The Stability of Intelligent Control Based on l^*-Module (1)*

Yuan Jian, Xu Yang
Department of Applied Mathematics
Southwest Jiaotong University
Chengdu 610031, Sichuan
P. R. China
Email: yxu@center2.swjtu.edu.cn

Abstract – It is well known that the Fuzzy Control and Network of Neurons play important roles in Artificial Intelligence and Intelligent Control. The Control procedure is often represented by language rules that different from the classical control. The classical stability theory is useless here. The stability of intelligent control is still an open problem. The key to discuss this problem is how to characterize and analysis the control procedure by an appropriate mathematical tool. For this reason, we introduce the analytical theory based on l^*-module. In this paper we first define a concept of lattice convergence, then discuss its properties, and finally we study the l^*-topology derived from the convergence and l^*-topological lattice group.

1 INTRODUCTION

It is well known that the Fuzzy Control and Network of Neurons play important roles in Artificial Intelligence and Intelligent Control. The control procedure is often represented by language rules that is different from the classical control. The classical stability theory is useless here. The stability of intelligent control is still an open problem. The key to discuss this problem is how to characterize and analysis the control procedure by an appropriate mathematical tool. For this reason, we introduce an analytical theory based on l*-module. In 1995, Mr. Xu Yang, in his doctoral dissertation thesis [1], first introduced a double lattice-ordered algebraic structure called l^*-module and discussed its properties, by which he dealt with the problem of lattice valued game and set up the elemental theory of lattice valued game. In this paper we first gave a definition of lattice-convergence on l^*-module and further discussed the properties of the lattice-convergence , finally we discuss the properties of the l^*-topology and l^*-topological group on l^*-module. The proofs in this paper which are not difficult are omitted.

Definition 1.1 [1] Let $(R, \vee, \wedge, \cdot, +, 1, \theta, \leq)$ be a lattice-ordered commutative unitary ring (l-ring), let $(M, \vee, \wedge, +, 0, \leq)$ be a lattice-ordered Abel group (l-group), if for $\forall \alpha, \beta \in R$, $\forall x, y \in M$ such that $\alpha x \in M$ and

(1) $\alpha(x+y) = \alpha x + \alpha y$,

(2) $(\alpha+\beta)x = \alpha x + \beta x$,

(3) $\alpha(\beta x) = (\alpha\beta)x$,

(4) $1x = x$

(5) $x > 0, \alpha > \theta \Rightarrow \alpha x > 0$

(6) M, R *are conditionally complete lattices.*

then the pair (M, R) denoted briefly by $M(R)$ is an l^*-module.

2 THE PROPERTIES OF LATTICE CONVERGENCE ON M

Definition 2.1 Let $\{s_n, n \in D\}$ be a net in M, let D be a directed set. If $\bigwedge_n \bigvee_{k\geq n} s_k$ and $\bigvee_n \bigwedge_{k\geq n} s_k$ exist, then called them respectively superior lattice-limit and inferior lattice-limit of the net $\{s_n\}$ denoted respectively by

$$\overline{\lim_n} s_n = \bigwedge_n \bigvee_{k\geq n} s_k \text{ and } \underline{\lim_n} s_n = \bigvee_n \bigwedge_{k\geq n} s_k .$$

A net $\{s_n\}$ lattice-converges to $s \in M$ if and only if $\overline{\lim_n} s_n = \underline{\lim_n} s_n = s$ denoted by $s_n \xrightarrow{L} s$ or $(L)\lim_n s_n = s$.

A net $\{s_n\}$ is increasing (decreasing) if and only if for $\forall m, n \in D$, if $m \geq n$, then $s_m \geq s_n$ ($s_m \leq s_n$) .

Theorem 2.1 Let net $\{s_n\}$ be bounded in M. If the net $\{s_n\}$ is increasing (decreasing), then

$$(L)\lim_n s_n = \vee_n s_n \quad ((L)\lim_n s_n = \wedge_n s_n).$$

Theorem 2.2 If $\overline{\lim_n} s_n$ and $\underline{\lim_n} s_n$ exist, then

1) $\overline{\lim_n} s_n = (L)\lim_n \vee_{k\geq n} s_k$,

2) $\underline{\lim_n} s_n = (L)\lim_n \wedge_{k\geq n} s_k$, and

3) $\underline{\lim_n} s_n \leq \overline{\lim_n} s_n$.

Theorem 2.3 $s_n \xrightarrow{L} s$ if and only if there are increasing net $\{y_n\}$ and decreasing net $\{z_n\}$ which satisfy that $y_n \leq s_n \leq z_n$, $\forall n \in D$ and $y_n \xrightarrow{L} s$, $z_n \xrightarrow{L} s$.

Proof: "$\Rightarrow$" The nets $\{A_n\}$ and $\{B_n\}$ where $A_n = \vee_{k\geq n} s_k$ and $B_n = \wedge_{k\geq n} s_k$ are decreasing and increasing respectively which satisfy conditions.

"$\Leftarrow$" Let nets $\{y_n\}$ and $\{z_n\}$ satisfy the given conditions. Then,

$$(L)\lim_n y_n = \vee_n y_n = s, \quad (L)\lim_n z_n = \wedge_n z_n = s,$$

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and $s_k \le z_k$, $y_k \le s_k$ from theorem 2.2. It follows that $\vee_{k\ge n} s_k \le \vee_{k\ge n} z_k = z_n$. It holds

$$\overline{\lim_n} s_n = \wedge_n \vee_{k\ge n} s_k \le \wedge_n z_n = s.$$

then $\overline{\lim_n} s_n = \wedge_n \vee_{k\ge n} s_k \ge \wedge_n \vee_{k\ge n} y_k = s$ from $\vee_{k\ge n} s_k \ge \vee_{k\ge n} y_k$, so $\overline{\lim_n} s_n = s$. Similarly we obtain $\underline{\lim_n} s_n = s$, namely, $(L)\lim_n s_n = s$.

Theorem 2.4 A net $\{s_n\}$ converges to s if and only if each subnet $\{s_{N(m)}\}$ of which converges to s.

Proof: "$\Leftarrow$" Obviously. The net $\{s_n\}$ is a special subset of itself.

"$\Rightarrow$" Let $\{s_{N(m)}\}$ be a subnet of $\{s_n\}$. As $s_n \xrightarrow{L} s$, then there are increasing net $\{y_n\}$ and decreasing net $\{z_n\}$ in M such that $y_n \le s_n \le z_n$, $\forall n \in D$, and $y_n \xrightarrow{L} s$, $z_n \xrightarrow{L} s$. Taking the same subscript, we have $y_{N(m)} \le s_{N(m)} \le z_{N(m)}$, $m \in E$, $N(m) \in D$, where E is a directed set. Then $\{y_{N(m)}\}$ and $\{z_{N(m)}\}$ are subsets of $\{y_n\}$ and $\{z_n\}$. Because $\forall k,\ m \in E,\ k \ge m,\ N(k) \in D$ such that $y_{N(k)} \le \vee_n y_n = s$, that is to say, $\vee_{k\ge m} y_{N(k)} \le s$, so $(L)\overline{\lim_m} y_{N(m)} \le \vee_n y_n = s$. As the $\{y_{N(m)}\}$ is a subnet of $\{y_n\}$, for $\forall n \in D$, $\exists m_0 \in E$ such that if $p \ge m_0$, then $N(p) \ge n$, so $y_n \le y_{N(p)}$ and

$$y_n \le \wedge_{k\ge m_0} y_{N(k)} \le \vee_m \wedge_{k\ge m} y_{N(k)} = \underline{\lim_m} y_{N(m)},$$

that is $s = \vee_n y_n \le \underline{\lim_m} y_{N(m)}$. From 3) of theorem 2.2, we obtain $(L)\lim_m y_{N(m)} = s$. Similarly we can show that $(L)\lim_m z_{N(m)} = (L)\lim_n z_n = s$. By theorem 2.3, it holds that $(L)\lim_m s_{N(m)} = s$.

Theorem 2.5 If $(L)\lim_n s_n \ne s$, then there is a subnet $\{s_{N(m)}\}$ of $\{s_n\}$, each subnet $\{s_{N\circ R(k)}\}$ of which holds $(L)\lim_k s_{N\circ R(k)} \ne s$.

Proof: If $(L)\lim_n s_n \ne s$, we discuss two cases. First, $(L)\lim_n s_n = t \ne s$. From theorem 2.4,

$$(L)\lim_m s_{N(m)} = t \ne s$$

for each subnet $\{s_{N(m)}\}$ of net $\{s_n\}$. Second, $(L)\lim_n s_n$ does not exist. a) Let $\overline{\lim_n} s_n$ and $\underline{\lim_n} s_n$ exist, but $\overline{\lim_n} s_n \ne \underline{\lim_n} s_n$. It is no problem to assume that $\overline{\lim_n} s_n = t \ne s$. From c) of lemma 2.2) in [2], there is a subnet $\{s_{N(m)}\}$ of $\{s_n\}$ such that $(L)\lim_k s_{N\circ R(k)} \ne s$ for each subnet $\{s_{N\circ R(k)}\}$ of which. b) If one of the $\overline{\lim_n} s_n$ and $\underline{\lim_n} s_n$ does not exist. By theorem 2.1 in [2], the net $\{s_n\}$ is unbounded. It is no problem to assume that it is unbounded from above, from a) of lemma 2.2) in [2], there is a increasing subnet $\{s_{N(m)}\}$ which is unbounded from above and each subnet $\{s_{N\circ R(k)}\}$ of net $\{s_{N(m)}\}$ is unbounded from above by b) of lemma 2.2) in [2], then $(L)\lim_k s_{N\circ R(k)} \ne s$ for each subnet $\{s_{N\circ R(k)}\}$ of $\{s_{N(m)}\}$. From above discussion, there is some subnet $\{s_n\}$, each subnet $\{s_{N\circ R(k)}\}$ of which holds that $(L)\lim_k s_{N\circ R(k)} \ne s$.

Theorem 2.6 Let D be a directed set, let E_m be a directed set for $\forall m \in D$ and Let F be the product $D \times \prod_m \{E_m,\ m \in D\}$ and for each (m, f) in F let $R(m,f)=(m, f(m))$. S is such a function that for each $m \in D$ and each $n \in E_m$, $S(m,n) \in M$. If

$$(L)\lim_m \lim_n S(m,n) = s,$$

then $S \circ R$ lattice-converges to s.

Proof: We know that the operations $\wedge$ and $\vee$ satisfy infinitely divisible law, so

$$\begin{aligned}
\overline{\lim_{(m,f)}} S\circ R(m,f) &= \wedge_{(m,f)} \vee_{(n,g)\ge(m,f)} S\circ R(n,g)\\
&= \wedge_m \wedge_f \vee_{(n,g)\ge(m,f)} S\circ R(n,g)\\
&= \wedge_m \wedge_f \vee_{n\ge m} \vee_{g\ge f} S\circ R(n,g)\\
&= \wedge_m \vee_{n\ge m} \wedge_f \vee_{g\ge f} S\circ R(n,g)\\
&= \wedge_m \vee_{n\ge m} \wedge_f \vee_{g(n)\ge f(n)} S(n,g(n))\\
&= \wedge_m \vee_{n\ge m} \overline{\lim_k} S(n,k)\\
&= \overline{\lim_m}\,\overline{\lim_n} S(m,n) = s.
\end{aligned}$$

Similarly, it could be shown that $\underline{\lim_{(m,f)}} S\circ R(m,f) = s$, hence $(L)\lim_{(m,f)} S\circ R(m,f) = s$.

Theorem 2.7 Let $\{x_n\}$, $\{y_n\}$ be bounded nets. Then

1) $\overline{\lim_n}(x + x_n) = x + \overline{\lim_n} x_n$;

   $\underline{\lim_n}(x + x_n) = x + \underline{\lim_n} x_n$.

2) If $x_n \le y_n$, for each $n \in D$, then

   $\overline{\lim_n} x_n \le \overline{\lim_n} y_n$, $\underline{\lim_n} x_n \le \underline{\lim_n} y_n$.

3) $\overline{\lim_n}(x_n \vee y_n) = \overline{\lim_n} x_n \vee \overline{\lim_n} y_n$;

   $\underline{\lim_n}(x_n \wedge y_n) \le \underline{\lim_n} x_n \wedge \underline{\lim_n} y_n$;

   $\underline{\lim_n}(x_n \vee y_n) \ge \underline{\lim_n} x_n \vee \underline{\lim_n} y_n$;

$\underline{\lim}_n (x_n \wedge y_n) = \underline{\lim}_n x_n \wedge \underline{\lim}_n y_n$.

4). If net $\{x_n\}$ is a bounded separate set, then

$$\overline{\lim}_n |x_n| = (\overline{\lim}_n x_n)^+ + (\underline{\lim}_n x_n)^-,$$

$$\underline{\lim}_n |x_n| = (\underline{\lim}_n x_n)^+ + (\overline{\lim}_n x_n)^-$$

5). If net $\{x_n\}$ is bounded, then

$$(L)\lim_n (-x_n) = -(L)\lim_n x_n.$$

Theorem 2.8 Let net $\{x_n\}$ be bounded. If there are nets $\{y_n\}$ and $\{z_n\}$ such that for each $n \in D$, $y_n \le x_n \le z_n$, and $(L)\lim_n y_n = (L)\lim_n z_n = x$, then $(L)\lim_n x_n = x$.

Theorem 2.9 If nets $\{x_n\}$ and $\{y_n\}$ are decreasing and $(L)\lim_n x_n = (L)\lim_n y_n = 0$, then the net $\{x_n + y_n\}$ is decreasing and $(L)\lim_n (x_n + y_n) = 0$.

Theorem 2.10 Let net $\{x_n\}$ be bounded in *M*. The following results are equivalent each other:

1) $(L)\lim_n x_n = x$,

2) $(L)\lim_n (x_n - x) = 0$,

3) There are increasing net $\{y_n\}$ and decreasing net $\{z_n\}$ such that $y_n \le x_n \le z_n$ for each $n \in D$, and $\underline{\lim}_n y_n = \overline{\lim}_n z_n = x$, and

4) There is a decreasing net $\{u_n\}$ such that $(L)\lim_n u_n = 0$ and $|x_n - x| \le u_n$ for each $n \in D$.

Theorem 2.11 If $(L)\lim_n x_n = x$ and $(L)\lim_n y_n = y$, then:

1) $(L)\lim_n (x_n + y_n) = x + y$,

2) $(L)\lim_n (x_n \vee y_n) = x \vee y$ and

$(L)\lim_n (x_n \wedge y_n) = x \wedge y$,

3) $(L)\lim_n x_n^+ = x^+$ and $(L)\lim_n x_n^- = x^-$,

4) $(L)\lim_n |x_n| = |x|$, and

5) $(L)\lim_n |x_n| = 0$ if and only if $(L)\lim_n x_n = 0$.

Theorem 2.12 Let net $\{x_n\}$ be bounded in M. $(L)\lim_n x_n = x$ if and only if $(L)\lim_n |x_n - x| = 0$.

Theorem 2.13 (Complete Theorem) $(L)\lim_n x_n$ exists if and only if $(L)\lim_n \bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m| = 0$.

Proof: "$\Rightarrow$" From theorem 2.12 we have that $x_n \xrightarrow{L} x \Leftrightarrow |x_n - x| \xrightarrow{L} 0$. Let $y_n = |x_n - x|$, we obtain that

$$(L)\lim_n \bigvee_{k\ge n} |x_k - x| = (L)\lim_n \bigvee_{k\ge n} y_k = \overline{\lim}_n y_n = \overline{\lim}_n |x_n - x| = 0.$$

It holds that

$$|x_k - x_m| \le |x_k - x| + |x_m - x|$$
$$\le \bigvee_{k\ge n} |x_k - x| + \bigvee_{m\ge n} |x_m - x|$$
$$= 2 \bigvee_{k\ge n} |x_k - x|$$

for each $k,\ m \ge n$. Then

$$\bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m| \le 2 \bigvee_{k\ge n} |x_k - x|,$$

hence $(L)\lim_n \bigvee_{k\ge n} \bigvee_{k\ge m} |x_n - x_m| = 0$.

"$\Leftarrow$" From $|x_n| = x_n^+ \vee x_n^- \ge x_n \vee (-x_n)$ it holds that for each $k,\ m \ge n$,

$$(x_m - x_k) \vee (x_k - x_m) \le |x_k - x_m| \le \bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m|,$$

then

$$x_k \le x_m + \bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m|, \text{ and}$$

$$x_m \le x_k + \bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m|$$

so

$$\bigvee_{k\ge n} x_k \le \bigwedge_{m\ge n} x_m + \bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m|,$$

namely, $0 \le \bigvee_{k\ge n} x_k - \bigwedge_{k\ge n} x_k \le \bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m|$. We have $(\bigvee_{k\ge n} x_k - \bigwedge_{k\ge n} x_k) \xrightarrow{L} 0$ from theorem 2.8. As $\bigvee_{k\ge n} \bigvee_{m\ge n} |x_k - x_m|$ is bounded, we have that the net $\{x_n\}$ is bounded. So $\underline{\lim}_n x_n$ and $\overline{\lim}_n x_n$ exist. From 1) of theorem 2.11 we obtain

$$\overline{\lim}_n x_n - \underline{\lim}_n x_n = (L)\lim_n \bigvee_{k\ge n} x_k - (L)\lim_n \bigwedge_{k\ge n} x_k$$
$$= (L)\lim_n (\bigvee_{k\ge n} x_k - \bigwedge_{k\ge n} x_k) = 0,$$

hence, $\underline{\lim}_n x_n = \overline{\lim}_n x_n$, then $(L)\lim_n x_n$ exists.

3 THE LATTICE CONVERGENCE PROPERTIES ON *l**-MODULE

Because the set *R* is a l-ring, we are able to establish the concept of lattice-convergence of a net in *R* similarly. The convergence properties of a net in *M* still hold in *R*. We denote the elements of *M* by *x*, *y*, *z*, ..., and the elements of *R* by α, β, γ ....

Theorem 3.1 Let $\{x_n\}$ be a bounded net in *M*. Then:

1) $\overline{\lim}_n \alpha x_n = \alpha \overline{\lim}_n x_n$, $\underline{\lim}_n \alpha x_n = \alpha \underline{\lim}_n x_n$ if α is positive and convertible,

2) $\overline{\lim}_n \alpha x_n = \alpha \underline{\lim}_n x_n$, $\underline{\lim}_n \alpha x_n = \alpha \overline{\lim}_n x_n$ if α is negative and convertible, and

3) $(L)\lim_n \alpha x_n = \alpha x$ if α is convertible and $\alpha \ne \theta$.

Theorem 3.2 Let net $\{x_n\}$ be bounded in M and net $\{\alpha_n\}$ be bounded in *R*. Then:

1) If x belongs to M and $x \neq 0$ such that $(L)\lim_n \alpha_n = \alpha$, then $(L)\lim_n \alpha_n x = \alpha x$,

2) If $(L)\lim_n \alpha_n = \theta$, then $(L)\lim_n \alpha_n x_n = 0$, and

3) If $(L)\lim_n x_n = x$ and $(L)\lim_n \alpha_n = \alpha$ such that α is convertible and $\alpha \neq \theta$, then $(L)\lim_n \alpha_n x_n = \alpha x$.

4 l^*-TOPOLOGY LATTICE GROUP

Definition 4.1 A subset $\overline{A}$, which satisfies that a point s belongs to $\overline{A}$ if and only if there is a net $\{x_n, n \in D\}$ in subset A such that $(L)\lim_n x_n = x$, is the closure of the subset A in M. A subset A is closed if and only if $A = \overline{A}$.

Definition 4.2 A subset A of M is open if and only if its relative complement $X \sim A$ is closed.

Theorem 4.1 Let $\Im$={ X, X is the open subset of M }. Then the pair $(M, \Im)$ is a topological space called l^*-topological space.

Definition 4.3 Let $(M, \Im)$ be a l^*-topological space. A set U in M is a neighborhood of a point x if and only if U contains an open set to which x belongs. The neighborhood system of a point x is the family of all neighborhoods of the point x.

Lemma 4.1 For each x and y which belong to M and x is more than or equal to zero, $|y| \leq x$ if and only if $-x \leq y \leq x$.

Theorem 4.2 Let

$$B(x, \varepsilon) = \{y, \ |y - x| \leq \varepsilon, \ 0 \leq \varepsilon \in M\}.$$

Then $B(x, \varepsilon)$ is a closed set called ε-closed ball.

Proof: If ε is zero then $B(x, \varepsilon)$ is a single point set $\{x\}$, it is obvious that $\{x\}$ is closed. Now let ε be positive. Obviously, it suffices to show that $\overline{B(x, \varepsilon)} \subseteq B(x, \varepsilon)$ where $\overline{B(x, \varepsilon)}$ is the closure of $B(x, \varepsilon)$. For each y in $\overline{B(x, \varepsilon)}$ there is a net $\{y_n, n \in D\}$ in $B(x, \varepsilon)$ such that $(L)\lim_n y_n = y$, then $|y_n - x| \leq \varepsilon$ for each n in D which is a directed set. From lemma 4.1 we have $-\varepsilon \leq y_n - x \leq \varepsilon$ for each n in D, that is $x - \varepsilon \leq y_n \leq x + \varepsilon$ for each n in D. Then

$$-\varepsilon \leq (L)\lim_n y_n - x \leq \varepsilon,$$

namely, $\left|(L)\lim_n y_n - x\right| \leq \varepsilon$, hence $y \in B(x, \varepsilon)$. We have shown that $B(x, \varepsilon)$ is closed.

Definition 4.4 Let $(M, \Im)$ be an l^*-topological space. Let $U(x)$ be the neighborhood system of x. If for each $U \in U(x)$, there is m in D such that $x_n \in U$ when $n \geq m$, then the net $\{x_n, n \in D\}$ converges to x associated with the l^*-topology denoted by $x_n \xrightarrow{T} x$.

Theorem 4.3 Let net $\{x_n, n \in D\}$ be in M. $x_n \xrightarrow{L} x$ if and only if $x_n \xrightarrow{T} x$.

Theorem 4.4 l^*-topological space is a Hausdoff topological space.

Theorem 4.5 Let x and ε be in M where $\varepsilon \geq 0$, then:

1) $B(x, \varepsilon) - x = B(0, \varepsilon)$, and

2) $B(x, \varepsilon) \subseteq B(0, \varepsilon + |x|)$.

Theorem 4.6 $A \subseteq M$ is closed (open) if and only if $\forall x \in M$, $x+A$ is closed (open).

Theorem 4.7 1) If A is open and B is a subset of M, then $A+B$ is open,

2) If A is closed, and B is a finite subset of M, then $A+B$ is closed, and

3) If A and B are closed, then $A \vee B$ and $A \wedge B$ are closed.

Theorem 4.8 $U(x)$ is a neighborhood system of point x in M if and only if $U - x \in U(0)$ for each U in $U(x)$ where $U(0)$ is the neighborhood system of point 0.

Proof: Let $U(x)$ be a neighborhood system of point x and $U(0)$ be a neighborhood system of point 0. If $U - x \in U(0)$, then there is an open set V such that V is contained in $U - x \in U(0)$, hence U contains $V+x$. From theorem 4.6 we know that $V+x$ is an open set, so U is a neighborhood of point x which contains point x. Conversely, if U is a neighborhood of point x, then there is an open set V which contains point x such that $V \subseteq U$, then $V - x \subseteq U - x$. From theorem 4.6 we know that $V - x$ is an open set which contains point 0, hence $U - x \in U(0)$.

Definition 4.5 The map f from l^*-topological space $(M,\Im)$ to l^*-topological space $(N, \wp)$ is continuous if and only if $f(\overline{A}) \subseteq \overline{f(A)}$ for each subset A in M.

Theorem 4.9 The group operation on M is continuous with respect to l^*-topology $(M,\Im)$.

Proof: Let f be a map from $M \times M$ to M such that $f(x, y)=x+y$ for each x and y in M, and g be a map from M to M such that $g(x) = -x$ for each x in M. Firstly, we show that the function f is continuous, that is $\overline{A} + \overline{B} \subseteq \overline{A + B}$ for each subsets A and B in M. For each z in $\overline{A} + \overline{B}$ there are x in $\overline{A}$ and y in $\overline{B}$ such that $z=x+y$, then there are nets $\{x_n, n \in D\}$ in A and net $\{y_m, m \in E\}$ in B where D and E are directed sets such that $(L)\lim_n x_n = x$ and $(L)\lim_m y_m = y$. The Cartesian product $F = D \times E$ is also a directed set by binary relation $\succ$ where $(n, m) \succ (p, q)$ if and only if $n \geq p$ and $m \succ q$ for each n in D and m in E, then it is a net in $A + B$. From theorem 2.11 we have

$$(L)\lim_n \lim_m (x_n + y_m) = (L)\lim_n (x_n + y) = x + y = z.$$

From theorem 2.6, we have that

$$(L)\lim_{(n,m)} z_{(n,m)} = (L)\lim_n \lim_m z_{(n,m)}$$

$$= (L)\lim_n \lim_m (x_n + y_m) = z$$

then z belongs to $\overline{A+B}$. It has been shown that $\overline{A}+\overline{B}\subseteq\overline{A+B}$. Secondly, we will show that the function g is continuous, that is $-\overline{A}\subseteq\overline{(-A)}$ for each subset A in M. For each x in $-\overline{A}$ there is y in $\overline{A}$ such that $x=-y$, then there is a net $\{y_n\}$ in A such that $(L)\lim_n y_n=y$, hence $\{x_n\}$ is also a net in $-A$ where $x_n=-y_n$ for each n in directed D. From 5) of theorem 2.7 we have that

$$(L)\lim_n x_n=(L)\lim_n(-y_n)=-(L)\lim_n y_n=-y=x,$$

then x belongs to $\overline{(-A)}$. We have shown that $-\overline{A}\subseteq\overline{(-A)}$.

Theorem 4.10 The lattice operations on M are continuous with respect to l^*-topology.

Theorem 4.11 The lattice ordered group M with l^*-topology $(M,\Im)$ is a topological lattice group called l^*-topological lattice group.

Theorem 4.12 Let $\Omega=\{B(0,\varepsilon),\ 0\le\varepsilon\in M\}$, then:

1) For each U and V in Ω, there is W in Ω such that $W\subseteq U\cap V$,

2) For each x in U which belongs to Ω, there is V in Ω such that $x+V\subseteq U$, and

3) For each U in Ω, there is V in Ω such that $-V\subseteq U$.

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