B-B Fuzzy Sets $\underset{\sim}{S}$ (II)*

Shi Kaiquan
Department of Automation Engineering,
Shandong University of Technology,
Jinan, Shandong 250061, P.R.China.

Abstract

This paper proposes B-B fuzzy set theory which is the continuation of [1].

This paper proposes union-ordinary resolution theorem, intersection-ordinary resolution theorem of B-B fuzzy set $\underset{\sim}{S}$ which establish contact between B-B fuzzy set $\underset{\sim}{S}$ and ordinary set S_λ. These results get ready for B-B fuzzy logic, B-B fuzzy reasoning in theory.

This paper points out that

1°. There exists union-ordinary resolution form $\underset{\sim}{S} = \bigcup_{\lambda\in[-1,1]} \lambda S_\lambda$ and intersection -ordinary resolution form $\underset{\sim}{S} = \bigcap_{\lambda\in[-1,1]} \lambda S_\lambda$ at the same time in B-B fuzzy set $\underset{\sim}{S}$, which give the important theoretic basis for B-B fuzzy logic and B-B fuzzy reasoning.

2°. Union-ordinary resolution theorem, intersection-ordinary resolution theorem of O-B fuzzy set $\underset{\sim}{A}$ (L.A. Zadeh Fuzzy Set $\underset{\sim}{A}$) are the special form of those of B-B fuzzy set $\underset{\sim}{S}$.

Keywords: B-B fuzzy set, Union-ordinary resolution theorem, Intersection-ordinary resolution theorem.

1. Introduction

Applying resolution method [3, 4], fuzzy set $\underset{\sim}{A}$ can be expressed by ordinary set A_λ in O-B fuzzy set $\underset{\sim}{A}$ (L.A.Zadeh fuzzy set $\underset{\sim}{A}$), which establish contact between fuzzy set $\underset{\sim}{A}$ and ordinary set A_λ. This is a larger success in the study of O-B fuzzy set $\underset{\sim}{A}$, and the study of O-B fuzzy set theory gets deep development .

Paper [1] proposes B-B fuzzy set $\underset{\sim}{S}$, then people propose such questions naturally : Can fuzzy set

* The project supported by Natural Science Foundation of Shandong Province . China

$\underset{\sim}{S}$ be expressed by ordinary set S_λ? Can fuzzy set $\underset{\sim}{S}$ make the resolution of S_λ?

Due to above questions, this paper proposes union-ordinary resolution theorem, intersection-resolution theorem of B-B fuzzy set $\underset{\sim}{S}$, which give following answers : There exist resolution relations between B-B fuzzy set $\underset{\sim}{S}$ and ordinary set S_λ, or we say, B-B fuzzy set $\underset{\sim}{S}$ can be expressed by ordinary set S_λ; union-ordinary resolution theorem, intersection-ordinary resolution theorem establish contact between B-B fuzzy set $\underset{\sim}{S}$ and ordinary set S_λ.

The results given in this paper make clear : union-ordinary resolution theorem, intersection-ordinary resolution theorem of O-B fuzzy set $\underset{\sim}{A}$ are the special forms of those of B-B fuzzy set $\underset{\sim}{S}$. Under certain conditions, union-ordinary resolution theorem, intersection-ordinary resolution theorem of B-B fuzzy set $\underset{\sim}{S}$ can be simplified into those of O-B fuzzy set $\underset{\sim}{A}$.

Appointed that signs and definitions in this paper without explanations can be found in [1]; X is a finite universe, $\mathscr{F}$(X) is the set of B-B fuzzy set $\underset{\sim}{S}$, and $\underset{\sim}{S}(x)$ is a fuzzy kiss function of x concerning $\underset{\sim}{S}$. B-B fuzzy set $\underset{\sim}{S}$ is the normal fuzzy set. For simplification and not arising confusion, $\forall\lambda_1 \in [0, 1]$, $\forall\lambda_2 \in [-1, 0]$ and $|\lambda_1| = |\lambda_2|$, it is denoted by $\lambda \in [-1, 1]$.

2. Union-ordinary resolution theorem of B-B fuzzy set

Definition 2.1 Let $S_\lambda^+ \in \mathscr{P}(X)$ be λ-cutset of $\underset{\sim}{S} \in \mathscr{F}(X)$ in $X^+ \cup X^\circ \subset X$, $S_\lambda^+(x)$ is said to be the characteristic function of x concerning $S_\lambda^+ \subset X^+ \cup X^\circ$, and :

$$S_\lambda^+(x) = \begin{cases} 1, & x \in S_\lambda^+ \subset X^+ \cup X^\circ \\ 0, & x \,\bar{\in}\, S_\lambda^+ \subset X^+ \cup X^\circ \end{cases} \tag{2.1}$$

Definition 2.2 Let $S_\lambda^- \in \mathscr{P}(X)$ be λ-cutset of $\underset{\sim}{S} \in \mathscr{F}(X)$ in $X^- \cup X^\circ \subset X$, $S_\lambda^-(x)$ is said to be the characteristic function of x concerning $S_\lambda^- \subset X^- \cup X^\circ$, and :

$$S_\lambda^-(x) = \begin{cases} -1, & x \in S_\lambda^- \subset X^- \cup X^\circ \\ 0, & x \,\bar{\in}\, S_\lambda^- \subset X^- \cup X^\circ \end{cases} \tag{2.2}$$

Definition 2.3 Let $\underset{\sim}{S} \in \mathscr{F}(X)$, $\lambda \in [-1, 1]$; Provided that $\lambda\underset{\sim}{S} \in \mathscr{F}(X)$, the fuzzy kiss function $\lambda\underset{\sim}{S}$ is defined as :

$$(\lambda\underset{\sim}{S})(x) = \lambda \wedge \underset{\sim}{S}(x) \tag{2.3}$$

If $S\in\mathscr{P}(X)$, then

$$(\lambda S)(x)=\lambda\wedge S(x) \tag{2.4}$$

where S is a ordinary set, $\mathscr{P}(X)$ is the set of S, $S(x)$ is the characteristic function of x concerning S.

From definition 2.3 of this paper and definition 3.1 of [1], let $\lambda_1, \lambda_2\in[-1, 1]$, we get :

1. 1°. given $\underset{\sim}{S}\in\mathscr{F}(X)$, if $\lambda_1<\lambda_2$; $\lambda_1, \lambda_2\in[0, 1]$; then

$$\lambda_1\underset{\sim}{S}\subseteq\lambda_2\underset{\sim}{S}$$

2°. given $\underset{\sim}{S}\in\mathscr{F}(X)$, if $\lambda_1<\lambda_2$; $\lambda_1, \lambda_2\in[-1, 0]$; then

$$\lambda_1\underset{\sim}{S}\subseteq\lambda_2\underset{\sim}{S}$$

2. given $\underset{\sim}{S}$, $\underset{\sim}{K}\in\mathscr{F}(X)$, $\lambda\in[-1, 1]$, $\underset{\sim}{S}\subseteq\underset{\sim}{K}$:

$$\lambda\underset{\sim}{S}\subseteq\lambda\underset{\sim}{K}$$

Theorem 2.1 Let $\underset{\sim}{S}\in\mathscr{F}(X)$ be the B-B fuzzy set in X , S_λ the λ-cutset of $\underset{\sim}{S}\in\mathscr{F}(X)$, $S_\lambda\in\mathscr{P}(X)$, then :

$$\underset{\sim}{S}=\bigcup_{\lambda\in[-1,1]}\lambda S_\lambda \tag{2.5}$$

Proof: 1°. Let $\lambda\in[-1, 0]$, for any $x\in X^-\cup X^\circ$, from definition 2.3 of this paper and definition 3.1, 3.3 of [1], we get :

$$\Big(\bigcup_{\lambda\in[-1,0]}\lambda S_\lambda\Big)(x)=\bigvee_{\lambda\in[-1,0]}(\lambda\wedge S_\lambda(x))$$

$$=\Big(\bigvee_{\lambda\in[-1,\underset{\sim}{S}(x)]}(\lambda\wedge S_\lambda(x))\Big)\vee\Big(\bigvee_{\lambda\in[\underset{\sim}{S}(x),0]}(\lambda\wedge S_\lambda(x))\Big)$$

$$=\bigvee_{\lambda\in[-1,\underset{\sim}{S}(x)]}(\lambda\wedge S_\lambda(x))=\bigvee_{\lambda\in[-1,\underset{\sim}{S}(x)]}\lambda=\underset{\sim}{S}(x)$$

so :

$$\underset{\sim}{S}=\bigcup_{\lambda\in[-1,0]}\lambda S_\lambda \tag{2.6}$$

2°. Let $\lambda\in[0, 1]$, for any $x\in X^+\cup X^\circ$, similar to 1° or see [3], we get

so :

$$\underset{\sim}{S}=\bigcup_{\lambda\in[0,1]}\lambda S_\lambda \tag{2.7}$$

For $\lambda\in[-1, 1]$, from 1°, 2°, we get :

$$\underset{\sim}{S}=\bigcup_{\lambda\in[-1,1]}\lambda S_\lambda \tag{2.8}$$

Theorem 2.2 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be the B-B fuzzy set in X, $\underset{\bullet}{S_\lambda}$ the λ-strong cutset of $\underset{\sim}{S} \in \mathscr{F}(X)$, $\underset{\bullet}{S_\lambda} \in \mathscr{P}(X)$, then

$$\underset{\sim}{S} = \bigcup_{\lambda \in [-1,\, 1]} \lambda \underset{\bullet}{S_\lambda} \tag{2.9}$$

Proof : It is similar to that of theorem of 2.1, so it is omitted.

Theorem 2.3 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be the B-B fuzzy set in X, and

$$\begin{aligned} H:\ [-1,\, 1] &\rightarrow \mathscr{P}(X) \\ \lambda &\rightarrow H(\lambda) \end{aligned}$$

satisfies $\lambda \in [-1,\, 1]$, $\underset{\bullet}{S_\lambda} \subseteq H(\lambda) \subseteq S_\lambda$

then
$$\underset{\sim}{S} = \bigcup_{\lambda \in [-1,\, 1]} \lambda H(\lambda) \tag{2.10}$$

Proof : 1°. Let $\lambda \in [-1,\, 0]$, from definition 3.5 of [1], we get $\underset{\bullet}{S_\lambda} \subseteq H(\lambda) \subseteq S_\lambda$, $\lambda \underset{\bullet}{S_\lambda} \subseteq \lambda H(\lambda) \subseteq \lambda S_\lambda$

so :
$$\underset{\sim}{S} = \bigcup_{\lambda \in [-1,\, 0]} \lambda \underset{\bullet}{S_\lambda} \subseteq \bigcup_{\lambda \in [-1,\, 0]} \lambda H(\lambda) \subseteq \bigcup_{\lambda \in [-1,\, 0]} \lambda S_\lambda = \underset{\sim}{S}$$

i.e.
$$\underset{\sim}{S} = \bigcup_{\lambda \in [-1,\, 0]} \lambda H(\lambda) \tag{2.11}$$

2°. Let $\lambda \in [0,\, 1]$, from definition 3.5 of [1], we get $\underset{\bullet}{S_\lambda} \subseteq H(\lambda) \subseteq S_\lambda$, $\lambda \underset{\bullet}{S_\lambda} \subseteq \lambda H(\lambda) \subseteq \lambda S_\lambda$

so :
$$\underset{\sim}{S} = \bigcup_{\lambda \in [0,1]} \lambda \underset{\bullet}{S_\lambda} \subseteq \bigcup_{\lambda \in [0,1]} \lambda H(\lambda) \subseteq \bigcup_{\lambda \in [0,1]} \lambda S_\lambda = \underset{\sim}{S}$$

i.e.
$$\underset{\sim}{S} = \bigcup_{\lambda \in [0,1]} \lambda H(\lambda) \tag{2.12}$$

For $\lambda \in [-1,\, 1]$, from (2.11), (2.12), we get

$$\underset{\sim}{S} = \bigcup_{\lambda \in [-1,\, 1]} \lambda H(\lambda) \tag{2.13}$$

Due to theorem 2.1, 2.2, 2.3, we get easily :

Theorem 2.4 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be B-B fuzzy set,

1°. if $\lambda_1 < \lambda_2$; $\lambda_1, \lambda_2 \in [0,\, 1]$, then

$$H(\lambda_1) \supseteq H(\lambda_2) \tag{2.14}$$

2°. if $\lambda_1 < \lambda_2$; $\lambda_1, \lambda_2 \in [-1,\, 0]$, then

$$H(\lambda_2) \supseteq H(\lambda_1) \tag{2.15}$$

Proof : 1°. if for λ_i, $\lambda_2 \in [0,\, 1]$ and $\lambda_1 < \lambda_2$, we have

$$H(\lambda_1) \supseteq \underset{\bullet}{S_{\lambda_1}} \supseteq H(\lambda_2) \supseteq \underset{\bullet}{S_{\lambda_2}}$$

then we get :
$$H(\lambda_1) \supseteq H(\lambda_2) \tag{2.16}$$

2° . if for λ_1 , $\lambda_2 \in [-1, 0]$ and $\lambda_1 < \lambda_2$, we have

$$H(\lambda_2) \supseteq \underset{\bullet}{S}_{\lambda_2} \supseteq H(\lambda_1) \supseteq \underset{\bullet}{S}_{\lambda_1}$$

then we get : $$H(\lambda_2) \supseteq H(\lambda_1) \qquad (2.17)$$

Theorem 2.5 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be B-B fuzzy set, we have :

1° . if $\lambda \in (0, 1)$, then

$$S_\lambda = \bigcap_{\alpha < \lambda} H(\alpha), \quad \lambda \neq 0 \qquad (2.18)$$

$$\underset{\bullet}{S}_\lambda = \bigcup_{\alpha > \lambda} H(\alpha), \quad \lambda \neq 1 \qquad (2.19)$$

2° . if $\lambda \in (-1, 0)$, then

$$S_\lambda = \bigcap_{\lambda < \alpha} H(\alpha), \quad \lambda \neq 0 \qquad (2.20)$$

$$\underset{\bullet}{S}_\lambda = \bigcup_{\alpha < \lambda} H(\alpha), \quad \lambda \neq -1 \qquad (2.21)$$

Proof : 1° . For $\lambda \in (0, 1)$, for any $\alpha < \lambda$, $H(\alpha) \supseteq \underset{\bullet}{S}_\alpha \supseteq S_\lambda$, so we get :

$$\bigcap_{\alpha < \lambda} H(\alpha) \supseteq S_\lambda$$

Due to $$\bigcap_{\alpha < \lambda} H(\alpha) \subseteq \bigcap_{\alpha < \lambda} S_\alpha = S_{(\bigvee_{\alpha < \lambda} \alpha)} = S_\lambda$$

we get : $$S_\lambda = \bigcap_{\alpha < \lambda} H(\alpha) \qquad (2.22)$$

For $\lambda \in (0, 1)$, for any $\alpha > \lambda$, similar to above, we get :

$$\underset{\bullet}{S}_\lambda = \bigcup_{\alpha > \lambda} H(\alpha) \qquad (2.23)$$

2° . For $\lambda \in (-1, 0)$, for any $\lambda < \alpha$, $S_\lambda \subseteq \underset{\bullet}{S}_\alpha \subseteq H(\alpha)$, so we get :

$$\bigcap_{\lambda < \alpha} H(\alpha) \supseteq S_\lambda$$

Due to $$\bigcap_{\lambda < \alpha} H(\alpha) \subseteq \bigcap_{\lambda < \alpha} S_\alpha = S_{(\bigwedge_{\lambda < \alpha} \alpha)} = S_\lambda$$

we get : $$S_\lambda = \bigcap_{\lambda < \alpha} H(\alpha) \qquad (2.24)$$

For $\lambda \in (-1, 0)$, for any $\alpha < \lambda$, similar to above, we get

$$\underset{\bullet}{S}_\lambda = \bigcup_{\alpha < \lambda} H(\alpha) \qquad (2.25)$$

Here we point out :

1° . B-B fuzzy set $\underset{\sim}{S}$ can be expressed by the union of ordinary set $S_\lambda \in \mathscr{P}(X)$: $\underset{\sim}{S} = \bigcup_{\lambda \in [-1, 1]} \lambda S_\lambda$;

Can B-B fuzzy set $\underset{\sim}{S}$ be expressed by the intersection of ordinary set $S_\lambda \in \mathscr{P}(X)$: $\underset{\sim}{S} = \bigcap_{\lambda \in [-1, 1]} \lambda S_\lambda$? If there exists $\underset{\sim}{S} = \bigcap_{\lambda \in [-1, 1]} \lambda S_\lambda$, then the two dual, supplementary forms bring great convenience for B-B fuzzy logic and B-B fuzzy reasoning theory.

2°. A simple fact accepted by all people explains why we want to find intersection-resolution forms of B-B fuzzy set $\underset{\sim}{S}$: many algebraic characters of the subsystem of an algebraic system are broken by union-operation, but they are conserved by intersection-operation. For example: the union of two subspaces is not necessarily so a subspace, the union of two subgroups is not necessarily so a subgroup,; On the contrary, the intersection of two subspace must be a subspace, the intersection of two subgroup must be a subgroup.

We carry these simple facts a step forward: there exist similar problems in B-B fuzzy logic and B-B fuzzy reasoning. So the intersection-ordinary resolution form of B-B fuzzy set $\underset{\sim}{S}$ is very important, and B-B fuzzy logic, B-B fuzzy reasoning will be discussed in later thesises.

3. Intersection -ordinary theorem of B-B fuzzy set

Definition 3.1 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be the B-B fuzzy set in X, $\lambda \in [-1, 1]$; Provided that $\lambda \underset{\sim}{S} \in \mathscr{F}(X)$, the fuzzy kiss function of $\lambda \underset{\sim}{S}$ is defined as :

$$(\lambda \underset{\sim}{S})(x) = \lambda \vee \underset{\sim}{S}(x) \qquad (3.1)$$

Theorem 3.1 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be the B-B fuzzy set in X, then

$$\underset{\sim}{S} = \bigcap_{\lambda \in [-1, 1]} \lambda S_\lambda \qquad (3.2)$$

Theorem 3.2 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be the B-B fuzzy set in X, then

$$\underset{\sim}{S} = \bigcap_{\lambda \in [-1, 1]} \lambda \underset{\bullet}{S_\lambda} \qquad (3.3)$$

In the following, we only give the proof of theorem 3.2 and that of theorem 3.1 is omitted.

Proof: 1°. Let $\lambda \in [0, 1]$, $\forall x \in X^+ \cup X^\circ$

$$(\bigcap_{\lambda \in [0, 1]} \lambda \underset{\bullet}{S_\lambda})(x) = \bigwedge_{\lambda \in [0, 1]} (\lambda \underset{\bullet}{S_\lambda})(x) = \bigwedge_{\lambda \in [0, 1]} (\lambda \vee \underset{\bullet}{S_\lambda}(x))$$

$$= (\bigwedge_{\lambda \in [0, \underset{\sim}{S}(x)]} (\lambda \vee \underset{\bullet}{S_\lambda}(x))) \wedge (\bigwedge_{\lambda \in [\underset{\sim}{S}(x), 1]} (\lambda \vee \underset{\bullet}{S_\lambda}(x)))$$

$$= \bigwedge_{\lambda \in [\underset{\sim}{S}(x), 1]} (\lambda \vee \underset{\bullet}{S_\lambda}(x)) = \bigwedge_{\lambda \in [\underset{\sim}{S}(x), 1]} \lambda = \underset{\sim}{S}(x)$$

so :

$$\underset{\sim}{S} = \bigcap_{\lambda \in [0, 1]} \lambda \underset{\bullet}{S_\lambda} \tag{3.4}$$

2°. Let $\lambda \in [-1, 0]$, for any $x \in X^- \cup X^\circ$, similar to 1° or see [4], we get

$$\underset{\sim}{S} = \bigcap_{\lambda \in [-1, 0]} \lambda \underset{\bullet}{S_\lambda} \tag{3.5}$$

For $\lambda \in [-1, 1]$, due to (3.4), (3.5), we get

$$\underset{\sim}{S} = \bigcap_{\lambda \in [-1, 1]} \lambda \underset{\bullet}{S_\lambda} \tag{3.6}$$

Theorem 3.3 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be the B-B fuzzy set in X, and

$$H: [-1, 1] \to \mathscr{P}(X)$$
$$\lambda \to H(\lambda)$$

satisfies : $\lambda \in [-1, 1]$, $\underset{\bullet}{S_\lambda} \subseteq H(\lambda) \subseteq S_\lambda$, then

$$\underset{\sim}{S} = \bigcap_{\lambda \in [-1, 1]} \lambda H(\lambda) \tag{3.7}$$

Proof : From definition 3.4, 3.5 of [1], we get :

1°. Let $\lambda \in [-1, 0]$, $\underset{\bullet}{S_\lambda} \subseteq H(\lambda) \subseteq S_\lambda$, $\lambda \underset{\bullet}{S_\lambda} \subseteq \lambda H(\lambda) \subseteq \lambda S_\lambda$

so

$$\bigcap_{\lambda \in [-1,0]} \lambda \underset{\bullet}{S_\lambda} \subseteq \bigcap_{\lambda \in [-1,0]} \lambda H(\lambda) \subseteq \bigcap_{\lambda \in [-1,0]} \lambda S_\lambda$$

Due to theorem 3.1, 3.2, we get :

$$\underset{\sim}{S} = \bigcap_{\lambda \in [-1,0]} \lambda S_\lambda \supseteq \bigcap_{\lambda \in [-1,0]} \lambda H(\lambda) \supseteq \bigcap_{\lambda \in [-1,0]} \lambda \underset{\bullet}{S_\lambda} = \underset{\sim}{S}$$

i.e.

$$\underset{\sim}{S} = \bigcap_{\lambda \in [-1,0]} \lambda H(\lambda) \tag{3.8}$$

2°. Let $\lambda \in [0, 1]$, $\underset{\bullet}{S_\lambda} \subseteq H(\lambda) \subseteq S_\lambda$, $\lambda \underset{\bullet}{S_\lambda} \subseteq \lambda H(\lambda) \subseteq \lambda S_\lambda$

so :

$$\bigcap_{\lambda \in [0,1]} \lambda \underset{\bullet}{S_\lambda} \subseteq \bigcap_{\lambda \in [0,1]} \lambda H(\lambda) \subseteq \bigcap_{\lambda \in [0,1]} \lambda S_\lambda$$

Due to theorem 3.1, 3.2, we get :

$$\underset{\sim}{S} = \bigcap_{\lambda \in [0,1]} \lambda \underset{\bullet}{S_\lambda} \subseteq \bigcap_{\lambda \in [0,1]} \lambda H(\lambda) \subseteq \bigcap_{\lambda \in [0,1]} \lambda S_\lambda = \underset{\sim}{S}$$

i.e.

$$\underset{\sim}{S} = \bigcap_{\lambda \in [0,1]} \lambda H(\lambda) \tag{3.9}$$

For $\lambda \in [-1, 1]$, from (3.8), (3.9), we get

$$\underset{\sim}{S} = \bigcap_{\lambda\in[-1,1]} \lambda H(\lambda) \tag{3.10}$$

From theorem 3.1, 3.2, we get directly :

Theorem 3.4 Let $\underset{\sim}{S} \in \mathscr{F}(X)$ be the B-B fuzzy set in X, $\underset{\sim}{S}(X) \subseteq Q \subseteq [-1, 1]$, $\underset{\sim}{S}(X) = \{\underset{\sim}{S}(x) | x\in X\}$, then

$$\underset{\sim}{S} = \bigcap_{\lambda\in Q} \lambda S_{\underset{\bullet}{\lambda}} \tag{3.11}$$

Proof : From definition 3.4, 3.5 of [1] and definition 2.1, 2.2, 2.3 of this paper ; $Q^{'} \subseteq [0, 1]$, $Q^{''} \subseteq [-1, 0]$, and $Q = Q^{'} \cup Q^{''}$

1°. Let $\lambda \in Q^{'}$, for any $x \in X^{+} \cup X^{\circ}$, we have

$$(\bigcap_{\lambda\in Q^{'}} \lambda S_{\underset{\bullet}{\lambda}})(x) = ((\bigcap_{\underset{\sim}{S}(x)>\lambda} \lambda S_{\underset{\bullet}{\lambda}}) \cap (\bigcap_{\underset{\sim}{S}(x)\le\lambda} \lambda S_{\underset{\bullet}{\lambda}}))(x)$$

$$= ((\bigcap_{\underset{\sim}{S}(x)>\lambda} \lambda S_{\underset{\bullet}{\lambda}})(x)) \wedge ((\bigcap_{\underset{\sim}{S}(x)\le\lambda} \lambda S_{\underset{\bullet}{\lambda}})(x))$$

$$= (\bigwedge_{\underset{\sim}{S}(x)>\lambda} (\lambda \vee S_{\underset{\bullet}{\lambda}}(x))) \wedge (\bigwedge_{\underset{\sim}{S}(x)\le\lambda} (\lambda \vee S_{\underset{\bullet}{\lambda}}(x)))$$

$$= \bigwedge_{\underset{\sim}{S}(x)\le\lambda} \lambda = \underset{\sim}{S}(x)$$

so :

$$\underset{\sim}{S} = \bigcap_{\lambda\in Q^{'}} \lambda S_{\underset{\bullet}{\lambda}} \tag{3.12}$$

2°. Let $\lambda \in Q^{''}$, for any $x \in X^{-} \cup X^{\circ}$, similar to 1°, we get

$$\underset{\sim}{S} = \bigcap_{\lambda\in Q^{''}} \lambda S_{\underset{\bullet}{\lambda}} \tag{3.13}$$

For $\lambda \in Q = Q^{'} \cup Q^{''}$, from (3.12), (3.13), we get

$$\underset{\sim}{S} = \bigcap_{\lambda\in Q} \lambda S_{\underset{\bullet}{\lambda}} \tag{3.14}$$

Here we point out :

1°. The second part of this paper gives three union-ordinary resolution theorems and some relational of B-B fuzzy set $\underset{\sim}{S}$. When $\lambda \in [-1, 1]$ turns into $\lambda \in [0, 1]$ (or $X^{-} = \varnothing$), union-ordinary resolution forms of B-B set $\underset{\sim}{S}$ turn into those of O-B fuzzy set $\underset{\sim}{A}$ given in [3].

2°. The third part of this paper gives three intersection-ordinary resolution theorems and some relational results of B-B fuzzy set $\underset{\sim}{S}$. When $\lambda \in [-1, 1]$ turns into $\lambda \in [0,1]$ (or $X^{-} = \varnothing$), intersection-ordinary resolution forms of B-B fuzzy set $\underset{\sim}{S}$ turn into those of O-B fuzzy set $\underset{\sim}{A}$ given in [4].

3°. Union-ordinary resolution forms, intersection-ordinary resolution forms of B-B fuzzy set $\underset{\sim}{S}$ constitute B-B fuzzy dual, supplementary ordinary resolution system.

From the above discussion of 2, 3, we give without proof :

4. Relational theorem of union-ordinary resolution and intersection-ordinary resolution of B-B fuzzy set

Theorem 4.1 Let $\bigcup_{\lambda\in[-1,1]} \lambda S_\lambda$, $\bigcap_{\lambda\in[-1,1]} \lambda S_\lambda$ be union-ordinary resolution, intersection-ordinary resolution of $\underset{\sim}{S}$, respectively ; for any λ, then

$$\bigcup_{\lambda\in[-1,1]} \lambda S_\lambda = \bigcap_{\lambda\in[-1,1]} \lambda S_\lambda \tag{4.1}$$

where S_λ is the λ-cutset of $\underset{\sim}{S}$, $\lambda\in[-1, 1]$, $S_\lambda\in\mathscr{P}(X)$, $\underset{\sim}{S}\in\mathscr{F}(X)$.

Theorem 4.2 Let $\bigcup_{\lambda\in[-1,1]} \lambda \underset{\bullet}{S_\lambda}$, $\bigcap_{\lambda\in[-1,1]} \lambda \underset{\bullet}{S_\lambda}$ be union-ordinary resolution, intersection-ordinary resolution of $\underset{\sim}{S}$, respectively ; for any λ, then

$$\bigcup_{\lambda\in[-1,1]} \lambda \underset{\bullet}{S_\lambda} = \bigcap_{\lambda\in[-1,1]} \lambda \underset{\bullet}{S_\lambda} \tag{4.2}$$

where $\underset{\bullet}{S_\lambda}$ is the λ-strong cutset of $\underset{\sim}{S}$, $\lambda\in[-1, 1]$, $\underset{\bullet}{S_\lambda}\in\mathscr{P}(X)$, $\underset{\sim}{S}\in\mathscr{F}(X)$.

Theorem 4.3 Let $\bigcup_{\lambda\in[-1,1]} \lambda H(\lambda)$, $\bigcap_{\lambda\in[-1,1]} \lambda H(\lambda)$ be union-ordinary resolution, intersection-ordinary resolution of $\underset{\sim}{S}$, respectively ; for any λ, then

$$\bigcup_{\lambda\in[-1,1]} \lambda H(\lambda) = \bigcap_{\lambda\in[-1,1]} \lambda H(\lambda) \tag{4.3}$$

where H(λ) is the λ-cutset of $\underset{\sim}{S}$, $\lambda\in[-1, 1]$, $H(\lambda)\in\mathscr{P}(X)$, $\underset{\sim}{S}\in\mathscr{F}(X)$.If B-B fuzzy set $\underset{\sim}{S}$ degenerates into O-B fuzzy set $\underset{\sim}{A}$, then theorem 4.1, 4.2, 4.3 degenerate into relational theorems of union-ordinary resolution and intersection-ordinary resolution of L.A.Zadeh fuzzy set ; $\lambda\in[-1,1]$ degenerates into $\lambda\in[0,1]$, $X=X^+\cup X^-\cup X^\circ$ degenerates into $X=X^+\cup X^\circ$, i.e.

Theorem 4.4 Let $\bigcup_{\lambda\in[0,1]} \lambda A_\lambda$, $\bigcap_{\lambda\in[0,1]} \lambda A_\lambda$ be union-ordinary resolution, intersection-ordinary resolution of $\underset{\sim}{A}$, respectively ; for any λ, then

$$\bigcup_{\lambda\in[0,1]} \lambda A_\lambda = \bigcap_{\lambda\in[0,1]} \lambda A_\lambda \tag{4.4}$$

where $\underset{\sim}{A}$ is L.A. Zadeh fuzzy set, A_λ is λ-cutset of $\underset{\sim}{A}$.

Theorem 4.5 Let $\bigcup_{\lambda\in[0,1]} \lambda \underset{\bullet}{A_\lambda}$, $\bigcap_{\lambda\in[0,1]} \lambda \underset{\bullet}{A_\lambda}$ be union-ordinary resolution, intersection-ordinary resolution of $\underset{\sim}{A}$, respectively ; for any λ, then

$$\bigcup_{\lambda\in[0,1]} \lambda \underset{\bullet}{A_\lambda} = \bigcap_{\lambda\in[0,1]} \lambda \underset{\bullet}{A_\lambda} \tag{4.5}$$

where $\underset{\sim}{A}$ is L.A. Zadeh fuzzy set , $\underset{\bullet}{A_\lambda}$ is the λ-strong cutset of $\underset{\sim}{A}$.

Theorem 4.6 Let $\bigcup_{\lambda\in[0,1]} \lambda H(\lambda)$, $\bigcap_{\lambda\in[0,1]} \lambda H(\lambda)$ be union-ordinary resolution, intersection-ordinary resolution of $\underset{\sim}{A}$, respectively ; for any λ, then

$$\bigcup_{\lambda\in[0,1]} \lambda H(\lambda) = \bigcap_{\lambda\in[0,1]} \lambda H(\lambda) \tag{4.6}$$

where $\underset{\sim}{A}$ is L.A. Zadeh fuzzy set, H(λ) is λ-cutset of $\underset{\sim}{A}$.

Clearly, theorem 4.4, 4.5, 4.6 are direct corollaries of theorem 4.1, 4.2, 4.3 ; in fact, theorem 4.4, 4.5, 4.6 can be gotten directly by [3], [4].

References

[1]. Shi Kaiquan. B-B Fuzzy Sets $\underset{\sim}{S}$ (I), BUEFAL, France, (to appear).

[2]. L.A.Zadeh, Fuzzy Sets, Information and Control, 8, 1965, 338 — 353.

[3]. Luo Chengzhong. Fuzzy Sets and Set Covers, Journal of Fuzzy Mathematics, 4, 1983, 113 — 126. (In Chinese)

[4]. Gu Wenxiang, Zhou Jun. A New Resolution Theory of Fuzzy Sets, Journal of Northeast Normal University, 2,1995, 6 — 7. (In Chinese)