A PROBLEM OF BIFUZZY PROBABILITY OF BIFUZZY EVENTS

Tadeusz Gerstenkorn and Jacek Mańko

Łódź University, Poland

Abstract: According to Yager's approach [5] to fuzzy probability for fuzzy events, we propose here a conception of a bifuzzy probability of bifuzzy events. We use the notion of a bifuzzy set [1]. We base ourselves on the notion of an (α, β) - level of bifuzzy set and the extension principle for bifuzzy sets [4].

Key words: bifuzzy set, α- level set of a fuzzy set, the resolution identity for fuzzy sets, the extension principle in Zadeh fuzzy set theory, probability of a fuzzy event, bifuzzy probability.

1. INTRODUCTION

In 1968 L.A.Zadeh published the first work treating of a probability of fuzzy random events [6]. According to him, the number

$$P(A) = \int_{\mathbf{R}^n} \mu_A(x) P(dx) \tag{1}$$

defined in a probability space $(\mathbf{R}^n, \mathcal{F}, P)$ measures the probability of a fuzzy event $A \in \mathcal{F}$ described by a measurable membership function $\mu_A : \mathbf{R}^n \to \langle 0,1 \rangle$.

In 1979 R.Yager [5] took notice of the inconsistency in defining the fuzzy probability as a real number. According to his suggestions, the probability of a fuzzy event should be described as a fuzzy set (more exactly - as a fuzzy number on $\langle 0,1 \rangle$). He used the notion of an α- level set of a fuzzy set, the resolution identity for fuzzy sets and the so-called extension principle [7]. Using the probability function (probability in the classical sense) R.Yager defined the probability of a fuzzy event A (an event in the sense of Zadeh) as

$$\widetilde{P}(A) = \bigcup_{\alpha \in \langle 0,1 \rangle} \alpha * P(A_\alpha) \tag{2}$$

where A_α denotes the α-level set of a fuzzy event A, i.e. the nonfuzzy set $A_\alpha = \{x \in \mathbf{R}^n : \mu_A(x) \geq \alpha\}$, and $\cup$ stands for a fuzzy union.

2. BIFUZZY SETS AND BIFUZZY EVENTS

In 1983 K.Atanassov proposed a generalization of a Zadeh fuzzy set and introduced the notion of an intuitionistic fuzzy set [1]. For some reasons, we call it bifuzzy set.

DEFINITION 1. By a bifuzzy set A in a space $X \neq \emptyset$ we mean the object

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\} \tag{3}$$

where the functions $\mu_A, \nu_A : X \to \langle 0,1 \rangle$ fulfil the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and describe, respectively, the degree of a membership and the degree of a nonmembership of an element x in a bifuzzy set A.

The family of all bifuzzy sets on universum X is denoted by $IFS(X)$. Relations and operations on bifuzzy sets can be found in [2].

In [3] one can find the first suggestion of measuring the probability of a bifuzzy event. It is based on Zadeh's approach [6]. So, let us consider a probability space $(E, \mathcal{F}, P)$ where $\mathcal{F}$ is a σ-field of subsets of E, P is a probability measure. In the family $IFS(E)$ let us consider only bifuzzy sets that have measurable functions μ and ν. Such bifuzzy sets are called bifuzzy events and their collection is denoted by $IM(E)$.

DEFINITION 2. The Lebesgue - Stieltjes integral

$$P(A) = \int_E \frac{\mu_A(x) + 1 - \nu_A(x)}{2} P(dx) \tag{4}$$

is called a probability of a bifuzzy event A.

It was shown [3] that formula (4) fulfils the Kolmogorov axioms of a probability and many other characteristics of the classical nonfuzzy probability.

3. THE RESOLUTION IDENTITY AND THE EXTENSION THEOREM

In paper [4] there are a conception of a generalization of the so-called α-level set of a fuzzy set, a generalization of the resolution identity and the extension principle that are adopted for bifuzzy set theory. We have [4]:

DEFINITION 3. For $\alpha,\beta \in \langle 0,1\rangle$ and $\alpha+\beta\le 1$ and for $A \in IFS(X)$, we define a product $(\alpha,\beta)* A$ as a bifuzzy set in the form

$$(\alpha,\beta)* A = \{(x,\alpha\cdot\mu_A(x),\beta+(1-\beta)\cdot\nu_A(x)): x\in X\}. \tag{5}$$

DEFINITION 4. By an (α,β)-level set of $A \in IFS(X)$ we mean the nonfuzzy set $A_{\alpha,\beta}$ in the form

$$A_{\alpha,\beta} = \{x\in X: \mu_A(x)\ge\alpha \text{ and } \nu_A(x)\le\beta\}. \tag{6}$$

DEFINITION 5. The set

$$N_{\alpha,\beta}(A) = \{(x,1,0): x\in A_{\alpha,\beta}\} \tag{7}$$

is called a bifuzzy analogue of the (α,β)- level set of A.

As is easy to notice, we have

$$(\alpha,\beta)* N_{\alpha,\beta}(A) = \{(x,\alpha,\beta): x\in A_{\alpha,\beta}\}.$$

Then the so-called resolution theorem states what follows.

THEOREM 1. For any bifuzzy set $A \in IFS(X)$,

$$A = \bigcup_{(\alpha,\beta)} (\alpha,\beta)* N_{\alpha,\beta}(A) \tag{8}$$

where the symbol $\bigcup$ denotes the operation of union in the bifuzzy sense.

Let now a function $f: E\to L$ (a function in the ordinary sense and E and L arbitrary non-fuzzy sets) be given. Let $A\in IFS(X)$. We extend the range of the function f to bifuzzy sets by means of the formula [4]

$$f(A) = \{(f(x),\mu_A(x),\nu_A(x)): f(x)\in L\} \tag{9}$$

or, equivalently, as

THEOREM 2 (the extension principle)

$$f(A)=\bigcup_{(\alpha,\beta)}(\alpha,\beta)*f(N_{\alpha,\beta}(A)). \tag{10}$$

As can be seen, the above extension principle allows one to extend a mapping defined on ordinary sets to a mapping on bifuzzy sets.

4. BIFUZZY PROBABILITY OF BIFUZZY EVENTS

Let, as before, $(E,\mathfrak{I},P)$ be an ordinary probability space. By $IM(E)$ we mean a family of bifuzzy events over E. Then applying the probability measure P to formula (10), we get

DEFINITION 6. For any $A\in IM(E)$, we define a bifuzzy probability of A as a bifuzzy event

$$\widetilde{P}(A)=\bigcup_{(\alpha,\beta)}(\alpha,\beta)*P(N_{\alpha,\beta}(A)). \tag{11}$$

It is worth seeing that the bifuzzy event given in (11) may be treated as a bifuzzy number over the interval $\langle 0,1\rangle$ and when the bifuzziness of the event A is reduced to the Zadeh fuzziness, the above conception is reduced to Yager's conception of fuzzy probability [5], and this one, in the complete lack of fuzziness, to the classical Kolmogorov probability.

5. EXAMPLE

Let $X=\{x_1,x_2,x_3,x_4,x_5\}$ be the set of five persons. Let us consider in X the bifuzzy set A of all people who are interested in bifuzzy set theory and assume that $A=\{(x_1,1,0),(x_2,0.8,0.1),(x_3,0.6,0.2),(x_4,0.5,0.5),(x_5,0,0.9)\}$. Let, in some random experiment, each of the persons x_i $(i=1,2,3,4,5)$ appear with the probability p_i, respectively: $p_1=0.4, p_2=0.1, p_3=0.1, p_4=0.3, p_5=0.1$. Then the probability of the bifuzzy event A consisting in the choice of a person interested in bifuzzy set theory equals, according to (4)

$$P(A)=\frac{1+1-0}{2}\cdot 0.4+\frac{1+0.8-0.1}{2}\cdot 0.1+\frac{1+0.6-0.2}{2}\cdot 0.1+$$
$$+\frac{1+0.5-0.5}{2}\cdot 0.3+\frac{1+0-0.9}{2}\cdot 0.1=0.71.$$

According to the procedure given in (11), the bifuzzy probability of the bifuzzy event A is counted in turns:

$$A_{0,0.9}=\{x_1,x_2,x_3,x_4,x_5\},$$

$$A_{0.5,0.5}=\{x_1,x_2,x_3,x_4\},$$

$$A_{0.6,0.2}=\{x_1,x_2,x_3\},$$

$$A_{0.8,0.1}=\{x_1,x_2\},$$

$$A_{1,0}=\{x_1\}$$

and

$$N_{0,0.9}(A)=\{(x_1,1,0),(x_2,1,0),(x_3,1,0),(x_4,1,0),(x_5,1,0\},$$

$$N_{0.5,0.5}(A)=\{(x_1,1,0),(x_2,1,0),(x_3,1,0),(x_4,1,0)\},$$

$$N_{0.6,0.2}(A)=\{(x_1,1,0),(x_2,1,0),(x_3,1,0)\},$$

$$N_{0.8,0.1}(A)=\{(x_1,1,0),(x_2,1,0)\},$$

$$N_{1,0}(A)=\{(x_1,1,0)\}$$

and

$$P(N_{0,0.9}(A))=1,$$

$$P(N_{0.5,0.5}(A))=0.9,$$

$$P(N_{0.6,0.2}(A))=0.6,$$

$$P(N_{0.8,0.1}(A))=0.5,$$

$$P(N_{1,0}(A))=0.4.$$

Then

$$\widetilde{P}(A)=(0,0.9)*P(N_{0,0.9}(A))\cup(0.5,0.5)*P(N_{0.5,0.5}(A))\cup$$

$$\cup(0.6,0.2)*P(N_{0.6,0.2}(A))\cup(0.8,0.1)*P(N_{0.8,0.1}(A))\cup(1,0)*P(N_{1,0}(A))=$$

$$=(0,0.9)*\{(1,1,0)\}\cup(0.5,0.5)*\{(0.9,1,0)\}\cup(0.6,0.2)*\{(0.6,1,0)\}\cup$$

$$\cup(0.8,0,1)*\{(0.5,1,0)\}\cup(1,0)*\{(0.4,1,0)\}=$$

$$=\{(1,0,0.9),(0.9,0.5,0.5),(0.6,0.6,0.2),(0.5,0.8,0.1),(0.4,1,0)\}.$$

The result obtained should be interpreted in the following way: when drawing from the persons $\{x_1,x_2,x_3,x_4,x_5\}$, we choose a person interested in bifuzzy set theory with probability 0 (respectively, with non-probability 0.9); if we choose from $\{x_1,x_2,x_3,x_4\}$ i.e. after the rejection of the decidedly „not interested" person, then such a probability increases to 0.5; similarly, rejecting the person x_4, such a probability is 0.6; rejecting the person x_3, we choose such a person from $\{x_1,x_2\}$ with probability 0.8 (with non-probability 0.1); and finely, rejecting the person x_2, we have only the person x_1 who is really „interested in bifuzzy set theory" and the probability for x_1 is, of course, 1.

References

[1] K.Atanassov, S.Stoeva, Intuitionistic fuzzy sets, Proc. Polish Symp. on Interval and Fuzzy Mathematics in Poznań 1983 (August 26-29), Inst. Math. Technical Univ. of Poznań, Poznań 1985, 23-26, ed. J.Albrycht and W.Wiśniewski.

[2] K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[3] J.Mańko, Probability, entropy and energy in bifuzzy set theory (in Polish), doctoral dissertation, Łódź University, 1992.

[4] D.Stojanova, Sets from (α,β)-level generated by intuitionistic fuzzy sets. Principle of generalization. Proc. of conference "Mathematical Foundations of Artificial Intelligence Seminar", Institute for Microsystems, Sofia, November 1990, pp. 44-46.

[5] R.R.Yager, A note on probabilities of fuzzy events, Inform. Sci. **18** (1979), 113-129.

[6] L.A.Zadeh, Probability measure of fuzzy events, J. Math. Anal. Applic. **23** (1968), 421-427.

[7] L.A.Zadeh, The concept of a linguistic variable and its applications to approximate reasoning, Part 1, Inform. Sci. **8** (1975), 199-249.