FUZZY LESS WEAKLY URYSOHN SPACES

Xiao Jia-Hong

Dept .of Mathematics,Ling Ling Teachers' College,

Yongzhou Hunan 425000,China

Abstract: In this paper, the concept of fuzzy less weakly Urysohn space is introduced. Its properties are systematically discussed

Key words: Fuzzy topological space, Less weakly Urysohn space, Semiopen set, Romote-neighborhood.

1.Introduction and preliminaries

In [6],Chen introduced the fuzzy Urysohn space in fuzzy topological space. And we introduced and studied the fuzzy less weakly Urysohn space in fuzzy topological space in [11]. In this paper, we introduce and study the fuzzy less weakly Urysohn space which is the weaker form of fuzzy weakly Urysohn space.

In this paper, (X,δ) will donote a Fuzzy topological space. A^0, A^- and A' will denote respectively the interior closure and complement of the fuzzy set A. Fuzzy set A is called fuzzy semiopen iff there is a $B\in\delta$ such that $B\le A\le B^-$ [1]. Fuzzy set A is called fuzzy strongly semiopen iff there in a $B\in\delta$ such that $B\le A\le B^{-0}$ [2]. Fuzzy set A is called semiclosed iff A' is semiopen. Fuzzy set A is called strongly semiclosed iff A' is strongly semiopen.

$A_0 = \bigcup\{B: B \le A, B$ fuzzy semiopen $\}$

$A_- = \bigcap\{B: B \ge A, B$ fuzzy semiclosed $\}$

$A^\Delta = \bigcup\{B: B \le A, B$ fuzzy strongly semiopen $\}$

$A^- = \bigcap\{B: B \ge A, B$ fuzzy strongly semiclosed $\}$

are called the semiinterior, semiclosure, strongly semiinterior and strongly semiclosure of A [1,2], respectively.

Definition 1.1[9] Let (X,δ) be a fuzzy topological space, e be a fuzzy point, $P \in \delta'$ and $e \notin P$.Then P is called a remote-neighborhood of e, and the set of all remote-neighborhood of e will be denoted by $\eta(e)$.

Definition 1.2[9, 6, 11] Let (X,δ) be a fuzzy topological space, (X,δ) is called a fuzzy Hausdorff (Urysohn, weakly Urysohn) space if for every pair of fuzzy points x_α and y_λ with $x \ne y$ there are $P \in \eta(x_\alpha)$ and $Q \in \eta(y_\lambda)$ such that $P \bigcup Q = 1$ ($P^0 \bigcup Q^0 = 1$, $P^\Delta \bigcup Q^\Delta = 1$).

2.Fuzzy less weakly Urysohn space

Definition 2.1 Let (X,δ) be a fuzzy topological space, If for every pair of fuzzy points x_α and y_λ with $x \ne y$ there are $P \in \eta(x_\alpha)$ and $Q \in \eta(y_\lambda)$ such that $P_0 \bigcup Q_0 = 1$.Then (X,δ) is called a fuzzy less weakly Urysohn space.

Obviously the following statements are valid:

fuzzy Urysohn space

$\Rightarrow$ fuzzy weakly Urysohn space

$\Rightarrow$ fuzzy less weakly Urysohn space

$\Rightarrow$ fuzzy Hausdorff space .

Theorem 2.2 Let (X,δ) be a fuzzy topological space, Then (X,δ) is a fuzzy less weakly Urysohn space iff for every pair of fuzzy points x_α and y_λ with $x \neq y$ and $\alpha,\lambda \in [0,1)$ there are fuzzy open neighborhoods V and W about x_α and y_λ, respectively, such that $V_- \cap W_- = 0$.

Proof. Necessity . Let (X,δ) be a fuzzy less weakly Urysohn space , x_α and y_λ be two fuzzy points in X with $x \neq y$ and $\alpha,\lambda \in [0,1)$.Choose two real numbers s and t satisfying $0 < s < 1-\alpha$ and $0 < t < 1-\lambda$.By Definition 2.1 there are $P \in \eta(x_s)$ and $Q \in \eta(y_t)$ such that $P_0 \cup Q_0 = 1$.Then P' and Q' are fuzzy open neighborhoods about x_α and y_λ, respectively, and

$$(P')_- \cap (Q')_- = (P_0)' \cap (Q_0)' = (P_0 \cup Q_0)' = 1' = 0 \ .$$

Sufficiency . Let the given condition hold.Suppose x_α and y_λ are two fuzzy points with $x \neq y$.Choose two real numbers s and t satisfying $1-\alpha < s < 1$ and $1-\lambda < t < 1$. In the light of the assumption , there are fuzzy open neighborhoods V and W about x_s and y_t, respectively, such that $V_- \cap W_- = 0$.

Then $x_\alpha \not\leq V'$ and $y_\lambda \not\leq W'$, i.e., $V' \in \eta(x_\alpha)$ and $W' \in \eta(y_\lambda)$, and

$$(V')_0 \cup (W')_0 = (V_-)' \cup (W_-)' = (V_- \cap W_-)' = 0' = 1.$$

Thus (X,δ) is a fuzzy less weakly Urysohn space.

Definition 2.3 Let (X,δ) be a fuzzy topological space, (X,δ) is called a semi-interior additive if $(A \cup B)_0 = A_0 \cup B_0$ for any two fuzzy sets A and B in X.

Theorem 2.4 Let (X,δ) be a fuzzy topological space, If (X,δ) is Hausdorff and semi-interior additive, then (X,δ) is a fuzzy less weakly Urysohn space.

Proof. This is immediate from Definition 2.2 and 2.3 .

Definition 2.5 Let x be a fuzzy point and $S = \{s(n), n \in D\}$ a fuzzy net [9] in (X,δ).Then x is called a Δ-limit point of S (or S Δ-converges to x) if for each $P \in \eta(x)$ we have eventually $s(n) \leq P_0$.

Theorem 2.6 (X,δ) is a fuzzy less weakly Urysohn space iff no fuzzy net in X can Δ-converges to two fuzzy point x_α and y_λ with $x \neq y$.

Proof. Necessity. Let $S = \{s(n), n \in D\}$ be a fuzzy net in X which

Δ-converges to a fuzzy point x_α, and y_λ be another fuzzy point with $x \neq y$.Because X is a fuzzy less weakly Urysohn space , there are $P \in \eta(x_\alpha)$ and $Q \in \eta(y_\lambda)$ such that $P_0 \cup Q_0 = 1$. Since eventually $s(n) \leq P_0$, therefore eventually $s(n) \leq Q_0$.Hence S dose not Δ-converge to y_λ .

Sufficiency. Assume that the condition is true and x_α and y_λ are two fuzzy points with $x \neq y$.If for every $P \in \eta(x_\alpha)$ and $Q \in \eta(y_\lambda)$, $P_0 \cup Q_0 = 1$, then there exists a fuzzy point $S(P,Q) \notin P_0 \cup Q_0$.Take

$$S = \{ S(P,Q) : \ (P,Q) \in \eta(x_\alpha) \times \eta(y_\lambda) \}$$

Then S is a net in X with the following relation:

$(P_1,Q_1) \leq (P_2,Q_2)$ iff $P_1 \subset P_2$ and $Q_1 \subset Q_2$

where (P_1,Q_1), $(P_2,Q_2) \in \eta(x_\alpha) \times \eta(y_\lambda)$

obviously, eventually $s(n) \nleq P_0$, so S Δ-converges to x_α.Similarly, S Δ-converges to y_λ as well .This contradicts the hypothesis.Conseguently, there are $P \in \eta(x_\alpha)$ and $Q \in \eta(y_\lambda)$ such that $P_0 \cup Q_0 = 1$.

Thus (X,δ) is a fuzzy less weakly Urysohn space.

Theorem 2.7 Let (X,δ) be a fuzzy less weakly Urysohn space and (Y,τ) a fuzzy topological space. If $f: (X,\delta) \to (Y,\tau)$ is a fuzzy homeomorphic mapping, then (Y,τ) is also a fuzzy less weakly Urysohn space .

Proof . Let y_α and y_λ^* be two fuzzy points in (Y,τ) with $y \neq y_\lambda^*$ .Then there are two fuzzy points x_α and x_λ^* in (X,δ) with $x \neq x^*$ such that

$$f(x_\alpha) = y_\alpha \quad \text{and} \quad f(x_\lambda^*) = y_\lambda^* \quad .$$

Since (X,δ) is a fuzzy less weakly Urysohn space , there are $P \in \eta(x_\alpha)$ and $Q \in \eta(x_\lambda^*)$ such that $P_0 \cup Q_0 = 1_x$,

Because f is a fuzzy homeomorphic mapping, we have

$$f(P) \in \eta(f(x_\alpha)) = \eta(y_\alpha)$$

and

$$f(Q) \in \eta(f(x_\lambda^*)) = \eta(y_\lambda^*) .$$

Again , Since fuzzy semiopen set in preserved under fuzzy homeomorphic mapping , we have

$$(f(P))_0 \cup (f(Q))_0 \geq (f(P_0))_0 \cup (f(Q_0))_0$$

$$= f(P_0) \cup f(Q_0) = f(P_0 \cup Q_0) = f(1_x) = 1_y$$

Thus (Y,τ) is a fuzzy less weakly Urysohn space.

References

[1]K.K. Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weak continuity, J.Math.Anal. Appl. 82(1981)14-32 .

[2]Bai Shi-Zhong, Fuzzy strongly semiopen sets and fuzzy strong

semicontinuity,Fuzzy sets and Systems 52 (1992) 345-356 .

[3] Bai Shi-Zhong, Fuzzy S-irresolute mappings, J. Fuzzy Math. 4 (1996) 2：397-411 .

[4] Bai Shi-Zhong, Q-Convergence of nets and weak separation axioms in fuzzy lattices, Fuzzy sets and Systems, 88 (1997) 3：379-386 .

[5] C.L.Chang, Fuzzy topological space, J-Math . Appl. 24 (1968) 182-190 .

[6] S.L.Chen, Fuzzy Urgsohn spaces and α-stratified fuzzy Urgsohn spaces, Fifth IFSA world congress (1993) 453-456 .

[7] J.L.Kelley, General Topology (princeton University press, princeton, NY 1955) .

[8] B.M.Pu and Y.M Liu, Fuzzy topology I, J. Math. Anal. Appl> 76 (1980) 571-599 .

[9] G.J.Wang , A new fuzzy compactress defined by fuzzy nets, J . Math . Anal. Appl. 94(1983) 1-23 .

[10] G.J.Wang , Theory of L-fuzzy topological spaces, pross of Shaanxi Normal University, Xian, 1988 .

[11] Xiao Jia –Hong, Fuzzy weakly Urysohn spaces, BUSEFAL 3 (1998) .

[12] L.A. Zadeh, Fuzzy sets, Inform and Control 8 (1965) 338-353 .