_\sim \text{ and } A\in\tau \\ 0,\text{otherwise} \end{cases}$$

It is easy to prove that δ_a is a gradation of openness on IFS(X) and supp$\delta_a=\tau$.

References

[1] Doğan Çoker,An introduction to intuitionistic fuzzy topological space,Fuzzy Sets and Systems 88(1997) 81-89.

[2] K.Atanassov, tIntuitionistic fuzzy ses,in:V.Sgurev,Ed,VIII TKR's Session.Sofia(June 1983 Central Sci. And Techn. Library,Bulg.Academy of Sciences,1984).

[3] K.Atanassov,Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986) 87-96.

[4] ,K.Atanassov,Review and new results on intuitionistic fuzzy ses,Preprint and IM-MFAIS-1-88,Sofia,1988.

[5] K.Atanassov and S.stoeva,Intuitionistic fuzzy sets,in:Polish Symp.on Interval & Fuzzy Mathematics,Poznan(August 1983) 23-26.