A Preliminary Study to Extend Binary Associative Memories to Fuzzy Systems *.

A. Blanco, M. Delgado, W. Fajardo.

Dpto. Ciencias de la Computación e Inteligencia Artificial.
Universidad de Granada. E.T.S.I. Informática. 18071 Granada.
Tlfno.: +58 243100 e-mail: aragorn@robinson.ugr.es

ABSTRACT

We present a preliminary study to extend a tradicional model of associative memory, particulary a Binary Associative Memory (BAM). The BAM is designed to work with crisp information. We will to study as extend this model to a fuzzy system. In this form, we will obtain a model of fuzzy system with all the functional characteristics of the original BAM.

1.- Introduction.

Actually we have a lot of crisp systems that work properly . These system are suficiently proved and we acepted them because they have an adecuate productivity level. Generally these systems have been researched in specific ways to do solution to specific problems and generaly, the designing idea and the final model are crips.

After work with they, we are interested in convert some of these models to fuzzy to reuse their characteristics in fuzzy systems.

One of these models is the Binary Associative Memory (BAM). A memory is a system that can learn information and can recall it after. Traditionally the memories had been classified depending on the access to the information that they store. An associative memory stores information learning by correlation or association, and access to this information using data or information, that is, its access maps relations data with data. The BAM is a very studied model with a big scope of application. It is particularly appropiated to work with partially altered information, either by noise or loss of detail. This mades very interesting its aplications to solve

* Paper financed by the DGICYT's proyect PB 92-0945.

problems in fuzzy enviroment. For this reason we are going to introduce a preliminary study over a method to extend the BAM to a fuzzy model, to obtain a perfectioned method of storage and process of vague and imprecise information.

In this paper we present a preliminary study to extend an Artificial Intelligence's classic model (BAM) to a fuzzy model. A computer is a discrete machine that works with discrete models in a natural form, it seems reasonable to think that the best form to research a computational model is to use a discrete representation of the linguistic information in models designed as crisp models. For this reason, it's obviously desirable to work with the fuzzy information with a discrete representation instead of the usual continuous representation.

According to what has been said before, we are going to apply a **method of imprecise data codification with linguistics labels** (Blanco, Delgado & Fajardo [1]) to use crisp models of tested efficacy (in this case a Binary Associative Memory) to obtain models that can work directly with linguistic information. For this, we work with linguistic labels with fundamentals in the fuzzy sets theory and we use a codification method (Fajardo [3]) that permits us to work with systems that were not designed for this proposal.

The paper is organized in four parts:

- A first part (paragraph 2) where we show how to codify and decodify in a discrete form a piece of linguistic information by using the **Incremental Discretization Method.**
- And a third part (paragraph 3) where we introduce an extension of the Binary Associative Memory to a **Linguistic Binary Associative Memory** that permits us to work with linguistic information using the extension of a crisp model.
- Finally we conclude with a conclusion set (paragraph 4) obtained from this paper and the bibliography (paragraph 5) used.

2.- Codifying Linguistic Variables by Incremental Discretization Method.

We start from a fuzzy information representation's method in the reasoning with fuzzy information (see Delgado [2], Pedrycz [4], [5], [6]) where any element u from V is represented with terms over $T(H)$ as:

$$u \equiv \alpha_1 t_1, \ldots, \alpha_n t_n; \ \alpha_i \in [0,1], \ t_i \in T(H) \tag{1}$$

where α_i notes the compatibility of u with t_i (it's calculated using an ad-hoc function, particularly the possibility). Founded on this representation we propose to codify the values' variable as follows.

Supposing that $T(H)$ is composed by n elements. We give to $T(H)$ an arbitrary order and every term from $T(H)$ is associated to a vector of dimension m (supposing the we want a global precision of order $\frac{1}{m}$, althought we can really determinate a different precision for each term). A vector of $m \times n$ dimension is associated to the totality of $T(H)$.

We are going to use a codification method (Blanco, Delgado & Fajardo [1]) that permits to represent any linguistic variable's value purely expressed or not in linguistic terms.

Then, we codify u with a binary (or bipolar) vector of dimension $m \times n$:

$$C(u) = (C_{11}, \ldots, C_{1m}, \ldots, C_{n1}, \ldots, C_{nm}) \tag{2}$$

where:

$$\begin{cases} \alpha_i = 0 \rightarrow C_{ij} = 0; \ i = 1, \ldots, n; \ j = 1, \ldots, m \\ \alpha_i \neq 0 \rightarrow \begin{cases} \exists j \ t.q. \ \frac{j}{m} \leq \alpha_i < \frac{j+1}{m} \\ C_{il} = \begin{cases} 1 \ si \ l \leq j \\ 0 \ si \ l > j \end{cases} \end{cases} \end{cases} \tag{3}$$

Now, we can express any element of the discourse universe V with terms from the set $T(H)$.

With this idea as basis we can extend the codification to the general case when the variable value is given by a fuzzy subset A over V. For this reason, we consider again α_i as the compatibility grade of A with t_i adequately measured.

In particular if $A = t_k$ then it's codified with:

$$C_{ij} = \begin{cases} 1 \ si \ i=k; \ \forall j \\ 0 \ in \ other \ case \end{cases} \qquad \textbf{(4)}$$

Here, when it is used only linguistic labels, it is redundant to use a high number of bits for label, for this, we can economize using only one bit ($m=1$) per term $t_i \in T(H)$.

In this case and in the precedent cases, when we fixed the bits' number to use in the codification, we are determining the maximum precision that we will permits in the problem.

3.- Extension of a Binary Associative Memory to a Fuzzy System.

We are going to extend a **Binary Associative Memory (BAM)** to a fuzzy system using the **incremental discretization method** in a Binary Associative Memory, this one used as host system.

A BAM works learning spatial pattern pairs, codified in a binary or bipolar form, using Hebbian learning. Suppose that we would like learning a system that it had been described by *m* rules expressed in the form (A_m, B_m), where A_i is described by *n* labels and B_j using *p* labels. If the cut precision in the labels of A is *l* and *k* to the labels of B. Then, (A, B) is expressed as a pair of vectors with dimension $n \times l$ and $p \times k$. We must generate the local minimums in the energy surface and that these have been the same that the *m* pattern to memorize. To do this, we use Hebbian learning:

$$M = \sum_{k}^{m} A_k^T B_k \qquad \textbf{(5)}$$

In this form, we will obtain the sinaptic conection matrix M that connect the process elements from the layer F_A with the process elements from the layer F_B, and in this form it is obtained:

$$M = \sum_{i=1}^{m} M_i \tag{6}$$

Where the M_i has the next form[1] showed in equation **(7)**.

Remembering the fundamental characteristics of the BAM, the correct recall of the learning information is conditioning by the necessity that each pair (A_k, B_k) must coincide with a local minimum of the energy surface.

$$M_i = \begin{pmatrix} a_{1_1}^i b_{1_1}^i & \cdots & a_{1_1}^i b_{1_k}^i & \cdots & a_{1_1}^i b_{p_1}^i & \cdots & a_{1_1}^i b_{p_k}^i \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{1_l}^i b_{1_1}^i & \cdots & a_{1_l}^i b_{1_k}^i & \cdots & a_{1_l}^i b_{p_1}^i & \cdots & a_{1_l}^i b_{p_k}^i \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{n_1}^i b_{1_1}^i & \cdots & a_{n_1}^i b_{1_k}^i & \cdots & a_{n_1}^i b_{p_1}^i & \cdots & a_{n_1}^i b_{p_k}^i \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ a_{n_l}^i b_{1_1}^i & \cdots & a_{n_l}^i b_{1_k}^i & \cdots & a_{n_l}^i b_{p_1}^i & \cdots & a_{n_l}^i b_{p_k}^i \end{pmatrix} \tag{7}$$

To research a fuzzy extension of a BAM we only must apply the incremental discretization method in a BAM. Let's see an example that clarifies how to do it.

Example.

Let's suppose that we pretend to implement a inverted pendulum control system with a fuzzy extension of a BAM. This pendulum control system is described by the linguistic rules from **Table I**.

---

[1] To don't uses three underscores, we have changed the underscore referred to the pattern identify to upperscore. In this form we note the pattern with the upperscore and order of the bits generated by the codification with the composed underscore.

Table I

[illegible]

We begin codifying the rules as cause-effect pattern pairs (A_k, B_k), corresponding A_k to the cause and B_k to the effect. If we are interested in a model that gives pondered responses in function from diverse appreciation levels over input variables, we must to use a determinate bits' number for label in the representation. In this form if we pretend a decimal precision level in the labels' membership grade, we must use 10 bits per label and we obtain a codification as the showed in **Table II**.

We can see that all the patterns are not linearly independent, for this we have to use the Wang, Cruz & Mulligan's methods. In this form we have to calculate extensions and we have to use multiple learning methods.When the codification of the information is correctly finished, we must go to the learning and recalling phases.

When we have obtained the sinaptic connection matrix, we can use them as a part of their corresponding system, this is prepared to give responses (effects) in function of inputs given to them.

Let us illustrate the above ideas by means of an example. Supose that we introduce to the system an input expressed a determinate position and velocity, that in terms of linguistics labels is equivalent to a position 0.3 Negative Short and 0.7 Zero, and a velocity 0.4 Negative Short and 0.6 Zero, and its codification would be:

0000000000111000000011111110000000000000000000000000 0000000000111100000011111100000000000000000000000000

Table II

Position					Velocity					Force				
NM	NS	ZE	PS	PM	NM	NS	ZE	PS	PM	NM	NS	ZE	PS	PM
0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	1111111111
0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000
0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000
1111111111	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000
0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000
0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000
0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000	0000000000	0000000000	1111111111	0000000000	0000000000
A										B				

The response given by the fuzzy extension of the BAM after process the input corresponding to this pair position, velocity[1] is (force variable):

0000000000 1111111111 1111111111 0000000000 0000000000

or the result equivalent to a force negative short and zero, in fact of:

0000000000 1111000000 1110000000 0000000000 0000000000

that corresponds to a force 0.4 negative short, 0.3 zero.

This can looks as estrange behavior, but really there isn't any mistake because the BAM *thinks* that the appreciations over the memberships 'grades of the linguistic labels are alterations given by non-desirable causes (noise). How it is a stronger system respect to the errors, it corrects automatically given the nearest memorized pattern in the learning phase. For this, this behavior it's correct and desirable.

If we like responses with a decimal precision referring to the compatibility grade with the linguistic labels, we must to learn with respect to this level. For this, we are obliged to present as learning pattern all and each one from the combinations that we like to recall posteriorly.

The **Table II** must been replaced with some more exhaustive table referring to the pattern codification, as we said before.

With the new pattern sets we must remember the restrictions of the host system respect to pattern to learning and recalling, therefore we must do a new analyze. It's easy to see that as we must give to the associative memory the exahustive pattern set that we like learning, the elements of this set aren't linearly independent. To solve this problem we can use the methods researched by Wan, Cruz & Mulligan [7], [8].

---

[1] We must say that as we only are interesting in the system at fire level, for this we consider process elements with threshold function.

If we go now to learning phase and use the Hebbian learning method with these patterns as in the formula (5) and posteriorly we present the corresponding inputs to the system, we will see that now obtain the synaptic connection matrixes, that give the desired outputs.

Now we can probe with the before example, and the output of the fuzzy extension of BAM that process the input corresponding to the position and velocity [1] is the force:

0000000000 1111000000 1110000000 0000000000 0000000000

that is equivalent to one force 0.3 negative short and 0.4 zero, now, as we expect, expressed in function of the codification of linguistic label using incremental codification method.

With all the say before, we can see that it is possible to use a BAM system as host system of a fuzzy system, therefore, we obtain with a relative low work a better system that can process information given at level of linguistic label.

4.- Conclusions.

- We represent information expressed in terms of a linguistic system with a binary or bipolar code.
- We can work on linguistic problems using procedures developed for crisp problems.
- The resulting extension of the BAM system obtains the characteristic typical of the host model.
- The most important advantage is the simplicity. The models obtained as a result are easy to implement because the method is easily intelligible and afterwards is implanted in a very studied host model.
- The versatility is not only limited to work with any system described by using linguistics labels, furthermore the resulting system after the codification can work with mixed systems. It can work with systems that use antecedents (or consequents) crisp and consequents(or antecedents) expressed using linguistics labels.
- The application of this representation to mechanisms of measure of uncertainty, fuzzy and non fuzzy, shows the utility of the presented numeric approximation. The idea of combining fuzzy and non fuzzy information can be useful when the number of non fuzzy pieces of information is limited

---

[1] As before, we considerate with process elements that use as process function a threshold function, because now we only are interested in the behavior of the system in the fire level.

by inputs problems (problems with the size of the measures and/or interferences). In these situations, the subjective linguistics appreciations given by an expert are very useful in the construction of fuzzy machines.

To finish, let us point out that to these advantages we must add all the specifically inherited from the host system. For example, we have presented as host system a BAM, which has a remarkable stability and capacity to work with heteroassociative pattern, characteristic to be inherited to the fuzzy system.

5.- Bibliography.

[1].- Blanco, A; Delgado, M; Fajardo, W; (1998). *Representation Model of Information in Linguistics Terms.* Fuzzy Sets & System, (to appear).

[2].- Delgado, M.; Verdegay, J. L.; Vila, M. A.; (1992). *A Linguistic Version of the Compositional Rule of Inference.* Preprints Sicica'92, Symposium of Intelligent Components and Instruments for Control Applications. 141 - 148.

[3].- Fajardo, W.; (1995). *Recuperación de Información con Memorias Asociativas Difusas.* Tesis Doctoral. Universidad de Granada.

[4].- Pedrycz, W.; (1983). *Numerical and Applicational Aspects of Fuzzy Relational Equations.* Fuzzy Sets and Systems, 11, 1-18.

[5].- Pedrycz, W.; (1985). *Applications of Fuzzy Relational Equations for Method of Reasoning in Presence of Fuzzy Data.* Fuzzy Sets and Systems, 16, 163-175.

[6].- Pedrycz, W.; (1995). *Fuzzy Sets Engineering.* CRC Presss, Inc.

[7].- Wang, Y. F.; Cruz, J. B.; Mulligan, J. H.; (1990). *Two Coding Strategies for Bidirectional Associative Memory.* IEEE Transaction on Neural Networks, vol. 1, 81 - 92.

[8].- Wang, Y.-F.; Cruz, J. B.; Mulligan, J. H.; (1991). *Guaranteed Recall of All Training Pairs for Bidirectional Associative Memory.* IEEE Transaction on Neural Networks, vol. 2, 559 - 567.

[9].- Zadeh, L. A.; (1975). *The Concept of a Linguistic Variable an its Applications to Approximate Reasoning, Part I.*, Information Sciences, vol. 8, 199 - 249, *Part II.* Information Sciences, vol. 8, 301 - 357, *Part III.* Information Sciences, vol. 9, 43 - 80.

[10].- Zadeh, L. A.; (1979). *Fuzzy Sets and Information Granularity.* Advances in Fuzzy Sets Theory and Applications, (M. M. Gupta et al. eds.), North Holland, Nueva York 3 - 19.