Fuzzy ideals in N(2,0) algebra

Jiang Zhao Lin

(Department of Mathematics, Lin yi Teacher's College, Shandong, 276005, P. R. China)

Deng Fang'an

(Department of Mathematics, HanZhong Teacher's College, Shaanxi, 723000, P. R. China)

Abstract: The fuzzy ideal of N(2,0) algebra is studied, some connectionsbetween fuzzy ideal and fuzzy N-ideal are discussed.

keywords: Fuzzy ideals; N(2,0) algebra; Monoid.

1. Introduction

Fuzzy ideals in Semigroups were introduced in [1] and discussed funther in[2]. In this note we shall describe the fuzzy ideals of N(2,0)algebra and investigate some connections between fuzzy ideals and fuzzy N-ideals of N(2,0) algebra.

2. preliminaries

We begin by recalling some definitions from [2] and [3].

Definition 2.1 [1] *Let S be a set with a constant 0, and the binaryoperation * subject to :*

(N1) $x*(y*z)=z*(x*y)$

(N2) $0*x=x$

for any $x, y, z \in S$, *then* $(S, *, 0)$ *is said to be a N(2,0) algebra.*

Remark 2.1 Let $(S, *, 0)$ *be a N(2,0) algebra, then the following identities hold for any* $x, y, z \in S$,

(1) $x*y=y*x$

(2) $(x*y)*z=x*(y*z)$

(3) $x*(y*z)=y*(x*z)$, $(x*y)*z=(x*z)*y$

(4) 0 is unit element .

hence N(2, 0) algebra is a Monoind

Definition 2. 2 Let S be a Set . A fuzzy set in S is a function $\mu: S \to [0, 1]$.

Definition 2. 3 Let μ be a fuzzy set in S . For $\alpha \in [0, 1]$, the set $\mu_\alpha = \{x \in S, \mu(x)\} \geq \alpha$ is called a level subset of μ.

Definition 2. 4 Let S be a N (2, 0) algebra , a function μ : $S \to [0, 1]$ is said to a fuzzy subalgebra of N (2, 0) algebra , if for any $x, y \in S$, $\mu(x*y) \geq \min\{\mu(x), \mu(y)\}$.

Definition 2. 5 Let S be a N(2, 0) algebra. A map $A: S \to [0, 1]$ is called fuzzy N-ideal of S , if for all $x, y \in S$.

$$A(x*(x*y)) \geq A(x)$$

Definition 2. 6 If S is a semigroup, then a fuzzy subset δ of S is called :

a fuzzy right ideal of S if $\delta(xy) \geq \delta(x)$ for all $x, y \in S$;

a fuzzy left ideal of S if $\delta(xy) \geq \delta(y)$ for all $x, y \in S$

a fuzzy ideal if it is both a fuzzy left ideal and a fuzzy right ideal.

Definition 2. 7 Let S be a N(2, 0) algebra. Let E(S) be the set idempotemts of S , ordered by the relation $a \leqslant b$ iff $a=b*c$, for any $a, b \in S$, there exists a unique element $c \in E(S)$.

It is easy to verify that relation "$\leqslant$" on S is a partial order. Hence $(S, *, \leqslant)$ partial order Monoid.

3. Fuzzy N-ideals and Fuzzy ideals

Theorem3. 1 Let A be a fuzzy N-ideal of S, for any $a, b \in S$, if $a \leqslant b$, then $A(a*b) \geq A(b)$.

Proof. Since $a \leqslant b$, there exists a unique element c in S such that $a=b*c$, so $A(a*b)=A((b*c)*b)= A(b*(b*c)) \geq A(b)$ (by A is a fuzzy N - ideal).

By Therem3. 1 we have the conclusion:

Every fuzzy N-ideal of N(2, 0) algebra with partail order $(S, *, \leqslant)$ is a fuzzy ideal.

Each subset I of S may be regarded as a fuzzy subset by identifing it with its characteristic function X_I *. IF I is any nonempty subset of S, then I is a ideal if and only if* X_I *is a ideal(see[2]). From above statement we have:*

Theorem3.2 E(s) is a ideal of S.

Proof. For any a, b in E(s), by $A(a*b)=A((a*a)*b)\geqslant A(a)$, $A(b*a)=A(a*b)\geqslant A(a)$ *So A is a fuzzy ideal of S, hence E(s) is a ideal of S.*

Theorem3.3 Let A be a fuzzy N-ideal of S, if $a\leqslant b$, *for any* $y\in S$, *the following hold:*

$a*y\leqslant b*y$ *and if* $a*b=0$ *then* $A(y)\geqslant A(b)$.

Proof. Since $a\leqslant b$, *there exists c in S, such that* $a=b*c$, *hence* $a*y=(b*c)*y=(b*y)*c$, *So* $a*y\leqslant b*y$, *in addition, if* $a*b=0$, *then* $A(y)=A(0*y)= A((a*b)*y)=A((b*c)*b*y))=A\ (b*(b*(c*y)))\geqslant A(b)$.

Theorem3.4 IF $a\leqslant b$ *and* $a*b=0$ *then* $A(a^2)=A(c)$ *(Where* $c\in E(S)$, *satisfying* $a=b*c$*).*

Proof. $A(a^2)=A(a*a)=A(a*(b*c))=A((a*b)*c)=A(0*c)=A(c)$

Theorem3.5 Let S be a N(2,0) algebra, for any a, b $\in S$, *if* $a*b=0$, *and* λ *is a fuzzy ideal of S, then* $\max\{\lambda(a), \lambda(b)\}\leqslant\lambda(0)$.

Proof. By $\lambda(o)=\lambda(a*b)=\lambda(b*a)\geqslant\lambda(a)$ *or* $\lambda(0)=\lambda(a*b)\geqslant\lambda(b*a)\geqslant\lambda(b)$ *we have:* $\lambda(0)\geqslant\max\{\lambda(a), \lambda(b)\}$.

Theorem 3.6 Let $(S, *, \leqslant)$ *is a N(2,0) algebra with a partail order"*$\leqslant$*", A is a fuzzy N-ideal of S, for any* $x, y\in S$, *if* $x\leqslant y$ *and* $x*y=0$ *then* $A(y)=A(0)$

Proof. By Reference[3] $A(y)\geqslant A(0)$, *on the other hand, by* $x\leqslant y$, *there exists a element z such that* $x=y*z$, *hence* $A(0)=A(x*y)=A((y*z)*y)=A(y*(y*z))\geqslant A(y)$ *(by A is a fuzzy N-ideal of S). Therefore* $A(y)=A(0)$.

Reference

[1] A. Rosenfeld, Fuzzy groups, J. Math. Anal. Appl. 35 (1971) 512-517.
[2] R. G. Mclean and H. Kummen, Fuzzy ideals in semigroups, Fuzzy sets and Systems 48(1992) 137-140.
[3] Deng Fang'an, Jiang Zhao lin, Xu Yang, Fuzzy N(2, 0) algebra, BUSEFAL (in France). Vol73(1997).
[4] Pratyayananda Das, Fuzzy regular and Inverse Subsemigroups, Fuzzy sets and systems 91(1997) 99-105.