States Knowledge–based Filtering of a Fuzzy State Model

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Abstract:
This paper deals with states calculation by a fuzzy state prediction model. The problem of vagueness reduction due to fuzzy arithmetics is solved by a modified algorithm of Sugeno controller. This access is problem oriented and it is based on expert knowledge about the concrete controlled system. The solving of this problem is showed on the example of an aeroplane.

Keywords: state description, fuzzy numbers, fuzzy arithmetics, Sugeno filter, similarity relations

1 Fuzzy state description of a system

The behaviour of a physical system is often dependent also on inner states of that one. Hence the information about inputs and outputs is not sufficient. The only system description involving also inner states is the state description (1), (2) that enables also prediction of the next state. The parameters used in such a description are calculated by linear or nonlinear relations. However, these relations are often inprecise or thay can be only estimate. The need of handling inprecise parameters arises. Convenient means for description of such parameters are fuzzy numbers and fuzzy arithmetics [3], [1].

There are two problems associated with fuzzy arithmetics:

1. calculation complexity — increase of calculation time
2. loss of information — average value decrease of grades of membership

The first problem can be solved using more efficient and quicker processors. The second problem is solved in this contribution.

1.1 Behaviour description problem of an aeroplane

An aeroplane is a nonlinear complex system. There are hundreds of variables and parameters that are changed during a flight either continuously (e.g. changes of velocity, height and angle of flight or fuel consumption) or steply (sudden wind changes or changes of wing profile using leeding edges or flaps). The nonlinearities affect behaviour robustness of such a system very negatively. This causes correct description of an aeroplane only in a very small interval around the set–point. The state description is the only possible way to describe such a system because the look at this system as a black box with its inputs and outputs is not sufficient. Therefore the information about inner states is necessary.

The behaviour of an aeroplane can be described by a set of parameters characterising its fuselage and air qualities. These are mostly measured in aerodynamical tunnels with limited precision. Concrete state values in time t_1 together with aeroplane state model parameters enable calculation of the aeroplane state in time t_2 ($t_2 > t_1$). If a sampling period T is given then $t_1 = k_1.T$ and $t_2 = k_2.T$ where k_1 and k_2 are sampling steps ($k_2 > k_1$). Using Z–transformation enables to describe a state model in the sampled time in the following form:

$$\begin{pmatrix} x_1((k+1)T) \\ x_2((k+1)T) \\ \vdots \\ x_n((k+1)T) \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} & \dots & F_{1n} \\ F_{21} & F_{22} & \dots & F_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ F_{n1} & F_{n2} & \dots & F_{nn} \end{pmatrix} . \begin{pmatrix} x_1(kT) \\ x_2(kT) \\ \vdots \\ x_n(kT) \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} . u(kT) \quad (1)$$

$$y(kT) = (C_1, C_2, \dots, C_n) . \begin{pmatrix} x_1(kT) \\ x_2(kT) \\ \vdots \\ x_n(kT) \end{pmatrix} + D . u(kT) \quad (2)$$

where x_1, x_2, ..., x_n, u and y are states, input and output of the described system, respectively. F_{ij}, Bi, Ci and D are characteristic parameters of an aeroplane.

1.2 Vagueness in a state model

Parameters and quantities of such a state model are often afflicted with certain errors manifesting in form of vaguenesses. The vaguenesses can by from principle devided in two groups:

1. vagueness of parameters

2. vagueness of measured variables

The first type of vagueness is due to insatisfactory knowledge about the described system, i.e. insatisfactorily precise model, e.g. inprecisely specified constants, neglecting nonlinearities, etc. Many aeroplane parameters are measured only under certain conditions (e.g. under certain velocities) not under all conditions. A need of approximation arises. This introduces also inprecision in the model. The second type of vagueness is due to measurement inprecision, noised signals among sensors and chip, etc. Going out from (1) and (2) the parameters F_{ij}, B_i, C_i and D belong to the first type and x_i, u belong to the second one.

To describe vaguenesses it is possible to use fuzzy sets for their representation [4]. The parameters and states can be in form of fuzzy numbers. The matrix representation of state model description enables to apply fuzzy arithmetics (by name multiplication and addition of fuzzy numbers). A number of multiplication and addition operations is to be performed to calculate the result. However, such a calculation causes loss of information. The support of membership functions is dilated and the average value of grades of membership falls down. It is lower and the peak of such a membership function is not more so expressive. Such a result is too vague and not more usable. The aim is to restrict these undesirable effects.

2 Filtering of membership functions

There are many designs of base operations in the fuzzy arithmetics [3] aiming to optimize the calculation with regard to the minimum support and the maximum peak expressivity of the membership function. These accesses are based on general mathematical principles of fuzzy sets theory. On the other side it is possible also another course based on modification of the already calculated membership function. In this case its shape is additionally narrowed using ad-hoc knowledge about the concrete system but not more general than in the first case. This method is similar to the filtering from the technical point of view.

A chart diagram in fig. 1 shows the design of the whole filtering feedback control circuit where the results of the fuzzy state prediction model are filtered to be got less vague values and then proceeded to a controller. In dependence of the controller used they can be defuzzified when a classical controller is used (e.g. a PID controller).

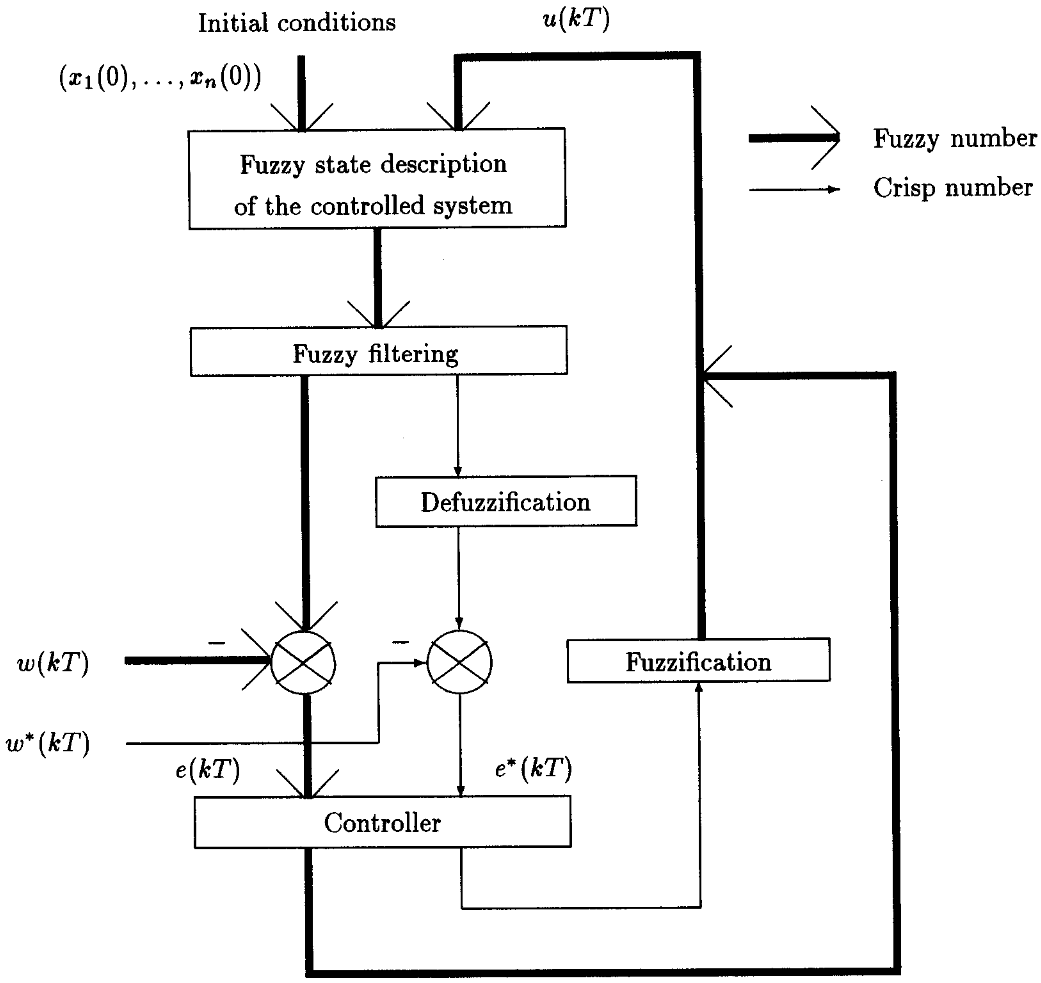

Figure 1: The chart diagram of filtering feedback control circuit (thick arrows — flow of fuzzy numbers, thin arrows — flow of crisp numbers).

2.1 Sugeno filter

The Sugeno filter is a special case of the Sugeno controller [5] belonging to fuzzy controllers. It arose as a modification of the Mamdani controller. The only difference is in form of fuzzy *IF - THEN* production rules.

Let be a system of m *IF - THEN* rules in form:

$$\begin{array}{llllllll} IF & x_1 \; is \; LX_1^1 & \& & \ldots & \& & x_n \; is \; LX_n^1 & THEN & u_1^* = f_1(x_1, \ldots, x_n) \\ IF & x_1 \; is \; LX_1^2 & \& & \ldots & \& & x_n \; is \; LX_n^2 & THEN & u_2^* = f_2(x_1, \ldots, x_n) \\ & & & & \vdots & & & \\ IF & x_1 \; is \; LX_1^m & \& & \ldots & \& & x_n \; is \; LX_n^m & THEN & u_m^* = f_m(x_1, \ldots, x_n) \end{array} \qquad (3)$$

LX are linguistic values and u_i^* are crisp (not more fuzzy) values of partial outputs for each rule i, $i = 1, 2, \ldots, m$, respectively. u_i^* are computed by analytical functions f_i. The total result u^* is then computed as the weighted average of u_i^*. The strengths of rules α_{u_i} are the weights:

$$u^* = \frac{\sum_{i=1}^{m} \alpha_{u_i}.u_i^*}{\sum_{i=1}^{m} \alpha_{u_i}} \tag{4}$$

Let us suppose a special case that f_i is a linear function, i.e.:

$$u_i^* = c_{1i}.x_1 + c_{2i}.x_2 + \cdots + c_{ni}.x_n, \tag{5}$$

where c_{ji} are constants, then:

$$u^* = \frac{\sum_{i=1}^{m} (c_{1i}.x_1 + c_{2i}.x_2 + \cdots + c_{ni}.x_n).\alpha_{u_i}}{\sum_{i=1}^{m} \alpha_{u_i}} \tag{6}$$

$$u^* = \frac{\sum_{i=1}^{m} c_{1i}.\alpha_{u_i}}{\sum_{i=1}^{m} \alpha_{u_i}}.x_1 + \frac{\sum_{i=1}^{m} c_{2i}.\alpha_{u_i}}{\sum_{i=1}^{m} \alpha_{u_i}}.x_2 + \cdots + \frac{\sum_{i=1}^{m} c_{ni}.\alpha_{u_i}}{\sum_{i=1}^{m} \alpha_{u_i}}.x_n \tag{7}$$

If we introduce a substitution from p_1 to p_n we get:

$$u^* = p_1.x_1 + p_2.x_2 + \cdots + p_n.x_n \tag{8}$$

The equation (8) gives us a linear Sugeno controller (regardless of that how we got the parameters p_i). If each state variable x_i has its derivatives in form $(x_i^{(0)}, x_i^{(1)}, x_i^{(2)}, \ldots, x_i^{(n)})$ (see (3)) and such a n-tity is the input to the Sugeno controller then it becomes a linear filter. The filtering quality depends directly proportional on the number of derivatives. We can so apply to output y and n state variables x_i (see (1) and (2)) $n+1$ different Sugeno filters. The condition of the derivatives existence for y and each x_i is but often very strict and cannot be fulfilled in many cases as too few derivatives are available and the filtering is little effective. Therefore a conception of a modified Sugeno filter is designed here to avoid the fulfilment necessity of this condition. The knowledge base of such a filter is case dependent and its results may be used as less inprecise inputs for a fuzzy controller of flight stability.

2.1.1 Modified Sugeno filter

A case dependent modification of Sugeno filter was designed to simplify and to enable its use also for cases when the classical Sugeno filtering is not effective. The main difference between this modification and the classical access with regard to generality of the method is that the classical Sugeno filtering is general with the same *IF - THEN* rules while the modified method is ad–hoc, in other words a new filter with case dependent rules must be designed for each another system to be controlled. The need of an expert arises which is able to create these rules. The modified method is explained under condition that the membership functions used are triangular (see fig. 2). Of course, also other shapes of membership functions can be used in general.

The *IF - THEN* rules are composed from one input (9) and one or three outputs (depending on that whether a fuzzy or a crisp number is to be calculated). Let be m filtering rules for the state variable x_i then the j-th rule looks:

$$IF \quad x_i \; is \; px_i^j \quad THEN \quad u_{iA}^{j*} = f_{iA}^j(x_i) \quad \& \quad u_{iB}^{j*} = f_{iB}^j(x_i) \quad \& \quad u_{iC}^{j*} = f_{iC}^j(x_i) \tag{9}$$

x_i is the result of the computational process of the fuzzy state model and its membership function has a wide support. It is simply too fuzzy. Functions $f_{iA}^j(x_i)$, $f_{iB}^j(x_i)$ and $f_{iC}^j(x_i)$ for calculating u_{iA}^{j*}, u_{iB}^{j*} and u_{iC}^{j*}, respectively define membership functions of so–called prominent values u_i^j.

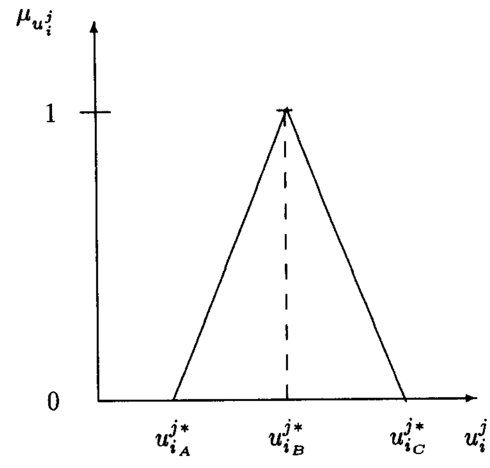

Figure 2: A triangular membership function with its prominent points $u_{i_A}^{j*}$, $u_{i_B}^{j*}$, $u_{i_C}^{j*}$.

The task is to narrow the support of x_i to get a less fuzzy number. The calculated x_i is compared with the image px_i^j of the prominent value u_i^j rule by rule (see fig. 3):

$$px_i^j \Leftrightarrow u_i^j \tag{10}$$

Prominent values are the relatively precisely measured values under certain standard conditions like e.g. some typical heights, air velocities etc. It is to be mentioned here that the aeroplane parameters are measured only at several values of hights or air velocities. If these parameters are measured at other values than the standard one then they can alter very sudden in large intervals and their evaluation is only more or less precise. It is the task of an expert to define the transformation relations (10). In other words, how can a prominent value u_i^j look when it proceedes to the fuzzy state prediction model and then becames more fuzzy, i.e. px_i^j? x_i, px_i^j and u_i^j have of course the same physical dimension and meaning. The sets of the *IF - THEN* rules are defined for each state x_i and output y in the same way too.

Both x_i and px_i^j are fuzzy numbers represented by membership functions. To calculate the measure of truth how much x_i is identical (similar) to px_i^j the so-called similarity relations are used [2], [6]. The result is a similarity index (a crisp number) directly proportional to similarity between x_i and px_i^j. The similarity index is also the strength of the competent rule, i.e. $\alpha_{u_i^j}$ for the i-th filter in this case and the total output $fu_{i_U}^*$ ($U = A, B, C$, respectively) is computed similarly to (4) to construct the filtered membership function fu_i:

$$fu_{i_U}^* = \frac{\sum_{j=1}^{m} \alpha_{u_i^j} . u_i^{j*}}{\sum_{j=1}^{m} \alpha_{u_i^j}} \tag{11}$$

If a crisp value is needed then the simplest way is to calculate only $fu_{i_B}^*$, i.e. $fu_{i_A}^* = fu_{i_B}^* = fu_{i_C}^*$.

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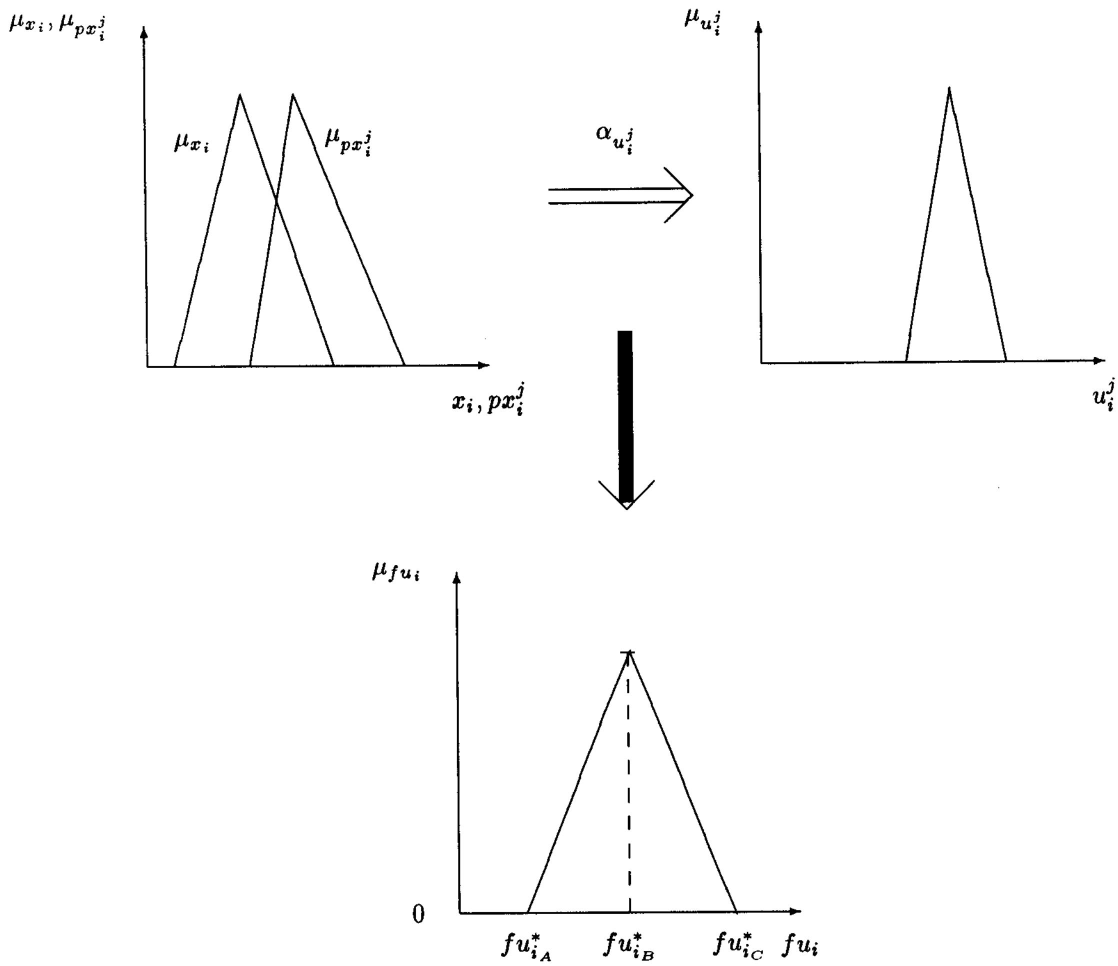

Figure 3: Evaluation of a rule by the modified filtering method.

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