An abductive diagnostic relational model based on possibility theory

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ABSTRACT

This paper presents a possibilistic relational model that is able to deal with the uncertainties of the entities and associations that are involved in diagnostic problem, using fuzzy relations. At the entities level of representation the fuzzy sets offer an integration of different type of diagnostic information: numerical values of manifestations/observations, linguistic values of manifestations/observations.
We have used a possibilistic representation of the model entities and relations in order to compute the plausible explanations for the present observations.

1. INTRODUCTION

Many researchers have proposed different approaches to capture uncertainties in the causal relational model defined by Peng and Reggia [Peng and Reggia, 1990]. Among these models, few of them have used a possibilistic approach in order to deal with different type of uncertainties:

- The certainty of the causal relation and the certainty of the presence of the manifestations [Cayrac, Dubois and Prade, 1996, Dubois and Prade, 1995]
- The certainty of the causal relations when the manifestations caused by a disorder are a matter of degree and the observations are crisp [Grabisch and Baran, 1998].

However, none of these models allows the expressions of both the lack of certainty of the observations and their graduality.

The relational model presented in this paper is able to take into account the uncertainty of the causal relation (through certainty levels) when the typical manifestations and the observations are fuzzy subsets. We have chosen such a representation for the typical values of the manifestation because usually these values are not precisely: they are intervals, subsets or even linguistically values.
The observations are also fuzzy subsets. The fuzziness is due either to the imprecision of measurements, either to the linguistically nature of the observations (ex:"increased liver")

The presence of a manifestation is no longer an all-or-nothing problem. Its level of presence is expressed by a matching method; the result of matching expresses the compatibility (consistency or inclusion) degree between the observations and the typical manifestations.

The solution of diagnosis is obtained incrementally, through hypothesis-and-test cycles. At the hypothesis generation step, we obtain a set of plausible explanation (diagnosis) for the present observation. At the second step we will perform discriminatory tests in order to obtain the "best" explanation.

In this paper we present only the model of the possibilistic causal network and the identification of the plausible diagnosis (hypothesis set).

In section 2 we briefly present the possibilisitc model developed by [Dubois and Prade, 1995; Cayrac, Dubois and Prade, 1996]

2. A POSSIBILISTIC DIAGNOSIS MODEL [Dubois and Prade, 1995; Cayrac, Dubois and Prade, 1996]

Let $D=\{d_1,...,d_n\}$ be the set of disorders and $M=\{m_1, ...,m_m\}$ be the set of manifestation. For each d there is a fuzzy subset of manifestation which are (more or less) certainly caused by d, denoted $M(d)^+$ and a fuzzy subset of manifestations that (more or less) certainly are not caused by d, denoted $M(d)^-$.

M^+ and M^- are the fuzzy sets of observations that are respectively (more or less) certainly present and (more or less) certainly absent.

Three types of observations define the solution of the diagnostic problem: *coherent explanations, relevant explanations, covering explanations.*

The coherent explanation are those disorders for which the typical caused manifestations are not in contradiction with observations (similar with $M(d)^+\cap M^-=\varnothing$ and $M(d)^-\cap M^+=\varnothing$) in the crisp case). D denotes the fuzzy subset of coherent disorders:

$$\forall d\in D,\ \mu_{D^\cdot}(d)=\min((1-\mathrm{cons}(M(d)^+,M^-),\ (1-\mathrm{cons}(M(d)^-,M^+)) \quad (1)$$

where $\mathrm{cons}(A,B)=\max_u(A(u), B(u))$ denotes the compatibility between two fuzzy sets.

In the fuzzy subset of relevant explanations, denoted by D^, are included those disorders for which their predicted manifestations are indeed (more or less) observed. The degree of relevance is:

$$\forall d\in D,\ \mu_{D^\wedge}(d)=\min(\mathrm{cons}(M(d)^+,M^+),\ \mathrm{cons}(M(d)^-,M^-)) \quad (2)$$

The covering explanations are those disorders for which the observations are included in the predicted manifestations

$$\forall d\in D,\ \mu_{D^\bullet}(d)=\min(\mathrm{incl}(M^+ ,M(d)^+),\ \mathrm{incl}(M^- ,M(d)^-)) \quad (3)$$

Where $\mathrm{incl}(A,B)=\min_u(\mu_A(u)\rightarrow\mu_B(u)$ and "$\rightarrow$" is the Godel implication.

These fuzzy subsets of possible explanations are used in order to perform more discriminatory tests the goals being to obtain the "most plausible". This explanation is the solution of the diagnosis problem.

Even for some disorder d we obtain a plausibility degree equal to 1, this do not mean that it is certainly present, it means only that it is fully possible to have d. However, if the degree of plausibility is 0 then the disorder can be discarded from the explanation set.

3. AN EXTENSION WITH FUZZY MANIFESTATIONS AND FUZZY OBSERVATIONS

There are real situations where we have to deal with fuzzy manifestations and fuzzy observations.

Lets call $M(d)^+$ the set of positive manifestations and $M(d)^-$ the set of negative manifestations. The set of the manifestations causal related with d is $M(d)=M(d)^+\cup M(d)^-$. Each one of its element is a fuzzy subset $M(d)_i$

corresponding to the manifestation m_i caused by d. In fact $M(d)_i$ can be considered as a flexible constraint on the values of m_i in the presence of d. . Let denote by X_i the univers (the domain of the possibles values) of the manifestation m_i

For the positive manifestation, the membership function ($\mu_{M(d)_i^+}$) of the fuzzy subset $M(d)_i^+$ expresses the possibility distribution of the values of m_i in the presence of d.

Case A. The is no uncertainty on causal realtion

In order to express the fuzzy manifestations that belong to $M(d)^-$ let us try to expound the corresponding rule.
The rule that expresses the negative manifestations is:

"if d is present then $m \notin M$"

(where M is fuzzy,) is the same as the rule

:"if d is present then $m \in \neg(M)$".

Saying that a manifestations m_k is not caused by d, means that there is a fuzzy subset $M(d)_k$ of the values of m_k that are not allowed in the presence of d. In order to have a coherent representation at this level, the elements of negative manifestation $M(d)^-$, are fuzzy subsets $M(d)_k^-$ with the membership function $1-\mu_{M(d)_i}$ where $\mu_{M(d)_i}$ is the posibility distribution of the values of m_k that are not allowed in the presence of d.

The observation set Mo, is in fact the set of fuzzy observation. So each observation mo_i is expressed by a fuzzy set Mo_i with the membership function μ_{Moi}. The fuzziness of the observations is due to the imprecision of the measurement or the linguistical nature of the observation. In both cases μ_{Moi} expresses a compatibility degree.

In order to obtain the set of plausible explanation for the set of present observation we will compute the plausibility of two types of explanation set:

Definition 1. The fuzzy set of the *consistent* diagnosis $D^\wedge$, is defined by

$$\begin{aligned}\forall d \in D,\quad \mu_{D^\wedge}(d) &= \mathrm{cons}(M(d), Mo)\\ &= \min(\min_{M(d)_i^+ \in M(d)^+}(\mathrm{cons}(M(d)_i^+, Mo_i), \min_{M(d)_k^- \in M(d)^-}(\mathrm{cons}(M(d)_k^-, Mo_K))\\ &= \min(\min_{M(d)_i^+ \in M(d)^+}(\max_{m \in X_i}(\min(\mu_{M(d)_{ii}^+}(m), \mu_{Mo_i}(m)),\\ &\qquad \min_{M(d)_i^- \in M(d)^-}(\max_{m \in X_k}(\min((1-\mu_{M(d)_k}(m), \mu_{Mo_k}(m))\end{aligned} \tag{4}$$

where $\mathrm{cons}(A, B) = \max_{u \in U}(\mu_A(u), \mu_B(u))$

$D^\wedge$ is the set of explanation that are (more or less) consistent with observations and $\mu_{D^\wedge}(d)$ expresses the consistency degree of d with the observations.

Important remark

The consistency degree is the same as the degree of relevant disorder obtained in the former model and in [Grabisch and Baran, 1998]. *Cons* expresses intersection degree of the two fuzzy subsets.
In order to obtain the degree of "absence" for an observation we define the measure of the absence as being the degree of contradiction between the present manifestation and the predicted one. In fuzzy theory the contradiction between two

fuzzy sets is contr(A,B)=1-cons(A,B). So contr($M(d)_k^+$,Mo_k) is equivalent with cons($M(d)^+$,M^-) from the relation (1) and then $D^\wedge$ is the same as D'.

Definition 2 The fuzzy set of plausible explanations that are covers of the present observations is defined as:

$$
\begin{aligned}
\forall d \in D,\quad \mu_{D^\wedge}(d) &= incl(Mo, M(d)) \\
&= \min(\min_{M(d)_i^+ \in M(d)^-}(incl(Mo_i, M(d)_i^+)), \min_{M(d)_i^- \in M(d)^-}(incl(Mo_K, M(d)_k^-)) \\
&= \min(\min_{M(d)_i^+ \in M(d)^+}(\min_{m \in X_i}(\mu_{Mo_i}(m) \rightarrow \mu_{M(d)_{ii}^+}(m)), \\
&\qquad \min_{M(d)_i^- \in M(d)^-}(\min_{m \in X_k}(\mu_{Mo_k}(m), \rightarrow (1 - \mu_{M(d)_k}(m))) \qquad (5)
\end{aligned}
$$

where $incl(A,B) = \min_{u \in U}(\mu_A(u) \rightarrow \mu_B(u))$ *and* "$\rightarrow$" is the Godel implication

The value $\mu_{D^\bullet}(d)$ expresses the coverage degree of d for the observation set.

Case B. The case of uncertainty of causal relation

Let consider now that a fuzzy manifestation m_i appear in the presence of d, with the certainty level $\alpha_i<1$. That means that there is a level of ignorance on the causal relation between disorder and manifestation bounded by $1-\alpha_i$. When $\alpha_i=0$ then we can not say anything about the causal relation between d and m. It is total uncertain that d cause m_i but it is posible. When $\alpha_i=1$ then there is no uncertainty on causal relation and we retrieve the case A.

In order to capture the uncertainty of the causal relation we have the following rule:

"If d is present then it is α_i certain that m_i appear with the linguistical value $M(d)_i$".

The rule above is a certainty rule. The possibility distribution of the causal relation is defined as being [Dubois and Prade, 1991]:

$$\mu_R(d,m_i)=\max(\mu_{M(d)i}(m), 1-\alpha_i).$$

Let denote by $M(d)_i^+$ the fuzzy subset with the membership function described as before.

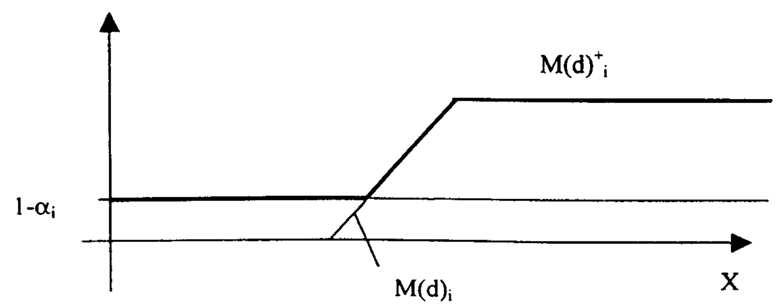

Fig.1. The possibility distributions of the positive m_i manifestation values in the presence of d where the causal relation is uncertain

If we have only positive manifestations then we have the following definition of the fuzzy subsets of the consistent and covering diagnosis:

Definition 3. The fuzzy set of the consistent diagnosis D^ is defined by

$$
\begin{aligned}
\forall d \in D, \quad \mu_{D^\wedge}(d) &= cons(M(d), Mo) \\
&= \min_{M(d)_i^+ \in M(d)^+} (cons(M(d)_i^{*+}, Mo_i)) \\
&= \min_{M(d)_i^+ \in M(d)^+} (cons(\max(1-\alpha_i, M(d)_i^+), Mo_i) \\
&= \min_{M(d)_i^+ \in M(d)^+} (\max(1-\alpha_i, cons(M(d)_i^+, Mo_i)) \\
&= \min_{M(d)_i^+ \in M(d)^+} (\max(1-\alpha_i, \max_{m \in X_i} (\min(\mu_{M(d)_i^+}(m), \mu_{Mo_i}(m))),
\end{aligned} \tag{4}
$$

where $cons(A, B) = \max_{u \in U}(\mu_A(u), \mu_B(u))$

Here $\mu_{D^\wedge}(d)$ is a plausibility degree of the d computed as a consistency index.

Definition 4. The fuzzy subset of the covering diagnosis D* is defined by:

$$
\begin{aligned}
\forall d \in D, \quad \mu_{D^\wedge}(d) &= \mathrm{incl}(Mo, M(d)) \\
&= \min_i (\mathrm{incl}(Mo_i, M(d)_i^{*+}) \\
&= \min_i (\mathrm{incl}(Mo_i, \max(1-\alpha_i, M(d)_i^+) \\
&= \min_i (\max(1-\alpha_i, \mathrm{incl}(Mo_i, M(d)_i^+)
\end{aligned} \tag{5}
$$

where $\mathrm{incl}(A, B) = \min_{u \in U}(\mu_A(u) \to \mu_B(u))$ and "$\to$" is Godel implication

Let us consider now the situation when there are fuzzy manifestations m_j that do not appear in the presence of d at a certainty degree β_j. The rule is as follows:

"*If d is present then it is β_j-certain that it will not occur the linguistical value $M(d)_j$ of m_j*"

that is equivalent with

"*if d is present then it is β_j–certain to have values of m_j outside the linguistical value M(d)*"

That is also a certainty rule and the possibility distribution of the corresponding causal relation is defined as:

$\mu_R(d, m_j) = \max(1-\beta_j, 1-\mu_{M(d)}(d))$ and the associated fuzzy subset is denoted by $M(d)^-_j$

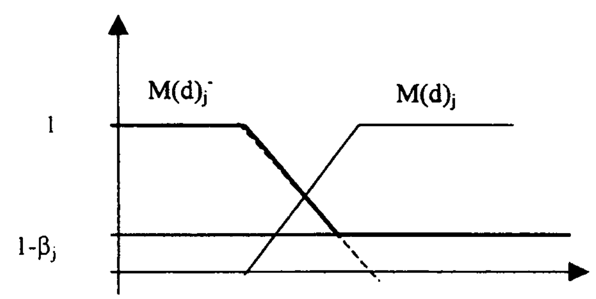

Fig.2. The possibility distribution of m_j negative manifestation values in the presence of d; β_j is the certainty level pervading to causal relation

In the presence of positive and negative information the *consistency degree* of a disorder d will be defined by:

Definition 5. The fuzzy set of the consistent diagnosis D^ is defined by

$$\forall d \in D,\quad \mu_{D^\wedge}(d) = cons(M(d), Mo)$$
$$= \min(\min_{M(d)_i^+ \in M(d)^+}(cons(M(d)_i^+, Mo_i), \min_{M(d)_j^- \in M(d)^-}(cons(M(d)_j^-, Mo_j))$$
$$= \min_{M(d)_i^+ \in M(d)^+}(cons(\max(1-\alpha_i, M(d)_i^+), Mo_i), \min_{M(d)_j^- \in M(d)^-}(cons(\max(1-\beta_j, M(d)_j^-), Mo_j))$$
$$= \min_{M(d)_i^+ \in M(d)^+}(\max(1-\alpha_i, cons(M(d)_i^+, Mo_i), \min_{M(d)_j^- \in M(d)^-}(\max(1-\beta_j, cons(M(d)_j^-), Mo_j)))$$
$$= \min(\min_{M(d)_i^+ \in M(d)^+}(\max(1-\alpha_i, \max_{m \in X_i}(\min(\mu_{M(d)_i^+}(m), \mu_{Mo_i}(m)),$$
$$\min_{M(d)_j^- \in M(d)^-}(\max(1-\beta_j, \max_{m \in X_j}(\min(1-\mu_{M(d)_j}(m), \mu_{Mo_j}(m)), \tag{6}$$

where $cons(A,B) = \max_{u \in U}(\mu_A(u), \mu_B(u))$

Observations:

- If $\alpha_i = 1$ and $\beta_j = 1$ then we are in the situation A
- If $\alpha_i = 0$ and $\beta_j = 0$ then $\mu_{D^\wedge}(d) = 1$, so d is a fully plausible candidate because we are in the situation of complete ignorance and everything is possible
- If $cons(Mo_i, M(d)_i^+) = 0$ and $cons(Mo_j, M(d)_j^-) = 0$, and both α_i and β_j are positive then $\mu_{D^\wedge}(d) = \min(\min_i(1-\alpha_i), \min_j(\min(1-\beta_j)) > 0$. Even if it seems to be unnatural this result expresses the fact that we cannot discard d from the set of plausible diagnosis because of a certain level of ignorance.

- If Mo is a set of crisp values x_i then $cons(Mo_i, M(d)_i^+)$ represents the values of the membership function $\mu_{M(d)_j}(x_j)$ and similar for the negative manifestation $cons(Mo_j, M(d)_j^-)$ is $1 - \mu_{M(d)_j}(x_j)$. Relation (6) becomes
$$\mu_D(d) = \min(\min_i(\max(1-\alpha_i, \min(\mu_{M(d)_i^+}(x_i),), \min_j(\max(1-\beta_j, 1-\mu_{M(d)_j^-}(x_j))$$
and we recover the same relation as in [Grabisch and Baran, 1998] for the expression of relevant diseases.

Definition 6. The fuzzy set of the disorders that covers of the present manifestations is defined by

$$\forall d \in D,\quad \mu_{D^*}(d) = incl(Mo, M(d))$$
$$= \min(\min_{M(d)_i^+ \in M(d)^+}(incl(Mo_i, M(d)_i^+), \min_{M(d)_j^- \in M(d)^-}(incl(Mo_j, M(d)_j^-)$$
$$= \min(\min_{M(d)_i^+ \in M(d)^+}(\max(1-\alpha_i, incl(Mo_i, M(d)_i),$$
$$\min_{jM(d)_j^- \in M(d)^-}(\max(1-\beta_j, incl(Mo_j, M(d)_j)) \tag{7}$$

where $incl(A, B) = \max_{u \in U}(\mu_A(u) \rightarrow \mu_B(u))$

D^* contains the set of all manifestations d that are (more or less) covers of the present observations.

It is easy to observe that D^* will be empty only in the situation where is no uncertainty pervading to causal relation.

4. Conclusion

In this paper we have presented a relational possibilistic model of a diagnosis problem that is able to deal with fuzzy manifestations, fuzzy observations and with the uncertainty of causal relation.

An interesting aspect of this model is that it is able to compute plausible explanation even in the situation when the knowledge about the domain is incomplete. In my opinion, saying that m is not causally related with d means that in the presence of d the possibility distribution of m values is one ($\mu_R(d,m)=1$) and that is consistent with human interpretation.

We can observe that in the situation in which the observation are not at all related with the possible disorder from our model , each of these disorder is equally plausible, their plausibility degree being 1. When we improve the information about the causal relation then the plausibility degree of the explanation is more precise.

In the future we will try to obtain the logical model that is equivalent with the relational one obtained above. We will also try to extend the model in the case of hypothesis with multiple disorders. An other important future research line is to define the test plan.

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References

[Cayrac, Dubois, Prade, 1996] Didier Cayrac, Didier Dubois, Henry Prade, Handling Uncertainty with Possibility Theory and Fuzzy Sets in a satelite Fault Diagnosis Application, *IEEE Transaction on Fuzzy Systems,* no.3, 1996

[Dubois, Prade, 1991] Didier Dubois, Henri Prade, Fuzzy sets in aproximate reasoning, Part 1: Inference with possibility distributions, *Fuzzy sets and Systems 40,* pp 143-202, 1991

[Dubois, Prade, 1995] Didier Dubois, Henri Prade, Fuzzy relation ecuation and causal reasoning, *Fuzzy sets and Systems 75,* 1995

[Grabisch, Baran, 1998] Michel Grabisch, David Baran, A diagnosis Based Approach to Image Interpretation, to appear October 16-20, IIZUKA'98 5th International Conference on Soft Computing.

IIZUKA'98 5th International Conference on Soft Computing. 1998, Iiuzisk? Yamakava sept oct

[Peng and Reggia, 1991] Yu Peng, James A Reggia, "Abductive Inference Models for Diagnostic Problem-Solving", SPRINGER VERLAG, 1991