The Basic Properties of Fuzzy Derivative (I)

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The fuzzy differentials theory is a very important and difficult subject in fuzzy mathematics. Now, we use caratheodory's derivative notion to define the fuzzy derivative. At the same time, we also prove a few basic properties of fuzzy derivative which generalizes the usually derivative theory.

Definition 1. Let E be a vector space over the field K of real or complex numbers, (E, T) be a fuzzy topological space, if the two mappings

(i) $\sigma : E \times E \to E, (x, y) \to x + y$

(ii) $\pi : K \times E \to E, (\alpha, x) \to \alpha x$

where K is the induced fuzzy topology of the usual norm, are fuzzy continuous. Then (E, T) is said to be a fuzzy topological vector space over the field K.

Definition 2 (Caratheodory). Let $f : (a, b) \subseteq R \to R, c \in$

(a, b), the function f is said to be differentiable at the point $c \in (a, b)$ if there exists a function ϕ_c that is continuous at $x = c$ and satisfies the relation

$$f(x) - f(c) = \phi_c(x)(x - c)$$

for all $x \in (a, b)$.

Definition 3. Let R be the field of real numbers and (R, T) be a fuzzy topological vector space over the field $R.f : R \to R, a \in R$, the function f is said to be fuzzy differentiable at the point a if there is a function ϕ that is fuzzy continuous at the point a, and have

$$f(x) - f(a) = \phi(x)(x - a)$$

for all $x \in R$. $\phi(a)$ is said to be the fuzzy derivative of f at a and denote $f'(a) = \phi(a)$.

Theorem 1. If f is fuzzy differentiable at the point a, then f is fuzzy continuous at the point a.

Theorem 2 (Chain Rule). If f is fuzzy differentiable at the point a and g is fuzzy differentiable at the point $f(a)$, then h = gof is also fuzzy differentiable at the point a and

$$h'(a) = g'(f(a))f'(a).$$

Theorem 3 (Mean Valued Theorem). If f is fuzzy continuous on $[a, b]$ and fuzzy differentiable on (a, b), then there exists $c \in (a, b)$ such that $f(b) - f(a) = f'(c)(b - a)$.

Theroem 4 (Critical Pooint Therrem). If f is fuzzy differentiable at the point a and $f(a)$ is an extreme value, then a is a critical point $(i.e., f'(a) = 0)$.

Theorem 5 (Inverse Function Theorem). Let f be fuzzy continuous and strictly monotonic on R and f be fuzzy differentiable at the point a, if $f'(a) \neq 0$, then $g = f^{-1}$ is fuzzy differentiable at the point $d = f(a)$ and $g'(d) = [f'(a)]^{-1}$.

Theorem 6. If f and g are fuzzy differentiable at a, k is a constant ,then

(1) $(kf + g)'(a) = kf'(a) + g'(a)$,

(2) $(fg)'(a) = f(a)g'(a) + g(a)f'(a)$.

Theorem 7. If f and g are fuzzy differentiable at a, $g(a) \neq 0$, then

$$(\frac{f}{g})'(a) = \frac{f'(a)g(a) - g'(a)f(a)}{g^2(a)}.$$

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