THE USE OF FUZZY NUMBERS IN FORECASTING STRONG MINING TREMORS

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Summary

In the thesis there is presented the use of fuzzy numbers type $\alpha-\beta$ in forecasting strong mining tremors. First numbers definitions and basic operations on these numbers were presented, then the mathematical model of tremor risk was given. The comparison between fuzzy numbers was used to define the accuracy of prediction. The calculations concerning the prediction of safe time, when there are not any tremors, were made for real tremor measurements in „Pstrowski" coal-mine. These measurements were carried out by means of the seismological and seismoacoustic methods.

1. Introduction

The fuzzy sets theory formulated by Zadeh [10] is being more and more widely used in different disciplines [1], [2], [3], [4], [5], [6], [7]. It is used in mechanics, steering, building network models, testing stability, drawing conclusions, taking decisions, medical diagnosis and others. In the thesis there is described the use of fuzzy sets theory in coal-mining. Fuzzy numbers are used here to predict strong mining tremors. Based on measurements, the quantity τ_{roz} was defined as a fuzzy number which characterizes a rock mass in relation to the mean time between successive strong mining tremors, and as a fuzzy number ϑ_{roz}, which characterizes the reversion time of a rock mass to the relative equilibrium state. This is a very important use in coal-mining because of the work safety of miners. The rock mass on which miners work cannot be precisely described by means of methematical equations. The relationships occuring in a rock mass can only be defined approximately. That is why it is vital to use the fuzzy sets theory in this important subject for coal-mining.

In the thesis there is predicted the safe time for miners, during which no tremors occur, by means of fuzzy numbers. The prediction is based on the measurements results of the mining tremors which took place in „Pstrowski" coal-mine. The measurements were contucted with two methods: a seismological one and a seismoacoustic one. In the thesis there is introduced the concept of the degree of prediction likelihood, while using the comparison between fuzzy numbers. The calculations carried out on the IBM PC computer for the real tremors measurements in this coal-mine showed a great number of accurate predictions.

2. Fuzzy numbers type $\alpha-\beta$

We would like to start with the general definition of a fuzzy number and the definition of a fuzzy number type $\alpha-\beta$. Such fuzzy numbers will be applied to predict the occurence of strong mining tremors. Then we will present some definitions of basic operations on fuzzy numbers type $\alpha-\beta$. These operations will be used in the following chapters.

Definition 1 [1] *(defining a fuzzy number).* The fuzzy number L is named as a convex and standard fuzzy set from space R, for which:

a) there is only one $x_0 \in R$, for which $\mu_L(x_0) = 1$; x_0 is called mean value L,
b) function $\mu_L(x)$ is upper semi-continuous

Definition 2 *(positive number).* The fuzzy number L is positive if:

$$\mu_L(x) = 0 \text{ for } x \leq 0. \tag{1}$$

Definition 3 *(negative number).* The fuzzy number L is negative if:

$$\mu_L(x) = 0 \text{ for } x \geq 0. \tag{2}$$

As defining of arithmetic operators and their use in practical application is very uncomfortable for numbers defined in such a way, we introduced fuzzy numbers with membership functions assigned in advance. They are numbers L-R (fig. 2) introduced by Dubois and Prade mentioned in the work [6] or numbers $\alpha-\beta$ discussed in the work [1]. We are going to concentrate on numbers $\alpha-\beta$, because they are convenient in application and often put into use. The number is represented by the quadruple of real numbers (a, b, α, β). Numbers a and b determine the interval in which a membership function reaches value 1. Numbers α and β determine the width of distribution on the left and right. The membership function $\mu_L(x)$ is defined as follows:

$$\mu_L(x) = \begin{cases} (1/a)(x-a+a) & \text{for} \quad x \in [a-a, a), \\ 1 & \text{for} \quad x \in [a, b], \\ (1/b)(b+b-x) & \text{for} \quad x \in (b, b+b], \\ 0 & \text{for} \quad x \notin [a-a, b+b]. \end{cases} \tag{3}$$

An example of the membership function $\mu_L(x)$ for the fuzzy number L is presented in the figure 1.

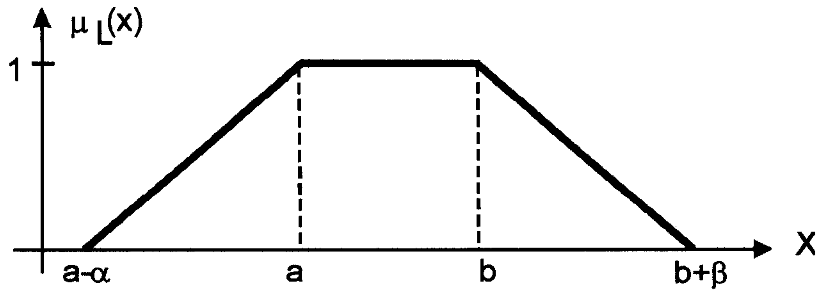

Fig. 1. Graph of example of membership function of fuzzy number α–β

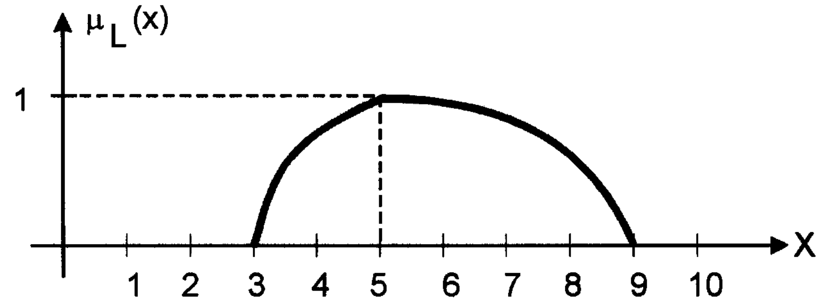

Fig. 2. The membership function of fuzzy number L-R (5,2,4)

***Definition 4** (fuzzy number with the minus sign)*. Fuzzy number with the minus sign is defined as follows:

$$-L = (-b, -a, \beta, \alpha) \tag{4}$$

***Definition 5** (inverse of fuzzy number)*. Inverse number is defined as follows:

$$1/L = (1/b,\ 1/a,\ \beta/(b(b+\beta)),\ \alpha/(a(a-\alpha))) \qquad \text{(for } L>0 \text{ or } L<0) \tag{5}$$

***Definition 6** (the sum of fuzzy numbers)*. Let us assume $L_1 = (a_1, b_1, \alpha_1, \beta_1)$ and $L_2 = (a_2, b_2, \alpha_2, \beta_2)$. The sum of fuzzy numbers is defined as follows:

$$L_1+L_2 = (a_1+a_2,\ b_1+b_2,\ \alpha_1+\alpha_2, \beta_1+\beta_2) \tag{6}$$

***Definition 7** (the difference of fuzzy numbers)*. The difference of fuzzy numbers is defined as follows:

$$L_1-L_2 = (a_1-b_2,\ b_1-a_2,\ \alpha_1+\beta_2,\ \alpha_2+\beta_1). \tag{7}$$

***Definition 8** (the product of fuzzy numbers)*. The product of fuzzy numbers is defined as follows:

$$L_1 \cdot L_2 = \begin{cases} (a_1a_2, b_1b_2, a_1\alpha_2 + a_2\alpha_1 - \alpha_1\alpha_2, b_1\beta_2 + b_2\beta_1 + \beta_1\beta_2) & \text{for} \quad L_1 > 0, L_2 > 0, \\ (a_1b_2, b_1a_2, b_2\alpha_1 - a_1\beta_2 + \alpha_1\beta_2, -b_1\alpha_2 + a_2\beta_1 - \beta_1\alpha_2) & \text{for} \quad L_1 < 0, L_2 > 0, \\ (b_1a_2, a_1b_2, b_1\alpha_2 - a_2\beta_1 + \beta_1\alpha_2, -b_2\alpha_1 + a_2\beta_2 - \alpha_1\beta_2) & \text{for} \quad L_1 > 0, L_2 < 0, \\ (b_1b_2, a_1a_2, -b_1\beta_2 - b_2\beta_1 - \beta_1\beta_2, -a_1\alpha_2 - a_2\alpha_1 + \alpha_1\alpha_2) & \text{for} \quad L_1 < 0, L_2 < 0. \end{cases} \tag{8}$$

Definition 9 (the quotient of fuzzy numbers). The quotient of fuzzy numbers is defined as follows:

$$\frac{L_1}{L_2}=\begin{cases}(a_1/b_2, b_1/a_2, (a_1\beta_2+b_2\alpha_1)/(b_2(b_2+\beta_2)), (b_1\alpha_2+a_2\beta_1)/(a_2(a_2-\alpha_2))) \text{ for } L_1>0, L_2>0,\\ (a_1/a_2, b_1/b_2, (a_2\alpha_1-a_2\alpha_2)/(a_2(a_2-\alpha_2)), (b_2\beta_1-b_1\beta_2)/(b_2(b_2+\beta_2))) \text{ for } L_1<0, L_2>0,\\ (b_1/b_2, a_1/a_2, (b_1\beta_2-b_2\beta_1)/(b_2(b_2+\beta_2)), (a_1\alpha_2-a_2\alpha_1)/(a_1(a_1-\alpha_1))) \text{ for } L_1>0, L_2<0,\\ (b_1/a_2, a_1/b_2, (-b_1\alpha_2-a_2\beta_1)/(a_2(a_2-\alpha_2)), (-a_1\beta_2-b_2\alpha_1)/(b_2(b_2+\beta_2))) \text{ for } L_1<0, L_2<0.\end{cases} \tag{9}$$

3. The mathematical model of tremor risk

In our considerations we are going to use the theory presented in the works [8], [9]. In these works it is assumed that the mathematical model of tremor risk for underground objects is the two-state Markov stochastic process. After suitable transformations the solution of the presented system of equations by Kolmogorov looks as follows:

$$P_0(t)=\frac{\tau}{\tau+\vartheta}+\frac{\vartheta}{\tau+\vartheta}\exp[-(\frac{1}{\tau}+\frac{1}{\vartheta})t], \quad P_1(t)=\frac{\vartheta}{\tau+\vartheta}[1-\exp(-(\frac{1}{\tau}+\frac{1}{\vartheta})t)]. \tag{10}$$

An interesting function is $P_1(t)$, which is interpreted as „the time probability of remaining of a rock mass in an active state and its ability to generate strong tremors" [8]. Function $P_0(t)$ characterizes the time probability in which a rock mass calms down. There occurs a relationship:

$$P_0(t)+P_1(t)=1. \tag{11}$$

In the formulas the following signs mean: „τ - the mean time between successive comparable strong mining tremors characterizing the instantaneous ability of a rock mass to generate them, ϑ - the time which characterizes the reversion of a rock mass to the state of relative equilibrium, t - the observation time of comparable dangerous mining tremors counted from t_0" [8].

The aim of the prediction was to determine the three quantities T_1, T_2, P_1 based on the formulas:

$$T_1=\frac{\tau\vartheta}{\tau+\vartheta}, \qquad T_2=\tau, \qquad P_1=\frac{\vartheta}{\tau+\vartheta}, \tag{12}$$

where:

„T_1 — the expected safe time, T_2-T_1 — the expected dangerous time, P_1 the — stationary probability"[8].

The values in the first four columns of tables 1 and 2 were the basis of the measurements. The time values TAU (τ) and TETA (ϑ) are prepared by the coal-mine crump section or they can be read from the interpretation of a seismoacoustic description [8]. Prediction effectiveness is determined by the accuracy of the relationship T_1<TAU. If this relationship is accurate, the predication is good. Otherwise the prediction is wrong. The interpretation of this inequality means

that no strong tremor should appear before the expiration of the predicated safe time T_1. A tremor appears during the time TAU and when TAU<T_1, the prediction did not verify (it is wrong). In the work [8] the calculation of T_1 was based on the values TAUŚR and TETAŚR, which mean the arithmetic means of the three previous values TAU and TETA.

Using fuzzy numbers and on the basis of measurements we are going to define the quantity τ_{roz} as a fuzzy number which characterizes a rock mass in relation to the mean time between successive strong mining tremors [7]. On the basis of measurements we will also define the fuzzy number ϑ_{roz}, which characterizes the reversion time of a rock mass to the relative equilibrium state. For a notation we will use the fuzzy numbers $\alpha - \beta$, which were described in chapter 2. So we define the fuzzy numbers τ_{roz} and ϑ_{roz} as follows [7]:

$$\tau_{roz} = (a_\tau, b_\tau, \alpha_\tau, \beta_\tau), \qquad \vartheta_{roz} = (a_\vartheta, b_\vartheta, \alpha_\vartheta, \beta_\vartheta). \tag{13}$$

The observation results of the seismological and seismoacoustic activities in wall 239, bed 620 in „Pstrowski" coal-mine were presented as histograms and diagrams [8]. The width of a class interval in the distributive series, on the basis of which histograms and diagrams were created, amounted to 8 hours. Therefore in our calculations we will assume for fuzzy numbers the following parameters $\alpha_\tau = \beta_\tau = \alpha_\vartheta = \beta_\vartheta = 8$. As we are interested in mean values τ_{sr} and ϑ_{sr}, so other parameters for fuzzy numbers τ_{roz} and ϑ_{roz} are assumed as $a_\tau = b_\tau = \tau_{sr}$, $a_\vartheta = b_\vartheta = \vartheta_{sr}$. Thanks to it we are going to get a simplified form of fuzzy numbers, whose membership function has got a triangle shape, not a trapezoid one. After these remarks fuzzy numbers can be presented as:

$$\tau_{roz} = (\tau_{sr}, \alpha_\tau, \beta_\tau), \qquad \vartheta_{roz} = (\vartheta_{sr}, \alpha_\vartheta, \beta_\vartheta). \tag{14}$$

The membership functions for these numbers are presented in figures 3 and 4.

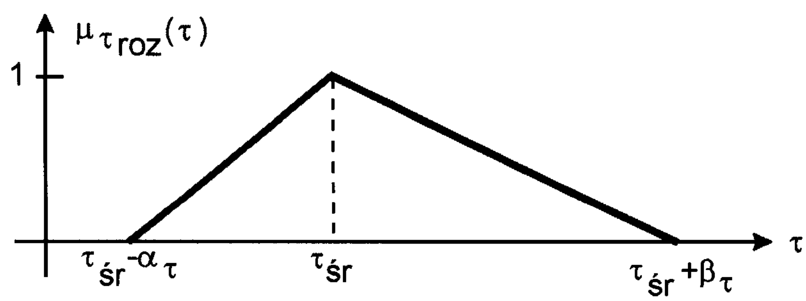

Fig. 3. Graph of membership function of fuzzy number τ_{roz}

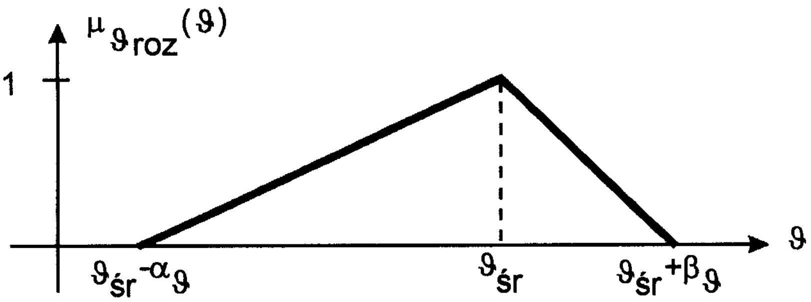

Fig. 4. Graph of membership function of fuzzy number ϑ_{roz}

Based on these fuzzy numbers we will define fuzzy number T_{1roz},, which will be characteristic for a given rock mass and will determine the safe time.

4. Defining fuzzy number T_{1roz}

To determine fuzzy number T_{1roz} we will use the formula (12):

$$T_{1roz} = \frac{\tau_{roz}\vartheta_{roz}}{\tau_{roz} + \vartheta_{roz}}. \tag{15}$$

For the calculation of T_{1roz} we will use the following operations on fuzzy numbers: the sum of fuzzy numbers, the product of fuzzy numbers and the quotient of fuzzy numbers.

Using formulas (3), (5), (6) and with assumptions (14) we get:

$$\tau_{roz} + \vartheta_{roz} = (\tau_{sr} + \vartheta_{sr}, \alpha_\tau + \alpha_\vartheta, \beta_\tau + \beta_\vartheta), \tag{16}$$

$$\tau_{roz} \cdot \vartheta_{roz} = (\tau_{sr}\vartheta_{sr}, \tau_{sr}\alpha_\vartheta + \vartheta_{sr}\alpha_\tau - \alpha_\tau\alpha_\vartheta, \tau_{sr}\beta_\vartheta + \vartheta_{sr}\beta_\tau + \beta_\tau\beta_\vartheta). \tag{17}$$

The division of two fuzzy numbers L=(a_l, α_l, β_l), and M=(a_m, α_m, β_m) is defined by the formula:

$$L/M = (a_l / a_m, (a_l\beta_m + a_m\alpha_l)/(a_m(a_m + \beta_m)), (a_l\alpha_m + a_m\beta_l)/(a_m(a_m - \alpha_m)). \tag{18}$$

Assuming that L=$\tau_{roz} \cdot \vartheta_{roz}$ and M=$\tau_{roz} + \vartheta_{roz}$ as well as using formula (15), we get:

$$T_{1roz} = (T_{1sr}, \alpha_{T_1}, \beta_{T_1}) = L/M =$$

$$= (a_l / a_m, (a_l\beta_m + a_m\alpha_l)/(a_m(a_m + \beta_m)), (a_l\alpha_m + a_m\beta_l)/(a_m(a_m - \alpha_m)). \tag{19}$$

where: $a_l = \tau_{sr}\vartheta_{sr}$, $a_m = \tau_{sr} + \vartheta_{sr}$, $\alpha_l = \tau_{sr}\alpha_\vartheta + \vartheta_{sr}\alpha_\tau - \alpha_\tau\alpha_\vartheta$, $\alpha_m = \alpha_\tau + \alpha_\vartheta$,

$\beta_l = \tau_{sr}\beta_\vartheta + \vartheta_{sr}\beta_\tau + \beta_\tau\beta_\vartheta$, $\beta_m = \beta_\tau + \beta_\vartheta$.

As the time goes by we get new information, that means new τ and new ϑ. So we must correct the already determined τ_{sr} and ϑ_{sr} according to the formulas:

$$\text{new } \tau_{sr} = (i \cdot \tau_{sr} + \tau)/(i+1), \qquad \text{new } \vartheta_{sr} = (i \cdot \vartheta_{sr} + \vartheta)/(i+1), \tag{20}$$

where i means the number of considered quantities τ and ϑ.

In other words, if the following quantities τ and ϑ are put into sequence $\{\tau_i\}$ and $\{\vartheta_i\}$ and we define:

τ_i - i-position of an element in sequence $\{\tau_i\}$,

ϑ_i - i-position of an element in sequence $\{\vartheta_i\}$,

τ_{sr_i} - τ_{sr} calculated out of "i" elements in sequence $\{\tau_i\}$,

ϑ_{sr_i} - ϑ_{sr} calculated out of "i" elements in sequence $\{\vartheta_i\}$,

then:

$$\tau_{sr_{i+1}} = (i \cdot \tau_{sr_i} + \tau_{i+1})/(i+1), \qquad \vartheta_{sr_{i+1}} = (i \cdot \vartheta_{sr_i} + \vartheta_{i+1})/(i+1). \tag{21}$$

The aim of the prediction is to determine quantity T_{1roz} on the basis of formula (19). This is the safe time during which no strong tremor should appear. If a strong tremor appears in time $\tau > T_{1roz}$, the prediction is accurate. If time τ of any following tremor is smaller than T_{1roz}, the prediction is wrong.

5. Defining the likelihood degree of a prediction

We deal here with the comparison between two numbers: a real one τ and a fuzzy one T_{1roz}. Generally, the comparison between fuzzy numbers is not as obvious as it seems to be. First we will determine the degree in which one fuzzy number is greater than the other one.

Definition 10 [6]. If we have two fuzzy numbers $A, B \subseteq R$, the degree in which fuzzy number A is greater than fuzzy number B is defined as:

$$v(A > B) = \max_{y>x;\ x,y \in R} (\mu_A(x) \wedge \mu_B(y)). \tag{22}$$

The degree $v(A > B)$ is graphically illustrated in Fig. 5. If the index is close to one, it is more difficult to answer the question whether A is greater than B. It can also mean that A is very close to B [6]. In our specific case, when we compare real number τ with fuzzy number T_{1roz} whose membership function has got a triangle shape, this problem does not occur. The value of index $v(\tau > T_{1roz})$ is specified as the value of the ordinate in the graph $\mu_{T1roz}(x)$ in the point $x = \tau$. It is presented in Fig. 6. In order to prove the prediction accuracy number τ is compared with real number T_{1sr} (as it was described in chapters 3 and 4). When membership functions of two fuzzy numbers $\mu_A(x)$ and $\mu_B(x)$ are disjoint (they have not got common domain of determinacy), then degree $v(A>B)=0$. In our case we want the difference between τ and T_{1sr} to be great, so that degree $v(\tau>T_{1roz})$ should equal zero or be close to zero. Then we achieve a better likelihood of a prediction. If $T_{1\acute{s}r}>\tau$, the prediction is wrong and $v(T_{1roz}>\tau)$ determines the degree in which T_{1roz} is greater than τ. Now we are going to define the factor of prediction reliability.

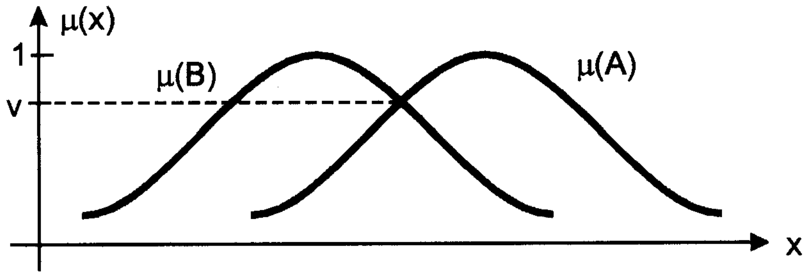

Fig. 5. Comparison of fuzzy numbers.

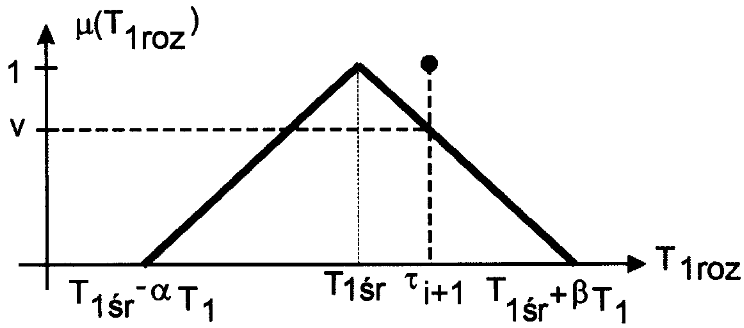

Fig. 6. Determining the accuracy of prediction (prediction good).

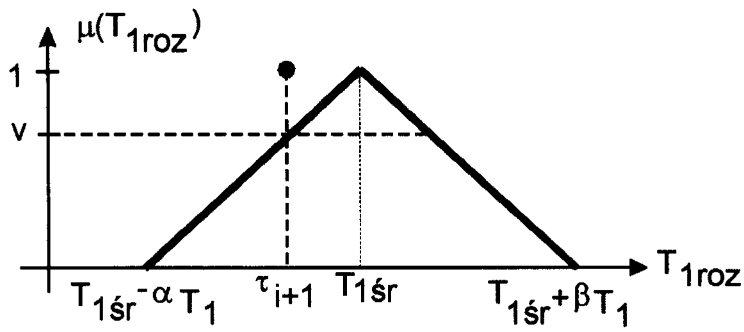

Fig. 7. Determining the accuracy of prediction (prediction wrong).

Definition 11. The factor of prediction reliability is determined by the formula:

$$W = 1 - v(\tau > T_{1roz}),\ \text{when}\ \tau > T_{1\acute{s}r}\ \text{or}\ W = 1 - v(T_{1roz} > \tau),\ \text{when}\ \tau < T_{1\acute{s}r} \quad (23)$$

Thanks to it we achieved some degree of prediction reliability. We can assign the above defined factor W both to good and wrong predictions and estimate how reliable the prediction is.

To each prediction we additionally assigned factor W which determines the degree of prediction reliability. Number P_1 characterizes the activity and susceptibility of a rock mass to generate strong tremors. This number as probability is from range [0, 1]. On the basis of this number we can estimate in percentages the safety of coal miners working directly on a given rock mass.

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