Fuzzy rings with operators

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Abstrct: In this paper, we further study the theory of the fuzzy ring and give some new concepts such as fuzzy ring with operators, fuzzy ideal with operators, fuzzy quotient ring with operators, etc. while their some elmentary proerties are discussed.

Keywords: Fuzzy ring; Fuzzy ring with operators; Fuzzy ideal with operators; Fuzzy quotient ring with operators; Homomorphism

1. Introduction

In 1982 W. J. Liu [1] introduced the concept of fuzzy ring. In 1985 Ren [2] studied the fuzzy ideal and fuzzy quotient ring. In this paper we further study the theory of the fuzzy ring and give some new concepts such as fuzzy ring with operators, fuzzy ideal with operators, fuzzy quotient ring with operators, etc. while their some elementary properties are discussed, some results in reference [1—2] are extended.

2. Preliminaries

For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1 (**Liu**[1]). Let X be a ring, A be a fuzzy set of X, A will be called a fuzzy ring of X, if

(1) $A(x-y) \geqslant A(x) \wedge A(y)$ for all x, y in G;

(2) $A(xy) \geqslant A(x) \wedge A(y)$ for all x, y in G.

Definition 2.2 (**Liu** [1]). Let X be a ring, a fuzzy ring A will be is an M-fuzzy ring of X iff for any $t \in [0,1]$, A_t is an M-subring of X when $A_t \neq \emptyset$.

Proposition 3. 3. Let X and X' both be M-ring and f an M-homomor phism from X onto X'. If A' is an M-fuzzy ring of X', then $f^{-1}(A')$ is an M-fuzzy ring of X.

Proposition 3. 4. Let X and X' both be M-ring, f an M-homomorphism from X into X', and A an M-fuzzy ring of X, then $f(A)$ is an M-fuzzy ring of X'.

4. *M*-fuzzy ideals

Definition 4. 1. Let X be an M-ring, A is said to be an M-fuzzy ideal of X if A is not only an M-fuzzy ring of X, but also a fuzzy ideal of X.

Proposition 4. 1. Let X be an M-ring, A and B both be M-fuzzy ideal of X. Then $A \cap B$ is an M-fuzzy ideal of X.

Proof. It is easy to know by propositions 3. 1 $A \cap B$ is an M-fuzzy ring of X, by lemma1 in [3], $A \cap B$ is an M-fuzzy ideal of X, hence $A \cap B$ is an M-fuzzy ideal of X.

Proposition 4. 2. Let X be an M-ring, A be a fuzzy set of X, then A is an M-fuzzy ideal of X iff for any $t \in [0,1]$, A_t is an M-ideal of X when $A_t \neq \emptyset$.

Proposition 4. 3. Let f be an M-homomorphism from the M-ring X to the M-ring X'. Then the preimage which can be written as $f^{-1}(A')$ of A' under f where A' is an M-fuzzy ideal of X' is an M-fuzzy ideal of X.

Proposition 4. 4. Let f be an M-homomorphim from the M-ring X to the M-ring X'. Then the imge which can be written as $f(A)$ of A under f is an M-fuzzy ideal in case of A being an M-fuzzy ideal of X.

5. *M*-fuzzy quotient ring

Let X be a M-ring, A an M-fuzzy ideal of X. Ren [2] had proved that X/A was a ring.

Proposition 5.1. X/A is an M-ring.

Proof. For all m in M, $x+A$, $y+A$ in X/A, we define

$$m(x+A)=mx+A$$

It is easy to prove the above definiens is reasonable.

Because

$$\begin{aligned} m((x+A)+(y+A)) &= m((x+y)+A) \\ &= m(x+y)+A \\ &= mx+my+A \\ &= (mx+A)+(my+A) \\ &= m(x+A)+m(y+A) \end{aligned}$$

$$\begin{aligned} m((x+A)(y+A)) &= m((xy)+A) \\ &= m(xy)+A \\ &= (mx)y+A \\ &= (mx+A)(y+A) \\ &= (m(x+A))(y+A) \\ &= (x+A)(m(y+A)) \end{aligned}$$

so

X/A is an M-ring.

Definition 5.1. The above M-ring X/A is called the M-fuzzy quotient ring of X with respect to A.

Now we define a fuzzy set on X/A. Let B be any M-fuzzy ring of X, B/A be a fuzzy set of X/A defined as follows:

$B/A : X/A \to [0,1]$ satisfying

$$B/A\ (a+A)=\sup_{x+A=a+A} B(x)$$

for all $a+A$ in X/A.

Proposition 5.2. The above fuzzy subset B/A is an M-fuzzy ring of X/A.

Definition 5.2. The above fuzzy ring B/A is called the M-fuzzy factor ring of B with respect to A.

Proposition 5.3. Let X be an M-ring, A is an M-fuzzy ideal of X.

called a fuzzy ideal of X, if $A(xy) \geqslant A(x) \vee A(y)$ for all x, y in X.

Definition 2.3 (**Xiong** [5]) Let X be a ring, M be a sets, if

(1) $ma \in X$ for all a in X, m in M;

(2) $m(a+b) = ma + mb$ for all a, b in X, m in M;

(3) $m(ab) = (ma)b = a(mb)$ for all a, b in X, m in M.

Then m is said to be a left operator of X, M is said to be a left operator sets of X, X is said to be a ring with operators. We use the phrase "X is an M-ring" instead of a ring with operators. If a subring of M-ring X is also M-ring, then it is said to be a M-subring of X.

Definition 2.4 (**Xiong** [5]). Let X and X' both be M-ring. f be a homomorphism from X onto X'. If $f(mx) = mf(x)$ for all x in X, m in M, then f is called an M-homomorphism.

3. *M*-fuzzy ring

Definition 3.1. Let X be an M-ring and A be a fuzzy ring of X, if

$$A(mx) \geqslant A(x)$$

hold for any $x \in X$, $m \in M$, then A is said to be a fuzzy ring with operators of X. We use the phrase "A is an M-fuzzy ring of X" instead of a fuzzy ring with operators of X.

It is clear that Definition 3.1 is the generalization of the general M-ring.

Proposition 3.1. Let X be an M-ring, A and B both be M-fuzzy ring of X. Then $A \cap B$ is an M-fuzzy ring of X.

Proof. It is clear that $A \cap B$ is a fuzzy ring of X. For any $x \in X$, $m \in M$,

$$\begin{aligned}(A \cap B)(mx) &= A(mx) \wedge B(mx) \\ &\geqslant A(x) \wedge B(x) \\ &= (A \cap B)(x)\end{aligned}$$

Hence $A \cap B$ is an M-fuzzy ring of X.

Proposition 3.2. Let X be an M-ring, A be a fuzzy set of X, then A

Let

$$f: X \to X/A,$$
$$x \to x + A$$

Then f is an M-homomorphism from X onto X/A.

References

[1] W.-J. Liu, Fuzzy invariant subgrups and fuzzy ideals, *Fuzzy Sets and Systems* 8(1982) 133—139.

[2] Y.-C. Ren, Fuzzy ideals and quotient rings, *Fuzzy Math* 4(1985) 19—26 (in chinese)

[3] T. K. Mvkherjee and M. K. Sen, on fuzzy ideals of a ring I, *Fuzzy Sets and Systems* 21(1987)99—104

[4] W. X. Gu, S. Y. Li and D. G. Chen, Fuzzy groups with operators, *Fuzzy Sets and Systems* 66(1994)363—371

[5] Q. Y. Xiong, Modern algebra, (Science and technology Publishment in Shang hai, 1978).