

SOME RELATION FOR POSSIBILITY INTERVAL , TRUTH INTERVAL AND CERTAINTY INTERVAL

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Based on [1],we discuss relation for possibility interval ,truth interval and certainty interval.

1. Introduction

The implementation of an expert system shell based on fuzzy logic virtually requires that certainty levels be assigned to complex logical statements, each element of which is a (fuzzy) logical assertion. This leads to interpreting a certainty level in the Sugeno sense [3], i.e. as a measure of belief that an assertion is true in the context of the current contents of the system's working memory. Interpreting certainty levels in this way, one can bring to bear the armamentarium of fuzzy set theory on the problem of constructing a fuzzy expert system shell.

Basing themselves on the fundamental work of Zadeh[4] on possibility theory Martin Clouaire and Prade [2] give an excellent discussion of fuzzy logic in which the certainty level is two-valued: the lower limit is the *necessary* of the datum, or the extent to which the data support the truth of the item; the upper limit is the *possibility* of a datum, or the extent to which the data do not refute the datum. Ruspini[5] uses

slightly different but isomorphic terms, plausibility instead of possibility, and support instead of necessity. Ruspini[5] also is concerned with the relationship between fuzzy logical operators (AND/OR/NOT) and prior associations between the logical operands, and the use of interval-valued logic in an expert system shell. D G Swartz[6] presented possibility interval sets and gave its application. Based on [1], we define certainty interval, and continue develop some relation between these measure.

2. possibility interval, truth interval and certainty interval

Assume V is a variable which can take values in the base set X . Let A and B be subsets of X . We define the truth of the statement

$$V \text{ is } A \text{ given } V \text{ is } B$$

$$\text{Truthi}[V=A|V=B]=\{B(x)/A(x)\} \text{ as}$$

Note. For notational convenience we shall denote $\text{Truthi}[V=A|V=B]$ as $\text{Truthi}_{A|B}$

Note. $T_{A|B}$ is fuzzy subset of the unit interval such that for any $i \in P([0, 1])$

$$\text{Truthi}(i) = \max_{x \in X: A(x)=I} [B(x)]^2$$

Note. $T_{A|B}$ is normal, has maximal interval membership grade of one, iff $B(x)$ is normal.

Example. For $X=\{a,b,c,d,e\}$ and

$$A=\{ 1/a, [0.6,0.8]/b, [0.6,0.8]/c, [0.2,0.3]/d,0/e\},$$

$$B=\{ 1/a, [0.4,0.5]/b, [0.7,0.8]/c,0/d,0/e\}$$

we have

$$T_{A|B}=\{ 1/1, [0.6,0.8]/[0.4,0.5], [0.6,0.8]/[0.7,0.8], [0.2,0.3]/[0.7,0.8], 0.2,0.3/0,0/1\}$$

In [7] Zadeh introduces the possibility interval that $V=A$ given $V=B$

as

$$\text{Possi}[V=A|V=B] = \max_{s \in X} [A(s) \wedge B(s)] \quad \wedge = \text{mim}$$

For notational convenience we shall denote $\text{Possi}[V=A|V=B]$ as $\Pi_{A|B}$

Note. $\Pi_{A|B}$ is always an element in the unit interval, $\Pi_{A|B} \in P([0, 1])$.

Example. In our preceding example,

$$\begin{aligned} \Pi_{A|B} &= \max\{1 \wedge 1, [0.6, 0.8] \wedge [0.4, 0.5], [0.6, 0.8] \wedge [0.7, 0.8], [0.2, 0.3] \wedge [0.7, 0.8], [0.2, \\ &0.3] \wedge 0, 0 \wedge 1\} \\ &= [1, 1] = 1 \end{aligned}$$

Note. $\Pi_{A|B} = 1$ iff there exists some element $x^* \in X$ such that $A(x^*) = B(x^*) = 1$

Note $\Pi_{A|B} = \Pi_{B|A}$

We shall now prove an important relationship between $\Pi_{A|B}$ and $T_{A|B}$

Lemma 1. $\Pi_{A|B} = \max_{x \in X} [A(x) \wedge B(x)]^{(1)}$

Let $i^* \in I$ be such that there exists some x such that $A(x) \in I^*$; then

$$\Pi_{A|B} = \max_{x \in X} [i \wedge T_{A|B}(i)]$$

If $i \in I - I^*$ then $T_{A|B} = 0$, hence

$$\Pi_{A|B} = \max_{i \in I} [i \wedge T_{A|B}(i)]$$

observation. $\Pi_{A|B}$ thus can be seen as the expected value of the possibility interval distribution for $T_{A|B}$ in the sense of using the Sugeno integral for the expected value [3]. Thus $\Pi_{A|B}$ is the mean value, in the fuzzy integral sense, of the truth of $V=A$ given $V=B$.

If T is a fuzzy subset of $P([0,1])$ and if we use $E_f[T]$ to indicate the fuzzy expected value in the sense of Sugeno, $E_f[T] = \max_{i \in I} [i \wedge T(i)]$, thus

$$\Pi_{A|B} = E_t[T]$$

Lemma 2. $\max_{i \in I} [i \wedge T_{A|B}(i)] = \max_{i \in I} [i \wedge T_{B|A}(i)]^{[1]}$

Assume again A and B are fuzzy subsets of $P([0,1])$ Then

$\text{Certi}[V=A | V=B] = 1 - \text{Possi}[V=A | V=B]$ where A is the negation of A^* ,
 $A^*(x) = 1 - A(x)$. Thus

$$\text{Certi}[V=A | V=B] = 1 - \Pi_{A|B} = 1 - \max_{i \in I} [i \wedge T_{A|B}(i)]$$

Before proceeding we recall that if T is a fuzzy interval truth value then the antonym of T^\wedge is defined as

$$T^\wedge(i) = T(1-i) \text{ for all } i \in I$$

Lemma 3. $T_{A^*|B} = T_{A|B}^\wedge$ the antonym of $T_{A|B}$.

Proof. Since

$$T_{A^*|B} = \{B(x)/(1-A(x))\}$$

it follows that

$$T_{A^*|B}(i) = \max_{x \in X: A(x)=1-i} [B(x)]$$

Hence

$$T_{A^*|B}(1-j) = \max_{x \in X: A(x)=j} [B(x)] = T_{A|B}(j)$$

Thus $T_{A^*|B}(i) = T_{A|B}(1-i)$.

Theorem 1. $\text{Certi}[V=A | V=B] = 1 - \max_{i \in I} [i \wedge T^\wedge(i)] = 1 - E_t[T^\wedge]$.

proof According to certainty interval and Lemma 2, theorem 1 is proved.

Theorem 2. If B is normal, then $\text{Certi}[V=A | V=B] = 1$ iff

$$T_{A|B} = 1.$$

Proof. (1) If $\text{Certi}(V=A | V=B) = 1$ then $\Pi_{A^*|B} = 0$, thus

$$\Pi_{A^*|B} = 0 = \max_{x \in X} [(1-A(x)) \wedge B(x)]$$

Hence for all x such that $A(x) \neq 1$, $B(x) = 0$. Since

$$T_{A|B} = \{A(x)/B(x)\}$$

for each $i \neq 1$ $T_{A|B}(i) = 0$. When $B(x) = 1$, $1-A(x) = 0$, thus $A(x) = 1$.

(2) If

$$T_{A|B} = 1.$$

then

$$\text{Certi}[V=A | V=B] = 1 - \max_{i \in I} [i \wedge T_{A|B}(1-i)]$$

but $T_{A|B}(1-i)=0$ for $i=0$. else it is zero.

3. Conclusion

We have investigated some relationships between possibility interval and truth interval. Most Significantly we have found that the possibility measure of A given B is the expected value of the truth of A given B , while the certainty A given B is the ones-complement of the antonym of the truth A given B

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