M-N-fuzzy normal subgroups

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Abstrct: The paper [1] introduces the concept of M-fuzzy groups. In this paper, based on this, the concept of M-N-fuzzy subgroup and M-N-fuzzy normal subgroup are given, and its some elmentary properties are discussed.

Keywords: Fuzzy subgroup; Fuzzy normal subgroup; M-N-fuzzy subgroup; M-N-fuzzy normal subgroup, M-N-homomorphism.

1. Introduction

In 1971 A. Rosenfeld [3] introduced the concept of fuzzy subgroup. In 1981 Wu [4] studied the fuzzy normal subgroup. W. X. Gu, S. Y. Li and D. G. Chen [1] further studied in 1994 the theory of the fuzzy groups and gave some new concepts such as M-fuzzy subgroup, M-fuzzy normal subgroup, etc.

In this paper, based on the reference [1], the concept of M-N-fuzzy subgroup and M-N-fuzzy normal subgroup are given, and its some elementary properties are discussed, some results in reference [1] are extended.

2. Preliminaries

For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1 (Rosenfeld [3]). A fuzzy set A of group G is called a fuzzy subgroup if

(1) $A(xy) \geqslant A(x) \wedge A(y)$ for all x, y in G;

(2) $A(x^{-1}) \geqslant A(x)$ for all x in G.

Definition 2.2 (Wu [4]). A fuzzy subgroup A of a group G is called a fuzzy normal subgroup if

$A(xyx^{-1}) \geqslant A(y)$ for all x, y in G.

Definition 2.3 (Xiong[5]) Let M is left operator sets of group G, N is right operator sets of group G. If

$(ma)n = m(an)$ for all a in G, m in M, n in N.

then G is said to be an M-N-group. If a subgroup of M-N-group is also M-N-group, then it is said to be an M-N-subgroup of G. If M-N-subgroup is also normal subgroup, then it is said to be M-N-normal subgroup.

Definition 2.4 (Xiong [5]). Let G and G' both be M-N-groups. f be a homomorphism from G onto G'. If $f(mx) = mf(x)$ and $f(xn) = f(x)n$ for all x in G, m in M, n in N, then f is called an M-N-homomorphism.

3. ***M*-*N*-fuzzy subgroups**

Definition 3.1. Let G be an M-N-group and A be a fuzzy subgroup of G. If

(1) $A(mx) \geqslant A(x)$;

(2) $A(xn) \geqslant A(x)$.

holds for any $x \in G, m \in M, n \in N$, then A is said to be an M-N-fuzzy subgroup of G.

It is clear that Definition 3.1 is the generalization of the general M-N-subgroup.

Proposition 3.1. Let G be an M-N-group, A and B both be M-N-fuzzy subgroups of G. Then $A \cap B$ is an M-N-fuzzy subgroup of G.

Proposition 3. 2. If A is an M-N-fuzzy subgroup of an M-N-group G, then the following statements hold for all x, y in G m in M, n in N:

(1) $A((m(xy))n) \geqslant A(x) \wedge A(y)$;

(2) $A((mx^{-1})n) \geqslant A(x)$.

Proposition 3. 3. Let G be an M-N-group, A be a fuzzy set of G, then A is M-N-fuzzy subgroup of G iff for any $t \in [0,1]$, A_t is an M-N-subgroup of G when $A_t \neq \varnothing$.

Proposition 3. 4. Let G and G' both be M-N-groups and f an M-N-homomorphism from G onto G'. If A' an M-N-fuzzy subgroup of G', then $f^{-1}(A')$ is an M-N-fuzzy subgroup of G.

Proposition 3. 5. Let G and G' both be M-N-groups, f an M-N-homomorphism from G onto G', and A an M-N-fuzzy subgroup of G, then $f(A)$ is an M-N-fuzzy subgroup of G'.

4. *M-N*-fuzzy normal subgroups

Definition 4. 1. Let G be an M-N-group. A is said to be an M-N-fuzzy normal subgroup of G if A is not only an M-N-fuzzy subgroup of G, but also a fuzzy normal subgroup of G.

Proposition 4. 1. Assume C is the characteristic function of A which is a nonempty subset of the M-N-group G. Then C is an M-N-fuzzy normal subgroup iff A is an M-N-normal subgroup of G.

Proposition 4. 2. Let A and B both be M-N-fuzzy normal subgroups of an M-N-group G. Then $A \cap B$ is an M-N-fuzzy normal subgroup of an M-N-group G.

Proposition 4. 3. If A is an M-N-fuzzy normal subgroup of the M-N-group G, then

(1) $(aA)((mx)n) \geqslant A(a) \wedge A(x)$ $a, x \in G$, $m \in M$, $n \in N$;

(2) $A((m(xy))n) = A((m(yx))n)$ $x, y \in G, m \in M, n \in N$.

Proposition 4. 4. Let A be a fuzzy set of the M-N-group G, then A is an M-N-fuzzy normal subgroup iff A_t is an M-N-normal subgroup of G for any $t \in [0,1]$ *when* $A_t \neq \emptyset$ *holds*.

Proposition 4. 5. Let A be an M-N-fuzzy subgroup of G while the identity operator is included in $M \cap N$. Then A is an M-N-fuzzy normal subgroup of G iff

(1) $A(m(xyx^{-1})) = A(my)$, $x, y \in G, m \in M$;

(2) $A((xyx^{-1})n) = A(yn)$, $x, y \in G, n \in N$.

Proposition 4. 6. Let A be an M-N-fuzzy subgroup of M-N-group G, while the identity operator is included in $M \cap N$, then A is an M-N-fuzzy normal subgroup of G iff $A(m(xy)) = A(m(yx))$ *and* $A((xy)n) = A((yx)n)$ *for all* x, y *in* G, m *in* M, n *in* n.

Proposition 4. 7. Let f be an homomorphism from the M-N-group G onto the M-N-group G'. Then the preimage which can be written as $f^{-1}(A')$ *of* A' under f where A' is an M-N-fuzzy normal subgroup of G' is an M-N-fuzzy normal subgroup of G.

Proposition 4. 8. Let f be an M-N-homomorphism from the M-N-group G to the M-N-group G'. Then the image which can be written as $f(A)$ of under f is an M-N-fuzzy normal subgroup in case of A beng an M-N-fuzzy normal subgroup of G.

Let G be a M-N-group and B an M-N-fuzzy normal subgroup of G. Wu [4] had proved that G/B was a group.

Propositoin 4. 9. G/B is a M-N-group.

Now we define a fuzzy set on G/B. Let A be any *M-N-fuzzy* group of G, A/B be a fuzzy set of G/B defined as follows:

$$A/B : G/B \to [0,1] \ \textit{satisfying}$$

$$A/B(aB) = \sup_{xB = aB} A(x)$$

for all aB in G/B.

Proposition 4. 10. The above fuzzy subset A/B is an M-N-fuzzy subgroup of G/B.

Proposition 4. 11. Let G be an M-N-group, A an M-N-fuzzy subgroup of G while B is an M-N-fuzzy normal subgroup of G. Let

$$r: G \rightarrow G/B$$
$$x \rightarrow xB.$$

Then r is an M-N-homomorphism from G onto G/B and $r(A)=A/B$.

References

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