# A Fuzzy Mathematical Model To DetermineDecrepit Degree

Li Su-Yun    Li Ping    Sun Shao-quan

*Dept. of Mathematics, Jilin Province college of Education, 130022, Changchun, China.*

**Abstract** In this paper, we give a fuzzy mathematical model to determine decrepit degree. The model is reasonable by means of verification of example, It will take important effect in the study of antiager.

**Keywords** Fuzzy mathematical model; decrepit degree; physiological age; gerontology; calendar age.

## 1. Introduction

Physiological age reflects decrepit degree of a person. Defining the physiological age is a problem about the study of gerontology. Gerontology worker tried to define the physiological age by mathematical means, such as setting up regression equation of age and physiological indexes by regression analysis to define the physiological age. Regression equation takes important effect in the study of antiager, such as judging if the medicine is useful according to the change of physiological age before and after having medicine.

Regression equation has a definite accuracy, but it is not quite good because the change of one person's physiological index values mean much, or not, after has the antiager, such as the blood pressure of a person of fifty is 135/90 before he has medicine, and after he has medicine it becomes 120/80, high blood pressure changes normal. This is a change that means much, it means the change of physiological age; If the blood pressure of the same person is 120/85 before he has medicine, and after he has medicine it becomes 115/80, the blood pressure before and after having medicine is both in the normal bounds, so this change is no point. Because the change does not mean that the physiological age changes, but according to the regression equation, the change will be considered that it causes the change of physiological age. So there will be some difficulty in the analysis about the result. This paper gives a Fuzzy mathematical model to determine decrepit degree in order to overcome the shortcoming of regression analysis.

## 2. Fuzzy mathematical model

We generally call the 45 to 59 age bracket"at the earlier stage of old age"and the more than 59 age bracket"at the stage of old age". In fact, "at the earlier stage of old age"and"at the stage of old age"are both Fuzzy sets, They are expressed by $\underset{\sim}{A}_1$ and $\underset{\sim}{A}_2$ respectively.

Aj(x)is defined as follow:

(1)We randomly choose N normal people(age$\geqslant$45) and determine their n physiological indexes $X_1, X_2, \cdots, X_n$ respectively. $x_k=(x_{k1}, x_{k2}, \cdots, x_{kn})$ expresses the check result of n physiological indexes of the person number k, $k=1,2,\cdots, N$.

(2)The extent of index $X_i$ is graded $t_i$ different $X_{1i}, X_{2i}, \cdots, X_{t_i i}, i=1,2,\cdots,n$.

(3)We divide N people into two groups: people of the 45 to 59 age bracket are in the group number 1 and the more than 59 age bracket are in the group number 2, they are expressed by $A_1$ and $A_2$ respectively. $X_k^{(j)}=(X_{k1}^{(j)}), X_{k2}^{(j)}, \cdots, X_{kn}^{(j)})$ expresses the value of n physiological index of the person number k in Aj, $k=1,2,\cdots,h_j, j=1,2, h_1+h_2=N$.

(4)We count frequencies $P_{1i}^{(j)}, P_{2i}^{(j)}, \cdots, P_{t_i i}^{(j)}$ that $X_{1i}^{(j)}, X_{2i}^{(j)}, \cdots, {}_{h_j i}^{(j)}$ fall into $X_{1i}, X_{2i}, \cdots, X_{t_i i}$ respectively, $j=1,2, i=1,2,\cdots,n$. Find bigest frequency.

$$P_{ti}^{(j)}=\max_{1\leqslant m\leqslant t_i} \{P_{mi}^{(j)}\}$$

Find the sum

$$S_j=\sum_{i=1}^{n} P_{ti}^{(j)} \tag{1}$$

For any $X=(X_1, X_2, \cdots, X_n)$, if $X_i \in X_{mi}, i=1,2,\cdots,n$.

then we find the sum

$$S_j(x)=\sum_{i=1}^{n} P_{mi}^{(j)} \qquad j=1,2.$$

The membership function of the fuzzy set $\underset{\sim}{A}j$ is defined as

$$\underset{\sim}{A}_j(x)=\frac{S_j(x)}{S_j} \qquad j=1,2. \tag{2}$$

Fuzzy set $\underset{\sim}{A}o$ corresponding to the less than 45 age bracket is defined as

$$\underset{\sim}{A}_0=\overline{\underset{\sim}{A}_1 \cup \underset{\sim}{A}_2}=\overline{\underset{\sim}{A}_1} \cap \overline{\underset{\sim}{A}_2}$$

Hence

$$\underset{\sim}{A}_0(x)=\min\{1-\underset{\sim}{A}_1(x),1-\underset{\sim}{A}_2(x)\} \quad (3)$$

(5)For check result $X=(X_1,X_2,\cdots,X_n)$ of n physiological indexes of anybody, according to formulas (2) and (3), we compute $\underset{\sim}{A}_j(x), j=0,1,2.$

if $\underset{\sim}{A}_{j_0}(x)=\max_{0\leqslant i\leqslant 2}\{\underset{\sim}{A}_j(x)\}$ then we judge that X belong in $\underset{\sim}{A}_{j_0}$ as compared with $\underset{\sim}{A}_j(j\neq j_0)$.

### 3. Example

We randomly take a sample with 170 normal people of calendar age $\geqslant 45$ some region and measure of their physiological indexes as follow: Systolic pressure ($X_1$), Diastolic pressure ($X_2$), Cholesterol ($X_3$), Three acid glyceride ($X_4$), $\beta$ proteinase ($X_5$), standing time with one foot ($X_6$).

$X_{mi}$ and $P_{mi}^{(j)}$, $1\leqslant m\leqslant t_i, 1\leqslant i\leqslant 6, t_1=11, t_2=8, t_3=13, t_4=17, t_5=8, t_6=12$, are shown in Table 1.

Table 1 The grades and frequencies of physiological indexes

| $X_{m1}$ | $P_{m1}^{(1)}$ | $P_{m1}^{(2)}$ | $X_{m2}$ | $P_{m2}^{(1)}$ | $P_{m2}^{(2)}$ | $X_{m3}$ | $P_{m3}^{(1)}$ | $P_{m3}^{(2)}$ |
|---|---|---|---|---|---|---|---|---|
| [0,90) | 0 | 1 | [0,60) | 1 | 0 | [0,140) | 0 | 5 |
| [90,100) | 2 | 3 | [60,70) | 4 | 7 | [140,150) | 2 | 9 |
| [100,110) | 6 | 6 | [70。80) | 16 | 26 | [150,160) | 6 | 12 |
| [110,120) | 17 | 17 | [80,90) | 23 | 33 | [160,170) | 7 | 15 |
| [120,130) | 10 | 19 | [90,100) | 14 | 22 | [170,180) | 5 | 9 |
| [130,140) | 14 | 16 | [100,110) | 10 | 9 | [180,190) | 11 | 7 |
| [140,150) | 9 | 14 | [110,120) | 1 | 3 | [190,200) | 6 | 5 |
| [150,160) | 3 | 5 | [120,∞) | 1 | 0 | [200,210) | 9 | 13 |
| [160,170) | 5 | 10 | | | | [210,220) | 9 | 12 |
| [170,180) | 4 | 3 | | | | [220,230) | 6 | 3 |
| [180,∞) | 0 | 6 | | | | [230,240) | 1 | 3 |
| | | | | | | [240,250) | 1 | 4 |
| | | | | | | [250,∞) | 7 | 3 |

| $X_{m4}$ | $P_{m4}^{(1)}$ | $P_{m4}^{(2)}$ | $X_{m5}$ | $P_{m5}^{(1)}$ | $P_{m5}^{(2)}$ | $X_{m6}$ | $P_{m6}^{(1)}$ | $P_{m6}^{(2)}$ |
|---|---|---|---|---|---|---|---|---|
| [40,50) | 3 | 9 | [0,300) | 7 | 13 | [0,1) | 0 | 8 |
| [50,60) | 7 | 3 | [300,400) | 6 | 13 | [1,2) | 2 | 32 |
| [60,70) | 9 | 4 | [400,500) | 16 | 24 | [2,3) | 6 | 22 |
| [70,80) | 5 | 8 | [500,600) | 17 | 20 | [3,4) | 9 | 13 |
| [80,90) | 11 | 8 | [600,700) | 14 | 14 | [4,5) | 11 | 12 |
| [90,100) | 9 | 8 | [700,800) | 5 | 13 | [5,6) | 5 | 4 |
| [100,110) | 5 | 13 | [800,900) | 3 | 3 | [6,7) | 5 | 3 |
| [110,120) | 3 | 8 | [900,∞) | 2 | 0 | [7,8) | 5 | 4 |
| [120,130) | 8 | 7 | | | | [8,9) | 8 | 0 |
| [130,140) | 1 | 5 | | | | [9,10) | 3 | 0 |
| [140,150) | 1 | 3 | | | | [10,11) | 8 | 1 |
| [150,160) | 0 | 6 | | | | [11,∞) | 8 | 1 |
| [160,170) | 1 | 5 | | | | | | |
| [170,180) | 1 | 1 | | | | | | |
| [180,190) | 3 | 4 | | | | | | |
| [190,200) | 0 | 2 | | | | | | |
| [200,∞) | 3 | 6 | | | | | | |

According to formula (1) and Table 1, we obtain

$$S_1 = P_{41}^{(1)} + P_{42}^{(1)} + P_{63}^{(1)} + P_{54}^{(1)} + P_{45}^{(1)} + P_{56}^{(1)}$$
$$= 17 + 23 + 11 + 11 + 17 + 11$$
$$= 90$$

$$S_2 = P_{51}^{(2)} + P_{42}^{(2)} + P_{43}^{(2)} + P_{74}^{(2)} + P_{35}^{(2)} + P_{26}^{(2)}$$
$$= 19 + 33 + 15 + 13 + 24 + 32$$
$$= 136$$

For example, a person's calendar age is 55, his 6 physiological indexes are $X = (130, 80, 200, 120, 750, 7.5)$. since $130 \in X_{61}, 80 \in X_{42}, 200 \in X_{83}, 120 \in X_{94}, 750 \in X_{65}, 7.5 \in X_{86}$, hence,

$$S_1(x) = P_{61}^{(1)} + P_{42}^{(1)} + P_{83}^{(1)} + P_{94}^{(1)} + P_{65}^{(1)} + P_{86}^{(1)}$$
$$= 14 + 23 + 9 + 8 + 5 + 5$$
$$= 64$$

$$S_2(x) = P_{61}^{(2)} + P_{42}^{(2)} + P_{83}^{(2)} + P_{94}^{(2)} + P_{65}^{(2)} + P_{86}^{(2)}$$
$$= 16 + 33 + 13 + 7 + 13 + 4$$
$$= 86$$

From formulas (2) and (3), we obtain

$$\underset{\sim}{A}_1(x) = \frac{S_1(x)}{S_1} = \frac{64}{90} \approx 0.71$$

$$\underset{\sim}{A}_2(x)=\frac{S_2(x)}{S_2}=\frac{86}{136}\approx 0.63$$

$$\underset{\sim}{A}_0(x)=\min\{1-\underset{\sim}{A}_1(x),1-\underset{\sim}{A}_2(x)\}=\min\{0.29,0.37\}=0.29$$

Since $\underset{\sim}{A}_1(x)=\max\{0.71,0.63,0.29\}=0.71$, we judge that x belong in $\underset{\sim}{A}_1$ as compared with $\underset{\sim}{A}_j(j=0,2)$. It accords with calendar age of the person. The conclusion shows that this person is normal aging.

For another example, a person's calendar age is 51, his 6 physiological indexes are x=140,90,185,90,500,1.5, appling the same method as above, we have

$$\underset{\sim}{A}_1(x)=\frac{62}{90}\approx 0.69$$

$$\underset{\sim}{A}_2(x)=\frac{103}{136}\approx 0.76$$

$$\underset{\sim}{A}_0(x)=\min\{0.31,0.24\}=0.24$$

$$\underset{\sim}{A}_2(x)=\max\{0.69,0.76,0.24\}=0.76$$

According to the bigest membership principle, we judge that x belong to $\underset{\sim}{A}_2$ as compared with $\underset{\sim}{A}_j(j=0,1)$. It does not accords with calendar age of the person. Now we will analyse all physiological indexes of the person. We find that two indexes are abnormal in six indexes one is that blood pressure is high, the standing time with one foot is short. This is the expression of early ageing. The conclusion shows that this person is older than normal people of same age. It corresponds to reality of the person. Above two examples show this fuzzy mathematical model is reasonable.

## 4. concluding remarks

(1) We must reasonably shoose a sample, that is people in the sample must be healthful because calendar age was treated as physiological age when we set up fuzzy matematical model. This is primal condition to raise acouracy of model.

(2) The objects of study are people of age $\geqslant 45$ in this paper, in the application of practice, we can widen age bracket.

## references

(1) Shi—quan Cen, The fuzzy patterns for the Diagnosis of occupational Diseases. Fuzzy mathematics, 2 (1983).

(2) Shock NW: physiological age and calendar age, Gerontology magazine 1 (1983)