SOME RELATION FOR POSSIBILIY INTERVAL AND TRUTH INTERVAL

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We define possibility interval and truth interval on fuzzy interval set, and discuss some relationships between the fuzzy subset concepts of possibility and truth . We show, for A and *B* fuzzy subsets of P([0,1]), that the possibility interval of A given B is related to the truth interval of A given *B* through a sugeno-like integral.

Keywords: Fuzzy interval sets, Measures of uncertainty interval , Possibility interval theory.

1.Introduction

The implementation of an expert system shell based on furry logic [1] virtually requires that certainty levels be assigned to complex logical statements, each element of which is a (fuzzy) logical assertion. This leads to interpreting a certainty level in the Sugeno sense [2], i.e. as a measure of belief that an assertion is true in the context of the current contents of the system's working memory. Interpreting certainty levels in this way, one can bring to bear the armamentarium of fuzzy set theory on the problem of constructing a fuzzy expert system shell.

Basing themselves on the fundamental work of Zadeh[3] on possibility theory, Martin-Clouaire and Prade [4] give an excellent discussion of fuzzy logic in which the certainty level is two-valued: the lower limit is the *necessity* of the datum, or the extent to which the data support the truth of the item; the upper limit is the *possibility* of a datum, or the extent to which the data do not refute the datum. Ruspini [5] uses slightly different but isomorphic terms, *plausibility* instead of possibility, and *support* instead of necessity. Ruspini also is concerned with the relationship between fuzzy logical operators (AND/OR/NOT) and prior associations between the logical operands, and the use of interval-valued logic[7][8] in an expert system shell. A.P Demp[6] presented possibility interval sets and gave its application. In this paper we will define possibility interval and truth interval ,and develop some relationships between these measures. In participation we show that the possibility interval of *A* given *B* is the same as the expected value of the truth interval of *A* given *B*.

2. possibility interval and truth interval

Assume *V* is a variable which can take values in the base set X. Let *A* and *B* be two fuzzy subsets of X. We defines the truth of the statement

V is A given V is B

as Truthi[V=A|V=B]={B(x)/A(x)}

Note. For notational convenience we shall denote

Truthi[V= A| V= B] as $\text{Truthi}_{A|B}$

Note. $T_{A|B}$ is fuzzy subset of the unit interval such that for any i∈P([0, 1])

$$\text{Truthi}(i)=\max_{x\in X:\ A(x)=1}[B(x)]^2$$

Note. $T_{A|B}$ is normal, has maximal interval membership grade of one, iff *B(x)* is normal.

Example. For X={a,b,c,d,e} and

A={1/a,[0.6,0.8]/b,[0.6,0.8]/c,[0.2,0.3]/d,0/e},B={1/a,[0.4,0.5]/b,[0.7,0.8]/c,0/d,1/e}

we have

$T_{B|A}$={1/1,[0.6,0.8]/[0.4,0.5],[0.6,0.8]/[0.7,0.8],[0.2,0.3]/[0.7,0.8],[0.2,0.3]/0,0/1}

In [9] J. Buckley and W. Siler introduces the possibility interval that

V=A given *V=B*

as

$$\text{Possi}[V=A|\ V=B]=\max_{x\in X}[A(x)\wedge B(x)]\qquad \wedge=\min$$

For notational convenience we shall denote Possi[V=A|V=B] as $\Pi_{A|B}$

Note. $\Pi_{A|B}$ is always an element in the unit interval, $\Pi_{A|B} \in P([0, 1])$.

Example. In our preceding example,

$\Pi_{A|B}$=max[1∧1,[0.6,0.8]∧[0.4,0.5],[0.6,0.8]∧[0.7,0.8],[0.2,0.3]∧[0.7,0.8],[0.2,0.3]∧0,0∧1]
=[1, 1]=1

Note. $\Pi_{A|B}=1$ iff there exists some element $x^* \in X$ such that $A(x^*) = B(x^*) = 1$.

Note. $\Pi_{A|B} = \Pi_{B|A}$;

We shall now prove an important relationship between $\Pi_{A|B}$ and $T_{A|B}$.

Theorem. $\Pi_{A|B} = max_{i \in P([0,1])}[i \wedge T_{A|B}(i)]$
$x \in X$: $A(x)=1$

Proof. $\Pi_{A|B} = max_{x \in X}[A(x) \wedge B(x)]$;let $i_x=A(x)$ then

$$\Pi_{A|B} = max_{x \in X}[i_x \wedge B(x)]$$

For each $i \in I$

Truthi(i)= max $[B(x)]^2$

Thus

$$\Pi_{A|B} = max_{x \in X}[i_x \wedge T_{A|B}(i_x)]$$

Let $I^* \subset I$ be such that there exists some x such that $A(x) \in I^*$; then

$$\Pi_{A|B} = max_{i \in I^*}[i \wedge T_{A|B}(i)]$$

If $i \in I - I^*$ then $T_{A|B}=0$, hence

$$\Pi_{A|B} = max_{i = I}[i \wedge T_{A|B}(i)].$$

Observation. $\Pi_{A|B}$ thus can be seen as the expected value of the possibility interval distribution for $T_{A|B}$ in the sense of using the Sugeno integral for the expected value [7]. Thus $\Pi_{A|B}$ is the mean value, in the fuzzy integral sense, of the truth of $V=A$ given $V=B$.

If T is a fuzzy subset of P([0,1]) and if we use $E_f[T]$ to indicate the fuzzy expected value in the sense of Sugeno, $E_f[T] = max_{i \in I}[i \wedge T(i)]$, thus

$$\Pi_{A|B} = E_f[T]$$

Theorem. $max_{i \in I}[i \wedge T_{A|B}(i)] = max_{i \in I}[i \wedge T_{B|A}(i)]$

Proof. $\Pi_{A|B} = max_{i \in I}[i \wedge T_{A|B}(i)]$ and $\Pi_{B|A} = max_{i \in I}[i \wedge T_{B|A}(i)]$.

so the result follows since $\Pi_{A|B} = \Pi_{B|A}$

Thus

$$E_f[T_{A|B}] = E_f[T_{B|A}]$$

Note. $\Pi_{A|B}=1$ iff $T_{A|B}=1$

3. Conclusion

We have investigated some relationships between possibility interval and truth interval. Most significantly we have found that the possibility measure of *A* given *B* is the expected value of the truth of *A* given *B*.

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