

MORTGAGE BANK MANAGEMENT DECISION-MAKING IN UNCERTAINTY

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Abstract

This paper deals with the fuzzy creation by a standard decision-making method analogy when the state of events are given as fuzzy sets. The proposed fuzzy model is applied to the economic system intended for the mortgage bonds issues amount decision-making. The order of fuzzy values is arranged while taking into account the risks following from financial decisions, especially considering the risk of the gained financial means unplacement, resp. the risk following from the decreasing marginal utility rule.

Keywords: Decision-making problem, extension principle, fuzzy state of events, risk.

1. Introduction

The decision-making support theory is widely applied in the economy, however, often facing a problem of too sharp input data not corresponding to the reality being modelled without rough distortion. One of potential alternatives how such insufficiency can be partially eliminated is to approach to the modelling in the fuzzy environment.

The aim of this paper is to set up a decision support model for mortgage bank management for mortgage bonds issue amounts for uncertain demands for mortgage loans in order to maximise the bank revenues.

In addition, a risk following from the decreasing marginal utility rule is quantified in the second part.

The stated system for a decision-making support is set up without taking into account the time development dynamics, i.e. immediate execution of all transactions without any delay is supposed, and the decision-making is based on the following year revenues from the executed actions.

2. The classic decision-making

The *classic* decision-making problem has the following characteristics [2]:

I. Predetermined values

- Ω Space of events (set of given situations, set of real numbers)
- numerable
 - not empty (included in R)

Ω_w State of events ($w = 1, 2, \dots, n$) (element of Ω)

- mutually incompatible
- not controllable by the decider

A Space of decision alternatives (set of alternatives)

- numerable
- not empty (included in R)

A_v Course of action ($v = 1, 2, \dots, m$), (element of A)

- mutually exclusive
- controllable by the decider

II. Space of consequences

(R, \leq) Space of consequences or of results (ordered set)

Result R_{vw} is constructed by a resulting function $c(A_v, \Omega_w) \in (R, \leq)$ for $\forall A_v, \forall \Omega_w$ (1)

III. *Decision matrix* is the form in which the decision model is expressed:

$$\begin{array}{c|cccccc}
 & \Omega_1 & \Omega_2 & \dots & \Omega_w & \dots & \Omega_n \\
 A_1 & R_{11} & R_{12} & \dots & R_{1w} & \dots & R_{1n} \\
 A_2 & R_{21} & R_{22} & \dots & R_{2w} & \dots & R_{2n} \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 A_v & R_{v1} & R_{v2} & \dots & R_{vw} & \dots & R_{vn} \\
 \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 A_m & R_{m1} & R_{m2} & \dots & R_{mw} & \dots & R_{mn}
 \end{array} \quad (2)$$

3. Decision-making for a space of events being a set of fuzzy number

Let $A = \{A_1, A_2, \dots, A_v, \dots, A_m\}$ be a set of decision alternatives (crisp numbers).

Let now the possible states of events be considered as a *fuzzy number*, e. g. $\tilde{\Omega}$ is a set of fuzzy numbers $\tilde{\Omega}_w$.

Let now it be $w = 1$, i. e. $\tilde{\Omega}_w = \tilde{\Omega}$.

A fuzzy number $\tilde{\Omega}$ is a special fuzzy subset of the real numbers [6]. Its *membership function* is defined by

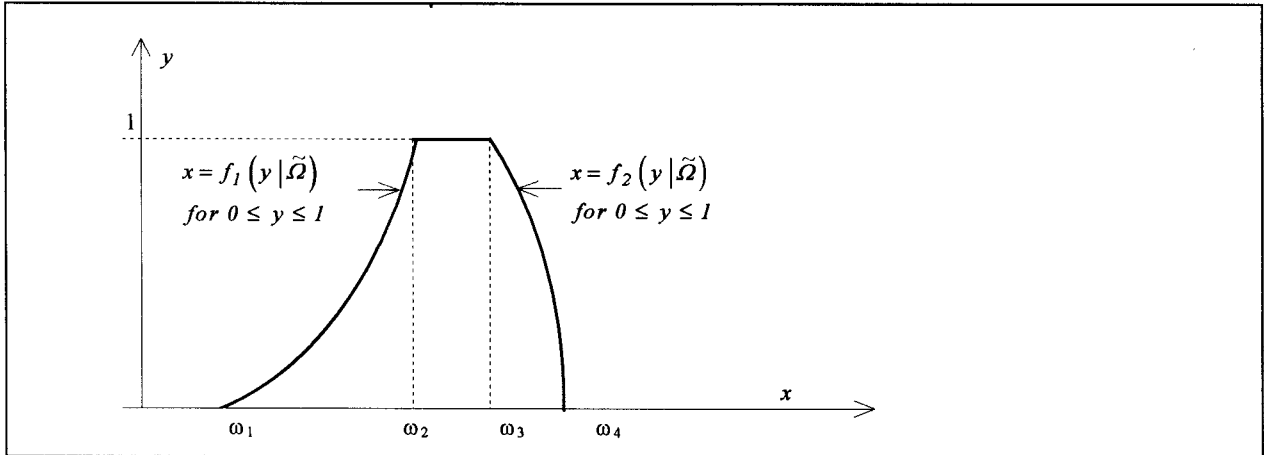
$$\mu(x | \tilde{\Omega}) = (\omega_1, f_1(y | \tilde{\Omega}) / \omega_2, \omega_3 / f_2(y | \tilde{\Omega}), \omega_4), \text{ where } \omega_1 < \omega_2 \leq \omega_3 < \omega_4, \quad (3)$$

$f_1(y | \tilde{\Omega})$ is a continuous increasing function of y for $0 \leq y \leq 1$

with $f_1(0 | \tilde{\Omega}) = \omega_1$ and $f_1(1 | \tilde{\Omega}) = \omega_2$,

$f_2(y | \tilde{\Omega})$ is a continuous decreasing function of y for $0 \leq y \leq 1$

with $f_2(0 | \tilde{\Omega}) = \omega_4$ and $f_2(1 | \tilde{\Omega}) = \omega_3$.



In order to discuss the graph of $\mu(x | \tilde{\Omega})$ let us assume that the x -axis is horizontal and the y -axis is vertical.

Then the graph of $y = \mu(x | \tilde{\Omega})$ is:

- (1) zero for $x \leq \omega_1$,
- (2) $x = f_1(y | \tilde{\Omega})$ for $0 \leq y \leq 1$,
- (3) one for $\omega_2 \leq x \leq \omega_3$,
- (4) $x = f_2(y | \tilde{\Omega})$ for $0 \leq y \leq 1$,
- (5) zero for $x \geq \omega_4$.

It is convenient to have x a function of y on $[\omega_1; \omega_2]$ and $[\omega_3; \omega_4]$. If it is eligible to have y a function of x , then it is possible to use the inverse functions $y = f_1^{-1}(x | \tilde{\Omega})$ for $\omega_1 \leq x \leq \omega_2$, and $y = f_2^{-1}(x | \tilde{\Omega})$ for $\omega_3 \leq x \leq \omega_4$.

Sometimes straight line segments for $\mu(x | \tilde{\Omega})$ on $[\omega_1; \omega_2]$ and $[\omega_3; \omega_4]$ are employed. In this case the membership function is denoted simply as $(\omega_1, \omega_2, \omega_3, \omega_4)$. (4)

If the state of events $\tilde{\Omega}$ is a fuzzy number, then the fuzzy result \tilde{R} must be modelled using the *extension principle* in (1). The space of results is an ordered set of *fuzzy numbers*:

$$(\tilde{R}; <) = \{\tilde{R}_{11}, \tilde{R}_{12}, \dots, \tilde{R}_{vw}, \dots, \tilde{R}_{mn}\}.$$

For each $\tilde{\Omega}_w$ ($w = 1, 2, \dots, n$) we obtained a fuzzy number \tilde{R}_v ($v = 1, 2, \dots, m$) which has a *membership function* defined by $\mu(x | \tilde{R}_v) = (r_{v1}, f_{v1}(y | \tilde{R}_v) / r_{v2}, r_{v3} / f_{v2}(y | \tilde{R}_v), r_{v4})$. (5)

As segments for membership function $\mu(x | \tilde{R}_v)$ is straight lines on $[r_{v1}; r_{v2}]$ and $[r_{v3}; r_{v4}]$, then

$$x = f_{v1}(y | \tilde{R}_v) = (r_{v2} - r_{v1}) \cdot y + r_{v1}, \quad x = f_{v2}(y | \tilde{R}_v) = (r_{v3} - r_{v4}) \cdot y + r_{v4}, \quad \text{for } 0 \leq y \leq 1. \quad (6)$$

If f_{vj} is the straight line for $j = 1, 2$ (\tilde{R}_v is a triangular fuzzy number or a flat fuzzy number), to organise obtained results $\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_v, \dots, \tilde{R}_m$ is proposed: the application of the semi-distance from the origin criterion if:

$$\tilde{R}_p < \tilde{R}_q \Leftrightarrow \frac{r_{p1} + r_{p2} + r_{p3} + r_{p4}}{4} < \frac{r_{q1} + r_{q2} + r_{q3} + r_{q4}}{4} \quad (7)$$

Let now be considered a continuous concave increasing function (*utility function*) u :

$$u(R) = U, \text{ where } R \text{ is a real number.} \quad (8)$$

Next the results \tilde{R} are fuzzy numbers and the utility \tilde{U} must be modelled using the *extension principle* in equitation (8).

If the function u is applied to a triangular or a flat fuzzy number \tilde{R} , the result \tilde{U} can be a fuzzy number which has lost their triangular or flat form, and which must be classified to obtain the order of preferences in another form. To organise the fuzzy number \tilde{U}_v with a membership function defined for $0 \leq y \leq 1$ by

$$\mu(x | \tilde{U}_v) = (u_{v1}, f_{v1}(y | \tilde{U}_v) / u_{v2}, u_{v3} / f_{v2}(y | \tilde{U}_v), u_{v4}) \quad (9)$$

it is suggested the following criterion (the criterion is suggested, for which the choice from many possibilities seems to be suitable after checking it on the economical applications given below):

the application of the centre of the gravity:

$$\tilde{U}_p < \tilde{U}_q \Leftrightarrow T_p < T_q, \text{ where } T_a = \frac{\int_0^1 [f_{a2}(y | \tilde{U}_v)^2 - f_{a1}(y | \tilde{U}_v)^2] dy}{2 \cdot \int_0^1 [f_{a2}(y | \tilde{U}_v) - f_{a1}(y | \tilde{U}_v)] dy} \quad \text{for } a = p, q \quad (10)$$

4. The mortgage bonds optimisation issues

The mortgage banks gain financial sources except others by placing and following sale of mortgage bonds issues on the capital market. A bank pays debit interests to the respective investors for those sources providing, and, therefore, its effort is aimed towards placing those sources without any time delay in such way that it provides mortgage loans to clients (credit interest). The revenues of a mortgage bank (in a simple way) are a difference between the credit and debit interests amount.

A risk of such transactions can be expected from several sides, e.g.

- a bank issues mortgage bonds in a larger volume than it then succeeds to contract mortgage loans,
- due to insufficient measures, it enables to repay mortgage loans in a shorter term, while it still has to pay the debit interests.

A mortgage bank management decision-making concerning a volume of mortgage bonds issues can be supported by quantitative methods, which, however, in their classical form face a fact the following demand for mortgage loans is only estimated, being uncertain. A possibility is offered to quantify such uncertainty in the fuzzy environment, thus gaining an efficient tool for decision-making support.

Let $A = \{A_1, A_2, \dots, A_v, \dots, A_m\}$ be a set of decision alternatives (crisp numbers). A_v represents the *decision of a banker* to issue mortgage bonds (e. g. to buy money) in v mil. CZK.

Let $\tilde{\Omega}$ is an expected *uncertainty demand after mortgage loans* (e.g. a *sale* of money by banker) in mil. CZK.

$\tilde{\Omega}$ is given as a flat fuzzy number (3).

Let i_c is a *credit interest rate* p.a. (crisp number) and i_d is a *debit interest rate* p.a. (crisp number).

If the state of events $\tilde{\Omega}$ is a fuzzy number, then the fuzzy revenues \tilde{R} of a bank in the course of A_v must be modelled using the *extension principle* in equitation (1). The membership function for $\tilde{R}_v = (r_{v1}, r_{v2}, r_{v3}, r_{v4})$ is determined by

$$\begin{aligned} r_{v1} &= f_{v1}(0 | \tilde{R}_v) = i_c \cdot \min(A_v, f_1(0 | \tilde{\Omega})) - i_d \cdot A_v, & r_{v2} &= f_{v1}(1 | \tilde{R}_v) = i_c \cdot \min(A_v, f_1(1 | \tilde{\Omega})) - i_d \cdot A_v \\ r_{v3} &= f_{v2}(1 | \tilde{R}_v) = i_c \cdot \min(A_v, f_2(1 | \tilde{\Omega})) - i_d \cdot A_v, & r_{v4} &= f_{v2}(0 | \tilde{R}_v) = i_c \cdot \min(A_v, f_2(0 | \tilde{\Omega})) - i_d \cdot A_v \end{aligned} \quad (11)$$

To organise the obtained fuzzy number \tilde{R}_v ($v = 1, 2, \dots, m$) the manager can use (7).

A banker decision-making is influenced by the decreasing marginal utility rule from the extent and thus related risk quantification in dependence on increasing investment amounts. This fact can be taken into account by applying the *utility function*. The *real variable* utility function is degressively increasing, i.e. the u function itself is increasing, however, its marginal function is decreasing. Several types of functions of required properties can be chosen, such as an increasing section of a concave quadratic function, logarithmic function, rational fraction function, etc. Let a required increasing concave function of a real variable is a function

$$u(R) = 2 \cdot R - R^2, \text{ for } R \in [0; 1]. \quad (12)$$

If the revenues \tilde{R} is a fuzzy number, then the utility \tilde{U} must be modelled using the *extension principle* in equitation (12):

$$u(\tilde{R}_v) = 2 \cdot \tilde{R}_v \oplus \left[-(\tilde{R}_v)^2 \right] \quad (13)$$

$$\text{for } \tilde{R}_v \text{ such that } \min_{v=1, \dots, m} (r_{v1}) \geq 0, \max_{v=1, \dots, m} (r_{v4}) \leq 1, \quad (14)$$

where a membership function $\mu(x | \tilde{R}_v)$ is given by the equitation (6) for $v = 1, 2, \dots, m$. In this paper the standard arithmetics of fuzzy numbers are used (the addition of fuzzy numbers is \oplus).

The membership function for \tilde{U}_v (9) is determined by

$$\begin{aligned} f_{vj}(y | \tilde{U}_v) &= 2 \cdot f_{vj}(y | \tilde{R}_v) - f_{v(3-j)}^2(y | \tilde{R}_v) \\ \text{for } j &= 1, 2; 0 \leq y \leq 1 \text{ and} \\ f_{v1}(0 | \tilde{U}_v) &= u_{v1}, f_{v1}(1 | \tilde{U}_v) = u_{v2}, f_{v2}(0 | \tilde{U}_v) = u_{v4}, f_{v2}(1 | \tilde{U}_v) = u_{v3}. \end{aligned} \quad (15)$$

To organise the obtained fuzzy number $\tilde{U}_v \in \tilde{U}$ ($v = 1, 2, \dots, m$) the manager can use (10).

5. Examples

Example 1: Modelling of risk following from changes of difference of credit and debit interest rates.

Example 1a:

- * Let a set of decision alternatives be $A = \{A_0, A_{50}, A_{100}, A_{150}, A_{200}, A_{250}, A_{300}\}$, where A_v represents the decision of banker to issue mortgage bonds on the capital market in v mil. CZK.
- * Let an expected uncertainty demand after mortgage loans be given as a flat fuzzy number $\tilde{\Omega} = (0, 150, 200, 300)$ in mil. CZK.
- * Let the credit interest rate be $i_c = 12\%$ p.a. and the debit interest rate $i_d = 8\%$ p.a., i. e. $i_c - i_d = 4\%$ p.a.

Provided a banker makes a decision to issue mortgage bonds in a value of v mil. of CZK, then the revenues of the bank for such decisions are given according to (11) by following values:

$$\tilde{R}_0 = (0.12 \cdot \min(0; 0) - 0.08 \cdot 0; 0.12 \cdot \min(0; 150) - 0.08 \cdot 0; 0.12 \cdot \min(0; 200) - 0.08 \cdot 0; 0.12 \cdot \min(0; 300) - 0.08 \cdot 0)$$

$$\tilde{R}_{50} = (0.12 \cdot \min(50; 0) - 0.08 \cdot 50; 0.12 \cdot \min(50; 150) - 0.08 \cdot 50; 0.12 \cdot \min(50; 200) - 0.08 \cdot 50; 0.12 \cdot \min(50; 300) - 0.08 \cdot 50)$$

$$\tilde{R}_{100} = (0.12 \cdot \min(100; 0) - 0.08 \cdot 100; 0.12 \cdot \min(100; 150) - 0.08 \cdot 100; 0.12 \cdot \min(100; 200) - 0.08 \cdot 100; 0.12 \cdot \min(100; 300) - 0.08 \cdot 100)$$

...

$$\tilde{R}_{300} = (0.12 \cdot \min(300; 0) - 0.08 \cdot 300; 0.12 \cdot \min(300; 150) - 0.08 \cdot 300; 0.12 \cdot \min(300; 200) - 0.08 \cdot 300; 0.12 \cdot \min(300; 300) - 0.08 \cdot 300)$$

According to (7) a representative value can be created for each fuzzy number according to which particular decisions can be ordered based on the resulting bank revenues. For a graphic depiction of the revenues related to particular decisions it is suitable to norm the values (the highest revenue amount would be evaluated as a number one, a zero revenue as a zero, losses would be attributed by corresponding negative values). The results are written in the following table and depicted in the graph.

Table 1: The order of alternatives:

$$\tilde{R}_{150} \succ \tilde{R}_{100} \succ \tilde{R}_{200} = \tilde{R}_{50} \succ \tilde{R}_0 \succ \tilde{R}_{250} \succ \tilde{R}_{300}$$

Stocks	Results \tilde{R}_v	Representants	Values of representants
A_0	(0;0;0;0)	0	0
A_{50}	(-4;2;2;2)	0.5	1/3
A_{100}	(-8;4;4;4)	1.0	2/3
A_{150}	(-12;6;6;6)	1.5	1
A_{200}	(-16;2;8;8)	0.5	1/3
A_{250}	(-20;-2;4;10)	-2	-4/3
A_{300}	(-24;-6;0;12)	-4.5	-3

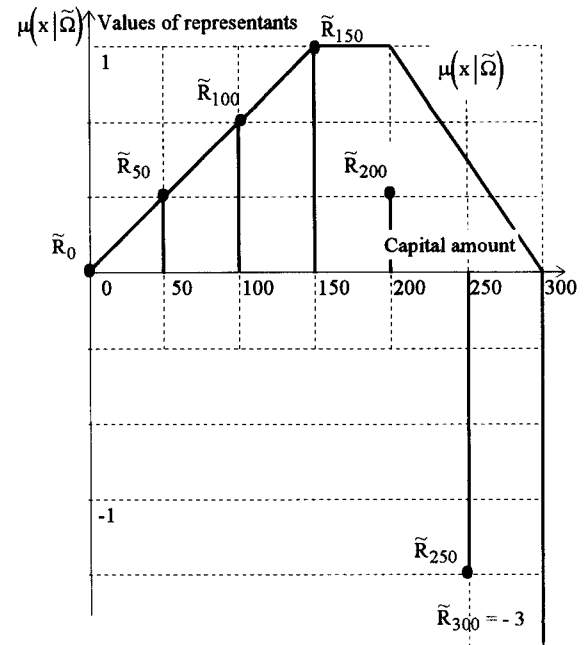


Fig. 1: The order of alternatives for $i_c - i_d = 4\%$ p.a.

$$\tilde{R}_{150} \succ \tilde{R}_{100} \succ \tilde{R}_{200} = \tilde{R}_{50} \succ \tilde{R}_0 \succ \tilde{R}_{250} \succ \tilde{R}_{300}$$

Evaluation of example 1a:

An estimated uncertain demand was chosen as very uncrisp, potential decision alternatives copied all the expected demand values. A difference between the credit and debit interest rates was chosen as medium. The alternatives order with positive revenues corresponds with the demand uncrisp, however, a risk of a too high number of mortgage bonds issue proved to be too high in relation to determined losses.

Example 1b.

* Let a set of decision alternatives be the same $A = \{A_0, A_{50}, A_{100}, A_{150}, A_{200}, A_{250}, A_{300}\}$.

* Let a flat fuzzy number be the same $\tilde{\Omega} = (0, 150, 200, 300)$.

* Let $i_c = 12\%$ p.a. and let i_d be a different one: $i_d = 5\%$ p.a., i. e. $i_c - i_d = 7\%$ p.a.

The values of revenues $\tilde{R}_0, \tilde{R}_{50}, \dots, \tilde{R}_{300}$ are numerated in the same way as in the example 1a.

Table 2: The order of alternatives:

$$\tilde{R}_{200} \succ \tilde{R}_{150} \succ \tilde{R}_{250} \succ \tilde{R}_{300} \succ \tilde{R}_{100} \succ \tilde{R}_{50} \succ \tilde{R}_0$$

Stocks	Results \tilde{R}_v	Representants	Values of representants
A_0	(0;0;0;0)	0	0
A_{50}	(-2.5;3.5;3.5;3.5)	2	0.3077
A_{100}	(-5;7;7;7)	4	0.6154
A_{150}	(-7.5;10.5;10.5;10.5)	6	0.9231

A_{200}	(-10;8;14;14)	6.5	1.0
A_{250}	(-12.5;5.5;11.5;17.5)	5.5	0.8462
A_{300}	(-15;3;9;21)	4.5	0.6923

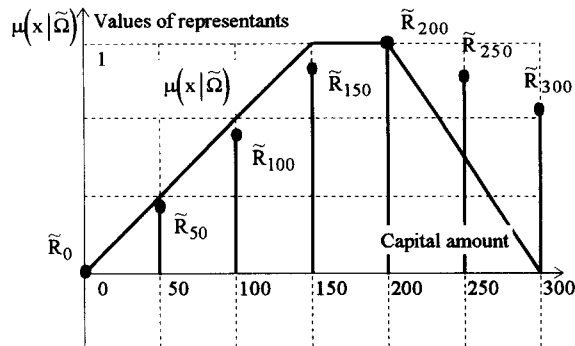


Fig. 2: Order of alternatives for $i_c - i_d = 7\%$ p.a.
 $\tilde{R}_{200} > \tilde{R}_{150} > \tilde{R}_{250} > \tilde{R}_{300} > \tilde{R}_{100} > \tilde{R}_{50} > \tilde{R}_0$

Evaluation of example 1b:

Due to increasing the difference between the credit and debit interest rates all alternatives became profitable, their order being changed so that they moved to higher issues amounts.

Example 1c.

* Let a set of decision alternatives be the same $A = \{A_0, A_{50}, A_{100}, A_{150}, A_{200}, A_{250}, A_{300}\}$.

* Let a flat fuzzy number be the same $\tilde{\Omega} = (0, 150, 200, 300)$.

* Let $i_c = 12\%$ p.a. and let i_d be a different one: $i_d = 10\%$ p.a., i. e. $i_c - i_d = 2\%$ p.a..

The values of revenues $\tilde{R}_0, \tilde{R}_{50}, \dots, \tilde{R}_{300}$ are numerated in the same way as in the example 1a.

Table 3: The order of alternatives (all of them are lossmaking):

$$\tilde{R}_0 > \tilde{R}_{50} > \tilde{R}_{100} > \tilde{R}_{150} > \tilde{R}_{200} > \tilde{R}_{250} > \tilde{R}_{300}$$

Stocks	Results \tilde{R}_v	Representants	Values of representants
A_0	(0;0;0;0)	0	0
A_{50}	(-5;1;1;1)	-0.5	-1/3
A_{100}	(-10;2;2;2)	-1.0	-2/3
A_{150}	(-15;3;3;3)	-1.5	-1
A_{200}	(-20;-2;4;4)	-3.5	-7/3
A_{250}	(-25;-7;-1;5)	-7	-14/3
A_{300}	(-30;-12;-6;-6)	-13.5	-9

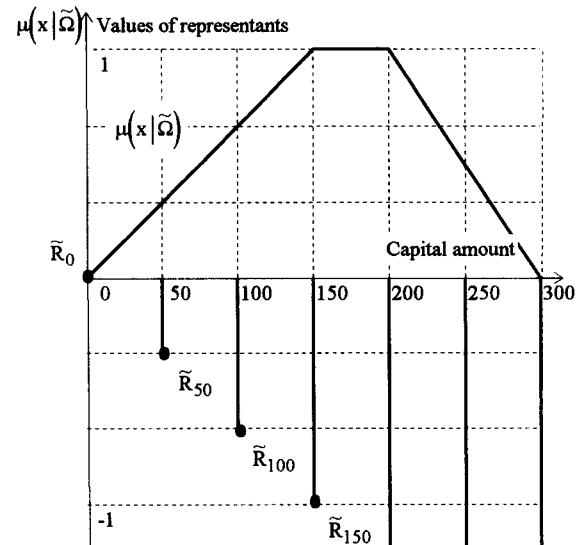


Fig. 3: Order of alternatives for $i_c - i_d = 2\%$ p.a.
 $\tilde{R}_0 > \tilde{R}_{50} > \tilde{R}_{100} > \tilde{R}_{150} > \tilde{R}_{200} > \tilde{R}_{250} > \tilde{R}_{300}$

Evaluation of example 1c:

A difference between the credit and debit interest rates being decreased, the mortgage bonds issue risk is increased so that all the alternatives seem to be loss-making. These results are supported by a fact the demand for mortgage loans is very uncertain (the choice of the demand from the example 1 is kept).

Example 2: Modeling of a financial decision risk following from the decreasing marginal utility rule.

Example 2a:

* Let a set of decision alternatives be the same $A = \{A_0, A_{50}, A_{100}, A_{150}, A_{200}, A_{250}, A_{300}\}$.

* Let a flat fuzzy number $\tilde{\Omega}$ be a different one: $\tilde{\Omega} = (150, 240, 260, 300)$.

* Let $i_c = 12\%$ p.a. and let i_d be the same as in example 1c, where all alternatives are lossmaking: $i_d = 10\%$ p.a., i. e. $i_c - i_d$ is only 2% p.a.

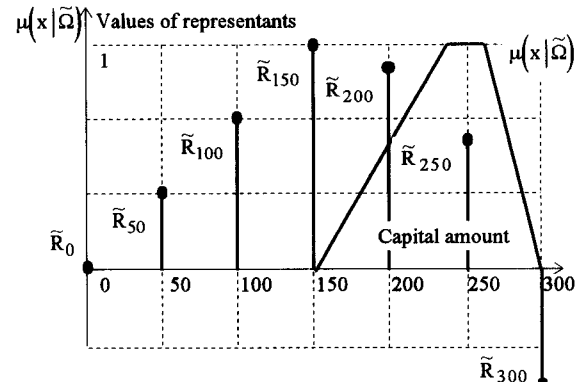
$$\tilde{R}_0 = (0.12 \cdot \min(0; 150) - 0.10 \cdot 0; 0.12 \cdot \min(0; 240) - 0.10 \cdot 0; 0.12 \cdot \min(0; 260) - 0.10 \cdot 0; 0.12 \cdot \min(0; 300) - 0.10 \cdot 0)$$

$$\begin{aligned}\tilde{R}_{50} &= (0.12 \cdot \min(50; 150) - 0.10 \cdot 50; 0.12 \cdot \min(50; 240) - 0.10 \cdot 50; 0.12 \cdot \min(50; 260) - 0.10 \cdot 50; 0.12 \cdot \min(50; 300) - 0.10 \cdot 50) \dots \\ \tilde{R}_{300} &= (0.12 \cdot \min(300; 150) - 0.10 \cdot 300; 0.12 \cdot \min(300; 240) - 0.10 \cdot 300; 0.12 \cdot \min(300; 260) - 0.10 \cdot 300; 0.12 \cdot \min(300; 300) - 0.10 \cdot 300).\end{aligned}$$

Table 4: The order of alternatives:

$$\tilde{R}_{150} \succ \tilde{R}_{200} \succ \tilde{R}_{100} \succ \tilde{R}_{250} \succ \tilde{R}_{50} \succ \tilde{R}_0 \succ \tilde{R}_{300}$$

Stocks	Results \tilde{R}_v	Represent ants	Values of representants
A_0	(0;0;0;0)	0	0
A_{50}	(1;1;1;1)	1	0.333
A_{100}	(2;2;2;2)	2	0.666
A_{150}	(3;3;3;3)	3	1
A_{200}	(-2;4;4;4)	2.5	0.833
A_{250}	(-7;3.8;5;5)	1.7	0.566
A_{300}	(-12;-1.2;1.2;6)	-1.5	-0.5

Fig. 4: Order of alternatives for $i_c - i_d = 2\%$ p.a. and for $\tilde{\Omega} = (150, 240, 260, 300)$ *Evaluation of example 2a:*

• Comparison with example 1c:

The mortgage loans fuzzy demand increasing and its movement to upper values resulted into profit achieved by most decisions, even the difference between the credit and debit interest keeps to be as small as in example 1c. However, a risk of investment to the higher amount than the lower exceeding fuzzy demand limit keeps to be too high. However, in case of the debit interest rate decreasing, the best solutions would be moved towards the higher mortgage bonds amount issues.

• Comparison with the example 2b:

Determination of alternatives order in example 2a quantifies just a risk following from the gained sources not placed on the mortgage loan market.

Example 2b:

Let all values from example 2a are holt.

In addition, a risk following from the decreasing marginal utility rule (increasing, concave utility function) can be quantified by finding of functional value of the fuzzy function u of variable \tilde{R}_v .

- * Revenues attributed to particular alternatives in the example 2a are first of all transformed to the interval $[0; 1]$ according to (14).
- * According to (6) the \tilde{R}_v fuzzy numbers are written down, applying α -cuts [1].
- * According to (13), (15) images of fuzzy numbers are found in a mapping $u: \tilde{R}_v \rightarrow \tilde{U}_v$.
- * The values are stated in Tables 5 and 6.
- * To organise the fuzzy numbers is used (10).

Table 5:

Stocks	Capital amounts in mil CZK	Transformed capital amounts
A_0	(0;0;0;0)	(2/3;2/3;2/3;2/3)
A_{50}	(1;1;1;1)	(13/18;13/18;13/18;13/18)
A_{100}	(2;2;2;2)	(7/9;7/9;7/9;7/9)
A_{150}	(3;3;3;3)	(5/6;5/6;5/6;5/6)
A_{200}	(-2;4;4;4)	(5/9;8/9;8/9;8/9)
A_{250}	(-7;3.8;5;5)	(5/18;79/90;17/18;17/18)
A_{300}	(-12;-1.2;1.2;6)	(0;3/5;22/30;1)

Table 6: The order of alternatives with an attitude to a risk: $\tilde{R}_{150} \succ \tilde{R}_{100} \succ \tilde{R}_{200} \succ \tilde{R}_{50} \succ \tilde{R}_0 \succ \tilde{R}_{250} \succ \tilde{R}_{300}$.

$\mu(x \tilde{R}_v)$	$\tilde{U}_v = 2 \cdot \tilde{R}_v \oplus [-(\tilde{R}_v)^2]$	Centre of the gravity
(2/3;2/3)	(0.8889; 0.8889)	0.8889
(13/18;13/18)	(0.9228; 0.9228)	0.9228
(7/9;7/9)	(0.9506; 0.9506)	0.9506
(5/6;5/6)	(0.9722; 0.9722)	0.9722
(1/3 α +5/9;8/9)	(0.6667 α +0.3210; -0.1111 α^2 -.3704 α +1.4691)	0.9357
(3/5 α +5/18;17/18)	(1.200 α -0.3364; -0.5289 α^2 -0.3333 α +1.8117)	0.8629
(3/5 α ;-8/30 α +1)	(-0.0676 α^2 +1.7200 α -1.0000; -0.5289 α^2 -.5200 α +2.0000)	0.6832

Evaluation of example 2b:

The quantification of a risk following not only from a potential not placement of the means (gained by the mortgage bonds sale) on the mortgage loans market, but also of a risk following from a degressive utility of increasing amounts of financial transactions resulted into the alternatives order re-arrangement. The alternative of the issue of 100 mil. CZK moved to the second order at the debit of the alternative issue of 200 mil. CZK, the alternative of 250 mil. CZK moved even by two positions back. In both the cases, the issue of 300 mil. CZK was kept on the last position, because a difference between the credit and debit interest rates was too low. This difference increasing would result into a choice of alternatives with larger amount issues.

The re-arrangement of the order is depicted in (Fig. 5).

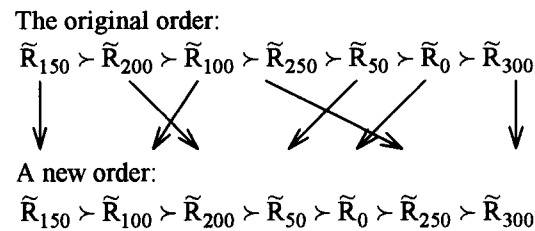


Fig. 5: A change in the alternatives order due to the quantification of the risk following from a degressive utility from increasing financial transaction amounts.

6. Conclusion

This paper presents a method for management decision-making support in relation to the mortgage bonds amount issues, when a demand for mortgage loans is based only on the following estimation, being uncertain. A decision-making function is modeled with regard to two risk groups:

- the risk of losses making because the gained financial sources were not placed,
- the risk following from decreasing marginal utility rule.

The examples demonstrate:

- the influence of a difference between credit and debit interest rates on the revenues of a bank,
- potential bad decisions when the decreasing marginal utility rule is not take into account.

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