INTRODUCTION TO α-subset
A WORKING METHOD ON SOFT SET

MICHELE PAPPALARDO

D I M E C -DEP. OF MECHANICAL ENGINEERING
UNIVERSITY OF SALERNO
VIA P. DON MELILLO 84084 FISCIANO (SA) ITALY
pappalar@bridge.diima.unisa,it

ABSTRACT

In this paper is studied the mathematical representation of information system in presence of uncertainty. Using Gentilhomme's definitions of the soft sets, with three values of truth, is proposed a new representation in which third the value of truth is the mean of membership functions of soft part in set. The new representation is easy and useful in classification, in design and measure of ambiguity in the set. The expressions of statistic and of the entropy are used for measuring uncertainty.

Key words: soft set, fuzzy set, entropy, classification, soft design

1-INTRODUCTION

The classification is the central problem dealing with uncertainty or vague knowledge in *AI* applications. The goal is to find information on the basis of propositions. The elements, screened with the same propositions, represent a set from which getting off numerical information or algebraic structure. From a universe Ω *with* uncertainty information, the set can be partitioned in crisp or/and soft subsets. In *Russell's* definition of sets, the universe U is partitioned on basis of *proposition functions P(x).* The propositions are used, in recursive structure in hierarchical organisation, as single or as collection of singleton functions.

$$x \underset{k}{\in} \left\{ \underline{E},\ \underset{false}{X} \right\} \ iff\ \mu(x) = K,\quad K = \begin{cases} 1 \ iff\ P(x)\ is\ true \\ 0 \ iff\ P(x)\ is\ false \end{cases}$$

In a finite nonempty set $U = \{x_i\}$, indicating with $(P(x), K)$ the *proposition functions*, and its criterion of verification $K \in \{0,1\}$, the subset $C \subseteq U$, can be represented as

$$C = \{\mu(x), x\} \quad \forall x \in U \quad , \quad U \overset{c}{\rightarrow} K$$

Where $\mu(x)$ is the *membership function.* If K can assume two values $K = \{0,1\}$, C is a *crisp set*, if K can assume infinite values between 0 and 1, $K \in [0, 1]$, C is a *fuzzy set* (Zadeh 1965)

$$x \underset{k}{\in} \tilde{C} \ iff\ \mu(x) = K, \ K \in [0, 1]$$

If K can assumes <u>*three values*</u> $K \in \left\{0, \frac{1}{2}, 1\right\}$ C is a *flou set* (Gentilhomme 1968). In Gentilhomme's original definition of soft sets, using propositions $P(x)$ with semantic ambiguity, a set U is partitioned in two parts $\left(\underline{E}, \underset{+}{E}\right)$, where $\underline{E}$ is the *certain* part, $\underset{+}{E}$ is the *possible (*equal to *certain plus flou)* part and $\left(\underset{+}{E} - \underline{E}\right)$ is the *flou* (lack certain) part.

The crisp subset is represented

$$x \in_k \left\{ \underline{E}, \underset{false}{X} \right\} \ \forall x U, \ \mu(x) = K, \ K = \begin{cases} 1 \ iff \ P(x) \ is \ true \\ 0 \ iff \ P(x) \ is \ false \end{cases}$$

the soft subset is represented

$$x \in_k \left\{ \underset{+}{E}, \underset{false}{X} \right\} \ \forall x U, \ \mu(x) = K, \ K = \begin{cases} 1 \ iff \ \ the\ truth\ of \ P(x) \ is > 0 \ and \leq 1 \\ 0 \ iff \ P(x) \ is \ false \end{cases}$$

<u>*The algebraic semantic of flou sets of Gentilhomme is with three values of truth.*</u> In the fuzzy logic the propositions have infinite values of truth including the crisp and the soft sets. In *flou* sets the third values of truth $\left[\frac{1}{2}\right]$ is a conventional value that contains the max uncertainty and the max entropy. Operating in real systems, portion of truth is well-known and all the events <u>*are not equally likely*</u>, therefore it is opportune to <u>introduce a definition of a new soft set</u> with the third values of truth that corresponds to the real measure of uncertainty in it contained.

<u>*2-DEFINITION OF α-subsets*</u>

In presence of semantic ambiguity in proposition, P(x) a *finite* universe U can be partitioned using the logic of flou sets in crisp and soft subsets $\left(\underline{E}, \underset{+}{E}\right)$.

If $X \subseteq U$, given by function $P(x)$, and $x \in U$

- If $x \in \underline{E}$ means that x has a *certain* property $P(x)$
- If $x \in \underset{+}{E}$ means that x has a *possible* property $P(x)$
- If $x \in \underset{false}{E}$ means that x have not property $P(x)$

In the subset $\left(\underline{E}, \underset{+}{E}\right)$, $\underline{E}$ is the *certain* part, $\underset{+}{E}$ is the *possible (*equal to *certain* plus *flou) part* and $\left(\underset{+}{E} \ \underline{E}\right)$ is the *flou* (uncertain) part. The set U is

$$U = \underline{E} \cup \underset{+}{E} \cup \underset{false}{E} = \underset{+}{E} \cup \underset{false}{E} \qquad \underline{E} \ \underset{+}{E} = \underset{flou}{E} \qquad U = \underline{E} \cup \underset{flou}{E} \cup \underset{false}{E} \qquad \underline{E} \cap \underset{flou}{E} = empty$$

The cardinality n_c of subset $\underset{flou}{E}$ is

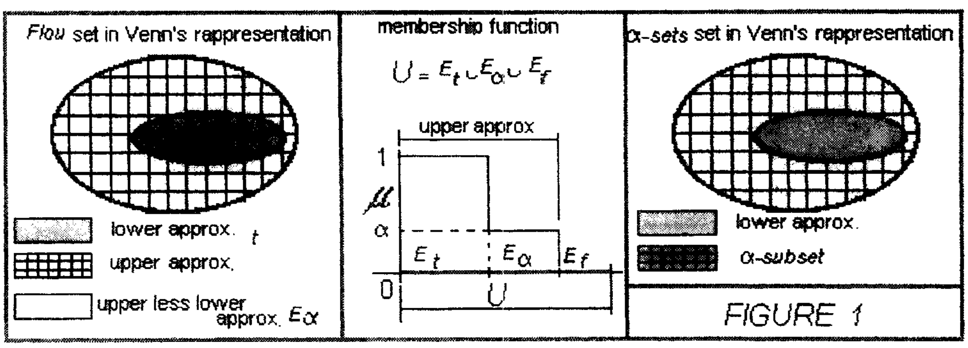


FIGURE 1

$$cardinality\left[\underset{flou}{E}\right] = \left(cardinality\left[\underset{+}{E}\right] - cardinality\left[\underline{E}\right]\right)$$

If the membership function is fuzzy, has infinite values of truth, than holds soft and crisp sets.

- For every $x \in \underset{false}{E}$ the membership fuzzy function is $\mu(x) = 0$
- For every $x \in \underline{E}$ the membership fuzzy function is $\mu(x) = 1$.
- For every $x \in \underset{flou}{E}$ has a membership function with values between zero and one $1 < \mu(x) > 0$.

If α is the *mean* of all membership functions of set $\underset{flou}{E}$

$$\alpha = \sum_{i=1}^{i=n_c} \mu(x_i) / n_c$$

it is possible to consider $\underset{flou}{E}$ as a pseudo crisp set with $\mu(x) = \alpha$

If $X \subseteq U$ given by function $P(x)$ and $x \in U$ it is possible to define the new soft subset $\underset{\alpha}{E}$ with membership function $\alpha(x) = \alpha$

$$x \in \underset{k\ \alpha}{X} \ \forall x U,\ \mu(x) = K,\ K = \begin{cases} 0 \text{ iff } P(x) \text{ is true} \\ \alpha \text{ iff the truth of } P(x) \text{ is} > 0 \text{ and} < 1 \\ 0 \text{ iff } P(x) \text{ is false} \end{cases}$$

in which α is the third conventional value of truth in the set.

I named $\underset{\alpha}{E}$ ***α-subsets*** and is a new soft set .

3- DEFINITION OF FINITE α-sets

Basic definitions. The basic idea of ***α-sets*** is to analyse, in a finite system S, one by one all the elements with the reference of a proposition functions $P(x)$, *using* the Gentilhomme's logic of *flou set* with three values of truth. The elements of a finite universe U are partitioned in three subset $U = \underline{E} \cup \underset{flou}{E} \cup \underset{false}{E}$. From the soft set $\underset{flou}{E}$ it is defined the pseudo crisp set *α-set* $\underset{\alpha}{E}$ containing all the uncertainty of the information system.

The consistence of an information system $S = (U, A, M, f_\mu)$ is defined by

- a finite nonempty set U of objects.
- a finite nonempty set A of attributes.
- M a finite nonempty set of values of membership functions $\mu(x)$

Where $f_\mu : U \times A \to M$ and for $a \in A$ and $x \in U$ exist $f(x,a) \in \mu(x)$

In the system S is partitioned in three subsets:

- if $\mu(x)=1$ than $x \in \underline{E}$, means that x has a *certain* property $P(x)$.
- if $\mu(x)= k$ $(0 < k \leq 1)$ than $x \in \underset{+}{E}$, means that x has a *possible* property $P(x)$.
- if $\mu(x)=0$ than $x \in \underset{false}{E}$ means that x have not property $P(x)$.

The flou set is $\underset{+}{E} \setminus \underline{E} = \underset{flou}{E}$

The cardinality n_c of subset $\underset{flou}{E}$ is: $n_c = \left(cardinality\left[\underset{+}{E}\right] - cardinality\left[\underline{E}\right] \right)$.

If α is the mean of all membership functions of $\underset{flou}{E}$ it is possible to define the flou set as pseudo crisp subset with $\mu(x) = \alpha$. The new pseudo crisp subset is $\underset{\alpha}{E}$ with membership function $\alpha(x) = \alpha$ containing all the uncertainty. If the proposition $P(x)$ *is composite* it is possible from the partition of U to obtain more than one *α-subset.*

It is possible to analyse a system $S = (\Omega, A, M, f_\mu,)$ *using flou logic* with three values of truth and α as third value of truth. The *α-subsets can be represented as*

$$x \in_k \left\{ \underline{E}, \underset{\alpha}{E}, \underset{false}{E} \right\} \ \forall x U,\ \mu(x) = K, \quad K = \begin{cases} 1 \text{ iff } P(x) \text{ is true} \\ \alpha \text{ iff the truth of } P(x) \text{ is} > 0 \text{ and} < 1 \\ 0 \text{ iff } P(x) \text{ is false} \end{cases}$$

The result of partition is

$$U = \underline{E} \cup \underset{flou}{E} \cup \underset{f}{E} \qquad \underline{E} \cap \underset{flou}{E} = empty$$

If is $X = \underline{E} \cup \underset{flou}{E}$ than the subsets *can be represented as*

$$x \in \left(\underset{k}{}\underset{\alpha}{E}\right) \; iff \; \mu(x) = K, \; \forall x X, \; K \in \{\alpha, 1\}.$$

Membership functions. Given a system $S = (\Omega, A, M, f_\mu,)$ let $C \subseteq \underset{+}{E}$, $D \subseteq \underline{E}$ and $\{C, D\} \subseteq \Omega$ the *flou membership function* $\mu(C, D) \in [0,1]$ can be defined on model of conditional probability

$$\mu_f(C,D) = \frac{|C \cap D|}{|C|} \qquad 0 \le \mu_f(C,D) \le 1$$

$$\mu_{\alpha-set} = \frac{\left|\underset{flou}{E}\right|}{\left|\underline{E} \cup \underset{flou}{E}\right|} \qquad \underline{E} \cap \underset{flou}{E} = empty$$

It is easy to extend to α-set all definitions of flou sets logic.

Approximation. . The *measure of approximation* of α-subsets, *in set X*, can be defined as

$$\beta = cardin.\left|\left(\underset{\alpha}{E}\right)\right| \Big/ cardin.\left|\left(\underline{E} \cup \underset{\alpha}{E}\right)\right|$$

If from proposition $P_1(x)$, the measure of approximation is β_1 and frm $P_2(x)$ the measure is β_2 and results $\beta_1 > \beta_2$ than proposition $P_1(x)$ is more accurate of $P_2(x)$.

4-α-sets AND STATISTIC.

Principle of Laplace-Bernuilli (LB). The LB principle, well known also as *principle of insufficient reason*, is: *In probable space , in absence of a priori reason, all events are equally likely.*

The LB principle is a *decision principle* because, in absence of a priori reason, *assign* to all the events equally the degree of belief.

On basis of LB principle can be assigned degrees of belief to a finite collections of events. In a probability finite space, Ω the probability measure must observe the Kolmogorov's axioms.

If in a probability space *(Ω,A,P)*

- Ω is a finite non empty set
- A is a *σ-algebra* of subset in Ω of events
- P is a probability measure on A

the applications of $P : A \to [0,1]$ must be under axioms

- $P(\Omega) = 1 \quad P(\phi) = \phi$
- $A_n \in A \;\; \forall n, \qquad A_m \cap A_n = \phi, \qquad m \ne n \Rightarrow P\left(\underset{n}{\cup} A_n\right) = \underset{n}{\sum} P(A_n)$

In a soft set *X*, on basis of LB principle, in absence of a priori reason, all events on elements have equal belief, then:

- *For all* $x \in X$ *of* $X \subseteq U$ *the probability is* $p(x) = \frac{1}{|X|}$.

The metric on set $U = \underline{E} \cup \underset{flou}{E} \cup \underset{false}{E}$ for $x \in U$ is:

- $P(U) = 1, \; p(x_i) = \frac{1}{|U|} \;\; P(\underline{E}) = \frac{|\underline{E}|}{|U|}, \; P\left(\underset{flou}{E}\right) = \frac{\left|\underset{flou}{E}\right|}{|U|} = P\left(\underline{E} \setminus \underset{+}{E}\right) \;, \; P\left(\underset{false}{E}\right) = \frac{\left|\underset{f}{E}\right|}{|U|}$

The metric on set $X = \underline{E} \cup \underset{flou}{E}$ for $x \in X$ is:

- $P(X)=1,\quad p(x_i) = \frac{1}{|X|},\quad P\left(\underset{flou}{E}\right) = \frac{\left|\underset{flou}{E}\right|}{|X|},\ P(\underline{E}) = \frac{|\underline{E}|}{|X|}$
- The degree of flou in X is $\beta(x) = P\left(\underset{flou}{E}\right)$ can be used as *quality of applications.*

5-ENTROPY

From Shannon's measure of information the entropy is

$$H = -\sum_i p_i \log p_i$$

Where $\{p_i\}$ are the statistic data on sets. In alpha-set the entropy is from statistic data of positions of elements in the sets and from the fuzziness of data in flou set.

1- *Entropy from metric* of elements in subset of U

$$H = -\left(P(\underline{E})\log P(\underline{E}) + P\left(\underset{flou}{E}\right)\log P\left(\underset{flou}{E}\right) + P\left(\underset{false}{E}\right)\log\left(\underset{false}{E}\right)\right)$$

2- *Entropy of fuzziness* of all the elements of the subset $\underset{flou}{E}$

$$H = -\sum_{1}^{n_c}\left(\mu(x_i)\log \mu(x_i) + (1-\mu(x_i))\log(1-\mu(x_i)) \right)$$

3- *Entropy as measure of degree* of approximation of $\underset{\alpha}{E}$

$$H = -(\alpha \log \alpha + (1-\alpha)\log(1-\alpha))$$

4-*Wiener's entropy* It is possible, using the same date, to calculate *entropy using Wiener's definition: "We may conceive this in following way: we know a priori that a variable lies between 0 and 1 and a posteriori that lies on interval [a,b] inside [0,1], then the amount of information we have from our a posteriori knowledgment is*

$$H = -\log_2(\text{measure of } [a,b]/\text{measure of } [0,1])$$

The measure of [a,b] =α and the measure of [0,1]=1

$$H = -\log_e \alpha$$

5-*Kulbak-Leibler's entropy.* If to $P_1(x)$ correspond values of statistics (α_1, β_1) and to $P_2(x)$ correspond (α_2, β_2) than the cross-entropy is $H_\alpha = \alpha_1 \log_2 \alpha_1/\alpha_2$ and $H_\beta = \beta_1 \log \beta_1/\beta_2$ and represent the distance $D(\alpha_1 : \alpha_2)$ and $D(\beta_1 : \beta_2)$.

6-APPLICATION IN DESIGN

The subset $\underset{\alpha}{E}$ contains all the uncertainty in terms of fuzzy membership functions. A finite fuzzy set $F = \{(\mu(x), x)\ \ \forall x \in A\}$ $\mu(x) \in [0,1]$ in different engineering discipline can to measure efficiency or quality in manufacturing process. The membership function $\mu(x) \in [0,1]$ represents quality. For an ideal process will be $\mu(x) = 1$. The value $\overline{\mu_i} = (\mu_i(x) - 1)$ represents quality loss. In the sum of square of quality-loss the term

$$D = \sqrt{\sum_i (\mu_i(x) - 1)^2}$$

represents a point on surface S_D of n dimensions hypersphere. On utilising the above statistical terms D is given by

$$D=\sqrt{n\left((\alpha-1)^2+\frac{n-1}{n}s^2\right)}$$

where α and s^2 are , respectively, the mean and the variance of $\mu(x)$ given by

$$\alpha=\frac{1}{n}\sum_i \mu(x) \qquad \sigma^2=\frac{1}{n-1}\sum_i(\mu(x)-1)^2$$

When n is large the equation can be written

$$D^2=n\left[(\alpha-1)^2+\sigma^2\right]$$

The quality lost is of two terms: $(\alpha-1)^2$ resulting from the deviation of the average value of $\mu(x)$ from the target $\mu_i(x)=1$, and σ^2 resulting from the mean squared deviation of $\mu(x)$ from its own mean. The value α of alpha-set E_α is the fundamental terms for measure the quality lost.

ENTROPY. The entropy is a generic value with very large means not connected to the probabilities. It is possible to define the idea of information for non-probabilistic events. It is possible to define entropy using the definitions $J(P(A))\overset{def}{=}-\log_a(P(A))$ in all situations. Using definition of the entropy on alpha-set is

$$H=-\log_2\left((\alpha-1)^2+\sigma^2\right)$$

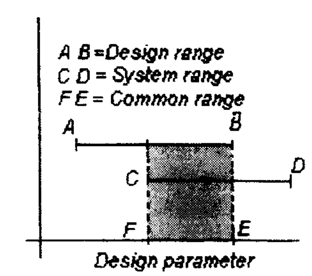
As example we analyse the relationship between designer's specification and a manufacturing system $S=(U,A,M,f_\mu)$ where

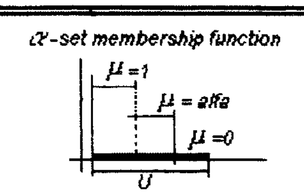
- U is a non empty set of machines *(system range)*
- A *is* a non empty set of designer's specification *(design range)*
- M *is* a non empty set of values of *membership functions* $\mu(x)$

For a set of machines range $T\subseteq U=\{x_i\}$ and a set design range $V\subseteq A=\{a_i\}$, where $f:T\times V\to M$ and for $x\in X$ and $a\in V$ than $p(x,a)\in\mu(x_i)$.

U	SYSTEM RANGE	DESIGN RANGE	$\mu(x_i)$
x_1	t_1	v_1	1
x_2	t_2	v_2	0.6
x_3	t_3	v_3	0.7
x_4	t_4	v_4	0
x_5	t_5	v_5	1
x_6	t_6	v_6	0.7
x_7	t_7	v_7	0.6
x_8	t_8	v_8	0.9
x_9	t_9	v_9	0
x_{10}	t_{10}	v_{10}	1

Figure 2





Indicating, as in figure 2, the common range and the system range the membership function is

$$\mu(x_i)=\frac{|common\ range|}{|system\ range|}=\frac{|\overline{FE}|\cap|\overline{CD}|}{|\overline{CD}|}.$$

- *If* $\mu(x_i)=1$ means that the manufacturing system achieve all designer's specification.
- *If* $\mu(x_i)=0$ means that the manufacturing system cannot achieve designer's specification.

- *If* $\mu(x_i) = z$ and 0>z<1 means that the manufacturing system has the possibility of achieve all designer's specification.

In flou set logic: $\underline{E} = \{x_1, x_5, x_{10}\}$; $\underset{+}{E} = \{x_1, x_2, x_3, x_5, x_6, x_7, x_8, x_{10}\}$; $\underset{false}{E} = \{x_4, x_9\}$

In α-set logic: $\underline{E} = \{x_1, x_5, x_{10}\}$; $\underset{\alpha}{E} = \{x_2, x_3, x_6, x_7, x_8\}$; $\underset{false}{E} = \{x_4, x_9\}; \alpha = 0.7$

The set $\underset{\alpha}{E} = \{x_2, x_3, x_6, x_7, x_8\}$ contains all uncertainty of the set *U*. The measure of uncertainty is $\alpha = 0.7$.

Utilising Wiener's expressions, in Jaynes' MaxEnt principle, it is possible to maximise entropy, in optimisation of processes, using the alpha-set's value of α.

7-APPLICATION IN CLASSIFICATION

In classification the result of a partition depends from semantic ambiguity of $P(x) =$ "*x has property....*". A measure of ambiguity is obtained from statistical value $\beta(x) = P\left(\underset{flou}{E}\right)$ or from the values of $\alpha = \frac{\sum_{i=1}^{i=n_c} \mu_i(x)}{n_c}$ mean of membership functions. In presence of uncertainty, the sense of propositions $P(x_i)$ can be vague, and the evaluation of objects results in ambiguity and often is not unique. Objects with the same attributes can be classified in different class. The ambiguity can be in

- subjects evaluating
- in objects evaluating
- in subjects and objects evaluating.

Using *soft set* it is possible to make a model for identification the evaluation's structure. In a finite set *X* with the function **g**, that makes the subsets *E* and *F* with $U \cap F = \varnothing$ $g(E) \in [0,1]$ and $g(F) \in [0,1]$, the measures are called fuzzy.

Table 1

Solution x_i	$\mu(cost)$	$\mu(affidability)$	$\mu(safety)$	*Average of* $\mu(a_i)$	*Decision* $\mu(decision)$
1	0.50	1.00	0.50	0.66	$\mu = 1 \rightarrow Yes$
2	0.50	0.50	0.25	0.41	$\mu = 0 \rightarrow No$
3	0.50	0.75	0.75	0.66	$\mu = 1 \rightarrow Yes$
4	1.00	0.25	0.75	0.66	$\mu = 0 \rightarrow No$
5	0.75	0.25	1.00	0.66	$\mu = 0 \rightarrow No$
6	1.00	0.50	0.75	0.75	$\mu = 1 \rightarrow Yes$
7	0.25	0.50	0.50	0.41	$\mu = 0 \rightarrow No$
8	0.25	0.50	0.50	0.41	$\mu = 1 \rightarrow Yes$
9	1.00	1.00	0.75	0.83	$\mu = 1 \rightarrow Yes$
10	0.50	1.00	0.50	0.66	$\mu = 1 \rightarrow Yes$

The property $g(\varnothing) = 0$, $g(X) = 1$ and $E \subset F$ $g(E) \leq g(F)$ are from monotonic of function **g**. If the measures are additive we have $\mu(E \cup F) = \mu(E) + \mu(F)$. In fuzzy measure we can have superior additivity or sub additivity. In general the fuzzy measure can be considered

$$g_\lambda(E \cup F) = g_\lambda(E) + g_\lambda(F) + \lambda g_\lambda(E) g_\lambda(F) \qquad -1 < \lambda < \infty$$

- *if* $\lambda > 0$ *then* $\rightarrow$ *superior additivity*

- *if* $\lambda = 0$ *then* $\rightarrow$ *additivity*
- *if* $\lambda < 0$ *then* $\rightarrow$ *sub additivity*

Considering the example in table 1. The data of a plant are reported in table and in presence of uncertain results with the evaluation structure must be identified.
Let be the set of attribute $A = \{cost(a_1), affidability(a_2), safety(a_3)\}$ and the domain of attribute a_i in membership functions $a_i \rightarrow \mu(a_i) \in [0,1]$. The dominion of decision attribute is $\{$ *reliable, unreliable* $\}$

$$\mu_{(dcision)} \in \{0,1\} = \begin{cases} Yes \text{ if the plant is reliable} \\ No \text{ if the plant is unreliable} \end{cases}$$

The results don't respect the additivity of membership functions and the decisions have an evaluation structure in solution x_7 *and* x_8 with ambiguity.

Table 2				
x_i	$\mu(cost)$	$\mu(affidability)$	$\mu(safety)$	Result $\mu(decision)$
1	*0.50*	*1.00*	*0.50*	$\mu = 1 \rightarrow Yes$
3	*0.50*	*0.75*	*0.75*	$\mu = 1 \rightarrow Yes$
6	*1.00*	*0.50*	*0.75*	$\mu = 1 \rightarrow Yes$
8	*0.25*	*0.50*	*0.50*	$\mu = 1 \rightarrow Yes$
9	*1.00*	*1.00*	*0.75*	$\mu = 1 \rightarrow Yes$
10	*0.50*	*1.00*	*0.50*	$\mu = 1 \rightarrow Yes$
Average of values of membership functions				
	0.625	0.79	0.625	*Yes*

The sets of <u>*Yes*</u> is $\{x_1, x_3, x_6, x_8, x_9, x_{10}\}$
The crisp set $\underline{E} = \{x_1, x_3, x_6, x_9, x_{10}\}$
The *flou set* is $\underset{flou}{E} = \{x_1, x_3, x_6, x_8, x_9, x_{10}\}$
<u>The data in table 2 are of the flou set</u> $\underset{flou}{E}$

From the values of average of membership functions it is possible deduce the dominant attributes in evaluation structure.
<u>*In structure of decision for **Yes** the value of membership* $\mu(affidability)$ *is dominant..*</u>

If to every interval of dominion of membership function correspond a verbal attribute

$$\{cost, affidability, safety\} = \begin{cases} for\, \mu = 0 \text{ to } 0.30 \rightarrow \{high, low, low\} \\ for\, \mu = 0.30 \text{ to } 0.65 \rightarrow \{mudium\ high, normal, regular\} \\ for\, \mu = 0.65 \text{ to } 0.75 \rightarrow \{good, good, good\} \\ for\, \mu = 0.75 \text{ to } 1 \rightarrow \{low, optimum, optimum\} \end{cases}$$

the result can be translate automatic in verbal reasoning.

A rule for identification the verbal structure is:

$$\textbf{If } P(x_i) \textbf{ then } D(y_i)$$

From data of table 2 we have:

If the cost is *medium* ***and*** the affidability is *optimum* ***and*** safety is *regular* ***then*** the remark of plant is ***reliable.***

8-CONCLUSIONS

In this paper is studied the mathematical representation of information system in presence of uncertainty. Using Gentilhomme's definition of flou sets, with three values of truth, is proposed a representation of new subset, alpha-subset, where the value of truth is the mean of membership functions. In alpha -subset is confined all the uncertainty. The operations of classification or design can be restricted in the universe U only on alpha-subset. The new representation is easy and useful in classification and in design. The entropy is used for measure of uncertainty. It is easy to calculate the statistic values on the sets.

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