Schein Decomposition of a F-matrix

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Abstract: the definition of Schein decomposition of a F-matrix was given in this paper. Principally, we were proved the following results:

let $A \in M(m\times n)$, $B \in M(m\times p)$ and $C \in M(p\times q)$ are non-zero.

(1) if $F \in M(m\times s)$ and $G \in M(s\times n)$ are a pair Schein decomposition matrices of A, then FP_s and $P_s^T G$ are also a pair Schein decomposition matrices of A.

(2) $\rho_s(P_m A) = \rho_s(A)$, $\rho_s(AP_n) = \rho_s(A)$.

(3) $\rho_s(ABC) \le \rho_s(A)$; $\rho_s(ABC) \le \rho_s(B)$; $\rho_s(ABC) \le \rho_s(C)$.

Keyword: Schein rank, Schein decomposition, Permutation matrix.

Definition 1[2] Let $A \in M(m\times n)$ are non-zero. The Schein rank ρ_s of matrix A is the least number of rank 1 matrices whose sum is A, where $M(m\times n)$ is a set of all $m\times n$ F-matrices.

Definition 2 let $A \in M(m\times n)$ are non-zero.,and $\rho_s(A)=s$,and let

$$A=\begin{bmatrix} c_{11} \\ \cdots \\ c_{m1}\end{bmatrix}(d_{11}\cdots d_{1n})+\cdots+\begin{bmatrix} c_{1s} \\ \cdots \\ c_{ms}\end{bmatrix}(d_{s1}\cdots d_{sn})$$

$$=\begin{bmatrix} c_{11} & \cdots & c_{1s} \\ \cdots & \cdots & \cdots \\ c_{m1} & \cdots & c_{ms}\end{bmatrix}\begin{bmatrix} d_{11} & \cdots & d_{1n} \\ \cdots & \cdots & \cdots \\ d_{s1} & \cdots & d_{sn}\end{bmatrix}=C_{m\times s}\,D_{s\times n} \qquad (※)$$

where all $c_{ij} \in [0,1]$ and $d_{ij} \in [0,1]$,and $C=(c_{ij})_{m\times s}$ and $D=(d_{ij})_{s\times n}$ are non-zero.(※) is called a Schein decomposition of A. And C and D are called a pair matrices of Schein decomposition of A.

Theorem 1. Let $A \in M(m\times n)$ are non-zero. If there exist a pair non-zero F-matrices $C_{m\times k}$ and $D_{k\times n}$ such that $A=C_{m\times k}\,D_{k\times n}$ then $\rho_s(A) \le k$.

The some columns of a s-order unit square matrix I_s are exchanged, we shall get a matrix P whose is called a permutation matrix. And $I_s(i,j)$ is a permutation matrix that column i and column j of a unit square matrix I_s are exchanged.

By proposition 1,2 and theorem 1.1 in reference [2] we get that

Lemma 1 $I_s(i,j)\, I_s(i,j)=I_s$.

Theorem 2 Let $A\in M(m\times n)$ are non-zero. If $C\in M(m\times s)$ and $D\in M(s\times n)$ are a pair matrices of Schein decomposition of A, then $C\, I_s(i,j)$ and $I_s(i,j)\, D$ are also a pair matrices of Schein decomposition of A.

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Theorem $2'$. Let $A\in M(m\times n)$ are non-zero. If $C\in M(m\times s)$ and $D\in M(s\times n)$ are a pair matrices of Schein decomposition of A, column i and column j of C are exchanged, we shall get C^*, at the same time row i and row j of D are exchanged, we shall get D^*, then C^* and D^* are also a pair matrices of Schein decomposition of A.

Lemma 2.[2] $PP^T = P^T P = I_h$, where P is any h-order permutation matrix.

Theorem 3. Let $A\in M(m\times n)$ are non-zero. If $C\in M(m\times s)$ and $D\in M(s\times n)$ are a pair matrices of Schein decomposition of A, where P is any s-order permutation matrix..

Theorem 4. Let $A\in M(m\times n)$ are non-zero symmetrical matrix. If $C_{n\times s}$ and $D_{s\times n}$ are a pair matrices of Schein decomposition of A, then D^T and C^T are also a pair matrices of Schein decomposition of A.

Theorem 5. Let $A\in M(m\times n)$ are non-zero. If $\rho_s(A)$=s,then $\rho_s(I_m(i,j)A) = s$ and $\rho_s(AI_m(i,j)) = s$.

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Theorem $5'$ Let $A\in M(m\times n)$ are non-zero matrix, and $\rho_s(A)$=s.

(1) Row i and row j of A are exchanged, we shall get A^*, then $\rho_s(A^*)$=s.

(2) Column i and column j of A are exchanged, we shall get A^*, then $\rho_s(A^*)$=s.

Theorem 6 Let $A\in M(m\times n)$ are non-zero. If $\rho_s(A)$=s, then $\rho_s(P_m A) = s$, $\rho_s(AP_n) = s$.

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Theorem $6'$. Let $A\in M(m\times n)$ are non-zero matrix, and $\rho_s(A)$=s.

(1) The some rows of A are exchanged, we shall get a matrix A^* then then $\rho_s(A^*)$=s.

(2) The some columns of A are exchanged, we shall get a matrix A^*

then then $\rho_s(A^*)$=s.

We can spread a property of Schein rank in reference [1] $\rho_s(AMB) \le \rho_s(M)$.

Theorem 7. If A1 $\in M(m_1 \times m_2)$,A2 $\in M(m_2 \times m_3)$,…,Ai $\in M(m_i \times m_{i+1})$, … ,Ak $\in M(m_k \times m_{k+1})$ are some non-zero fuzzy matrices,then $\rho_s(A_1 \cdots A_i \cdots A_k) \le \rho_s(A_i)$ (i=1,…,k).

Proof. Let $\rho_s(A_i)$=s,and Schein decomposition of A_i is $A_i = C_{m_i \times s} D_{s \times m_{i+1}}$ then

$A_1 \cdots A_i \cdots A_k = (A_1 \cdots A_{i-1} C)_{m_1 \times s} (D A_{i+1} \cdots A_k)_{s \times m_{k+1}}$ therefore $\rho_s(A_1 \cdots A_i \cdots A_k) \le s = \rho_s(A_i)$ (i=1,…,k) i.e. $\rho_s(A_1 \cdots A_i \cdots A_k) \le \rho_s(A_i)$.

References

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