Lower and upper weakly semi-continuous fuzzy multifunctions

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Abstract:

This paper is devoted to the introduction and study of fuzzy lower and upper weakly semi-continuous multifunctions between a general topological space (X,T) and a fuzzy topological space (Y,T_1) .Using the mapping F_* and F^* several equivelent conditions of a lower and upper fuzzy weakly semi-continuous multifunction.

Key words: Fuzzy semi-open(closed) sets; Fuzzy pre-semiopen (pre-semiclosed) sets; Fuzzy lower (upper) semi-continuous multifunctions; Fuzzy lower (upper) weakly semi-continuous multifunctions.

1.Preliminaries

Throughout the paper , by (X,T) or simply by X we will mean a topological space in classical sense, and (Y,T_1) or simply by Y will stand for a fuzzy topological space (fts, for short) as defined by Chang's [1].Fuzzy sets in Y will be denoted A,B,U,V etc. and interior(semi-interior) and closure (semi-closure) and complement of a fuzzy set A in an fts Y will be denoted by $Int\,A$ $(SIntA)$ $Cl\,A\,(SClA)$ and $1-A$ respectively.when a fuzzy set A is quasi-coincident (q-coincident, for short) with a fuzzy set B in (Y,T_1), we shall write AqB , if A and B are not q-coincident, denoted by $A\,\bar{q}B$ .The words 'neighborhood' and 'fuzzy topological space' will be abbreviated as 'nbd', and 'fts' respectively.

Definition 1.1. Let (X,T) be a topological space in the classical sense and (Y,T_1) be a fuzzy topological space. $F:X \to Y$ is called a fuzzy multifunction iff for each $x \in X$, $F(x)$ is a fuzzy set in Y.

Definition 1.2. For a fuzzy multifunction $F:X \to Y$, the $F_*(A)$ and $F^*(A)$ of a fuzzy set A in Y are defined as follows:

$$F_*(A) = \{x \in X: F(x) q A\}, \qquad F^*(A) = \{x \in X: F(x) \leq A\}$$

Theorem 1.3.[4] For a fuzzy multifunction $F:X \to Y$ we have $F_*(1-A)$ $= X - F^*(A)$, for any fuzzy set A in Y.

Definition 1.4. For a fuzzy multifunction $F:X \to Y$ is called :

(a) fuzzy lower semi-continuous (f.l.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F_*(V)$, there exists a semi-open nbd U of x_0 in X such that $U \subset F_*(V)$,

(b) fuzzy upper semi-continuous (f.w.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F^*(V)$), there exists a semi-open nbd U of x_0 in X such that $U \subset F^*(V)$,

(c) f.l.s.c. (f.u.s.c.) on X iff it is respectivety so at each $x_0 \in X$,

(d) f.s.c. on X iff it is f.l.s.c. and f.u.s.c. .

2. Lower and upper weakly semi-continuous fuzzy multifunctions

Definition 2.1. For a fuzzy multifunction $F:X \to Y$ is called :

(a) fuzzy lower weakly semi-continuous (f.l.w.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F_*(V)$, there exists a semi-open nbd U of x_0 in X such that $U \subset F_*(SClV)$,

(b) fuzzy upper weakly semi-continuous (f.u.w.s.c., in short) at a point $x_0 \in X$ iff for every fuzzy open set V in Y will $x_0 \in F^*(V)$, there exists a semi-open nbd U of x_0 in X such that $U \subset F^*(SClV)$,

(c) f.l.w.s.c. (f.u.w.s.c.) on X iff it is respectivety so at each $x_0 \in X$,

(d) f.w.s.c. on X iff it is f.l.w.s.c. and f.u.w.s.c. .

Remark 2.2. It is clear from Difinition 2.1 and 1.4 that every fuzzy semi-continuous (weakly semi-continuous, clmost semi-continuous [7]) multifunctions is f.w.s.c. where as the converse is not true.

Theorem 2.3. Let $F:X \to Y$ is f.l.w.s.c.(f.u.w.s.c.) iff for every fuzzy open set V in Y, $F_*(V) \subset Int F_*(SClV)$ (respectively, $F^*(V) \subset Int F^*(SClV)$).

Proof. we prove the theorem for the case of f.l.w.s.c. of F only; there other case is quite similar.

Let F be f.l.w.s.c. and V be f.open in Y .If $x \in F_*(V)$, then there exists semi-open nbd U of x in X such that $U \subset F_*(SClV)$ and hence $U \subset Int\, F_*(SClV)$.

Conversely , let $x \in X$ and V be fuzzy open set in Y such that $x \in F_*(V)$.Then $x \in F_*(V) \subset Int\, F_*(SClV) = U$ (say). Thus U is a semi-open nbd of x such that $U \subset F_*(SClV)$ and consequently, F is f.l.w.s.c. .

Theorem 2.4. If $F{:}X \to Y$ is f.l.w.s.c., for every fuzzy pre-semiopen set V in Y ,$F_*(V) \subset Int\, F_*(SClV)$.

Proof. Let F be f.l.w.s.c. and V be any fuzzy pre-semiopen set in Y .For any $x \in F_*(V)$, then there exists semi-open nbd U_x of x in X such that $U_x \subset F_*(SClIntSCl\, V) \subset F_*(SClV)$ and hence $F_*(V) \subset \cup \{U_x : x \in F_*(V)\} \subset F_*(SClV)$, and hence $F^*(V) \subset Int\, F^*(SClV)$.

Theorem 2.5. If $F{:}X \to Y$ is f.u.w.s.c., for every fuzzy pre-semiopen set V in Y, $F^*(V) \subset Int\, F^*(SClV)$.

Proof. Similar to Theorem 2.4. and omitted .

Theorem 2.6. If a fuzzy multifunction $F{:}X \to Y$ is f.l.w.s.c. on X, then $SCl\, F_*(V) \subset F_*(SClV)$ for any fuzzy open set V in Y.

Proof. Let $x \notin F_*(SClV)$, where V is a fuzzy open set in Y. Then $F(x)\; \bar{q}\; SCl\, V$ so $F(x) \leq 1 - SCl\, V$ .By fuzzy lower weakly semi-continuity of F ,there exists a semi-open nbd U of x in X such that $F(z) \leq SCl(1 - SCl\, V) \leq 1 - V$ for all $z \in U$ .Thus $F(z)\; \bar{q} V$,i.e $z \notin F_*(V)$, for all $z \in U$, hence $U = \cap F_*(V) = \varnothing$. Since U is a semi-open nbd of x, it then follows that $x \notin F_*(SClV)$.

Theorem 2.7. If a fuzzy multifunction $F{:}X \to Y$ is f.u.w.s.c. on X, then $SCl\, F^*(V) \subset F^*(SClV)$ for any fuzzy open set V in Y.

Proof. Similar to Theorem 2.6. and omitted .

Theorem 2.8. Let $F:X \to Y$ is fuzzy multifunctions then following are equivalent:
    (a) F is f.l.w.s.c.,
    (b) for any fuzzy open set V in Y, $F_*(V) \subseteq Int\, F_*(SCIV)$ and $F^*(V) \subseteq F^*(SCIV)$.

Proof. Evident.

Theorem 2.9. Let $F:X \to Y$ is fuzzy lower weakly semi-continuity if for each fuzzy open set V, $F_*(CIV)$ is fuzzy semi-open set.

Proof. Straightforward.

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