FUZZY PI+PD CONTROLLER

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Abstract: A setting of fuzzy controllers is more complicated process than for classical ones, because the fuzzy controller, like non-linear, have seemingly more freedom degrees. However its exploitation is often misguided. Fuzzy controllers are often realised with two or three inputs and one output. If these inputs consist from a variable and its derivations, this controller is similar to classical PI/PD/PID controller. A fuzzy PID controller is physically related to classical PID controller. A setting of these controllers is based on deep common physical groundwork, which is described in the article. Parameters of the fuzzy controller in presented papers are adjusted in physical meaning of classical PID controllers. A newly introduced method with a unified universe range considerably simplify the setting and realisation of fuzzy controllers.

Keywords: Fuzzy control, Fuzzy PI controller, Fuzzy PD controller, Fuzzy PI+PD controller

1. A NORMALISED UNIVERSE

For simpler design, universe ranges for inputs and outputs are normalised in interval <-1, 1> (*Fig. 1*). An input value or an output value is multiplied by a constant which indicates a real range of the universe. If the error value is multiplied by coefficient 5 before fuzzification, the real range of the universe for the error is $e \in$ < -0.2, 0.2>. For coefficient 0.1 the range is $e \in$<-10, 10>. It is evident there's no conflict with commonness and this procedure leads to the large simplicity for the fuzzy controller design as will be demonstrated.

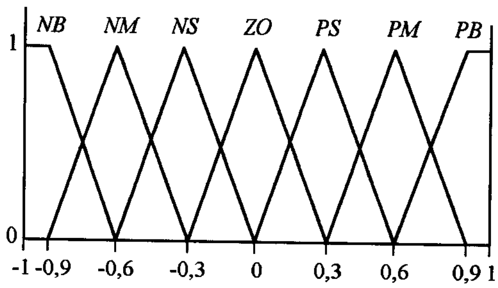

Fig. 1 Symmetrical membership function lay-out

2. FUZZY PI CONTROLLER DESIGN

A classical PI controller is described by equation (1) where K is gain of PI controller, T_I is a integral constant, $e(t)$ is a error signal, $e(t) = w(t) - y(t)$, $w(t)$ is desired value, $y(t)$ is output from process and $u(t)$ is output from controller - action.

$$u(t) = K\left(e(t) + \frac{1}{T_I}\int_0^t e(\tau)\mathrm{d}\tau\right) \tag{1}$$

When we derive (1) and we suppose zero initial condition, we get

$$\dot{u}(t) = K\left(\dot{e}(t) + \frac{1}{T_I}e(t)\right) \tag{2}$$

For a local extreme location we put

$$\dot{u}(t) = K\left(\dot{e}(t) + \frac{1}{T_I}e(t)\right) = 0 \tag{3}$$

A solution of equation (3) is

$$\dot{e}(t) = -\frac{1}{T_I}e(t) \tag{4}$$

because the PI controller gain have to stand $K > 0$. A line equation (4) depends only on the PI controller integral time constant. Its physical meaning lies in a fact that it determines a border where the action derivation changes a sign from positive to negative, if a state trajectory of the control system intersects the line from right to left (or from above to down) in a state space $\dot{e}(t)$, $e(t)$. The situation is displayed in *Fig. 2*. When the state trajectory passes from left to right (or from below to up) when it intersect the line, the sign of the action derivation changes from negative to positive. So there is a place in the step response where the state characteristic intersects the line (for $\dot{u}(t) = 0$) and the action derivation $\dot{u}(t)$ changes its sign. If we translate the equation (3) to the discrete form, we get a equation of a discrete PI controller

$$\Delta u(k) = K\,(\Delta e(k) + \frac{1}{T_I} e(k)) \qquad (5)$$

where $\Delta u(k) = (u(k) - u(k\text{-}1))\,/T$ T is the sampling period, k is the step.

$\Delta e(k) = (e(k) - e(k\text{-}1))\,/T$

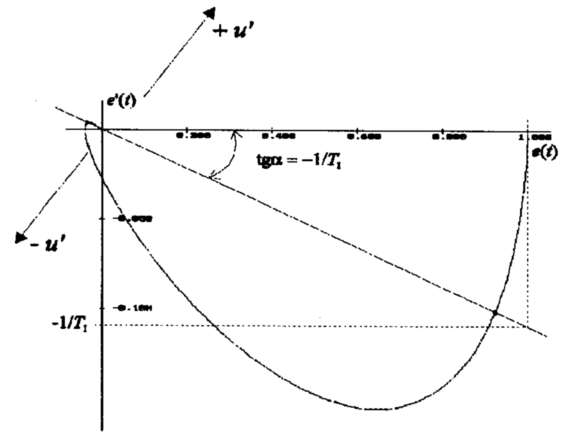

Fig. 2 State trajectory of the control system with the PI controller

From *Fig. 2* is obvious the time constant T_I have relation to the change-in-error. Therefore we modify the equation (5) for a fuzzy PI controller derivation

$$\Delta u(k) = K\frac{1}{T_I}\,(T_I\,\Delta e(k) + e(k)) \qquad (6)$$

In the next step it is necessary to map the rule base to the discrete state space $\Delta e(k)$, $e(k)$. We define a scale factor M for the universe range, $M > 0$. This scale factor sets the universe ranges for the error and its first differential (*Fig. 3*). We extend the equation (6) and get

$$\Delta u(k) = K\frac{M}{T_I}\,(\frac{T_I}{M}\,\Delta e(k) + \frac{1}{M} e(k)) \qquad (7)$$

We apply fuzzification at input variables and after defuzzification we get the equation

$$\Delta u(k) = K\frac{M}{T_I} \boldsymbol{D}\{\, \boldsymbol{F}\{\, \frac{T_I}{M}\,\Delta e(k) + \frac{1}{M} e(k)\}\} \qquad (8)$$

where $\boldsymbol{F}$ is an operation for fuzzification and $\boldsymbol{D}$ for defuzzification.

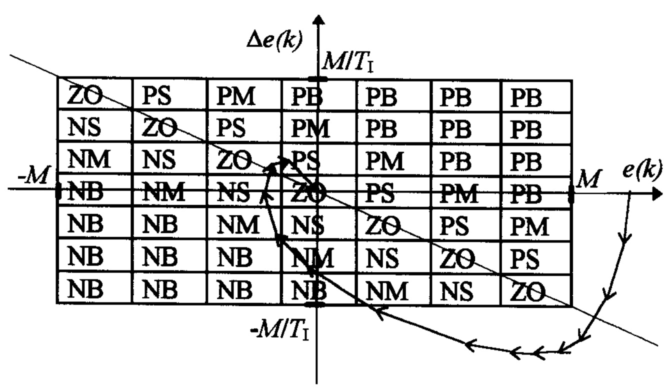

Fig. 3 A fuzzy PI controller rule base mapping to the discrete state space

We put an expression instead $\Delta u(k)$

$$\Delta u(k) = \frac{u(k)-u(k-1)}{T} = K\frac{M}{T_I}D\{\,F\{\,\frac{T_I}{M}\,\Delta e(k) + \frac{1}{M}e(k)\}\} \tag{9}$$

A result value of the fuzzy PI controller action in the step k is

$$u(k) = K\frac{MT}{T_I}D\{\,F\{\,\frac{T_I}{M}\,\Delta e(k) + \frac{1}{M}e(k)\}\} + u(k\text{-}1) \tag{10}$$

A realisation of the fuzzy PI controller according (10) is in *Fig. 4*.

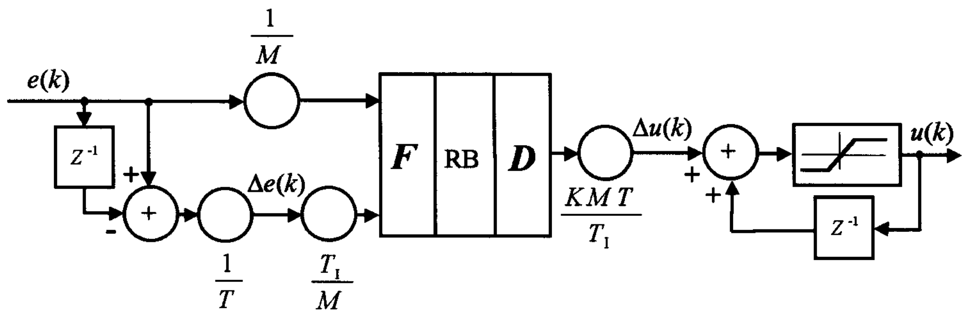

Fig. 4 Fuzzy PI controller structure with the normalised universe range

3. FUZZY PD CONTROLLER DESIGN

A classical PD controller is described:

$$u(t) = K\,(\,e(t) + T_D\dot{e}\,(t)\,) \tag{11}$$

We derive (11) if zero initial conditions are supposed

$$\dot{u}\,(t) = K\,(\dot{e}\,(t) + T_D\dot{e}\,(t)\,) \tag{12}$$

For location of the local extreme we put (if $K > 0$)

$$\dot{e}\,(t) + T_D\,\ddot{e}\,(t) = 0 \tag{13}$$

The convenient solution we get by integration of the equation (13). It is

$$\dot{e}\,(t) = -\frac{1}{T_D}e(t) \tag{14}$$

The line equation (14) depends only on the derivational time constant of the PD controller and its physical meaning is similar like for the PI controller. If we transfer the equation (11) to the discrete form, we get a equation of the discrete PD controller

$$u(k) = K\,(e(k) + T_D\,\Delta e(k)) \tag{15}$$

where $\Delta e(k) = (e(k) - e(k\text{-}1))\,/T$ and T is the sample period.

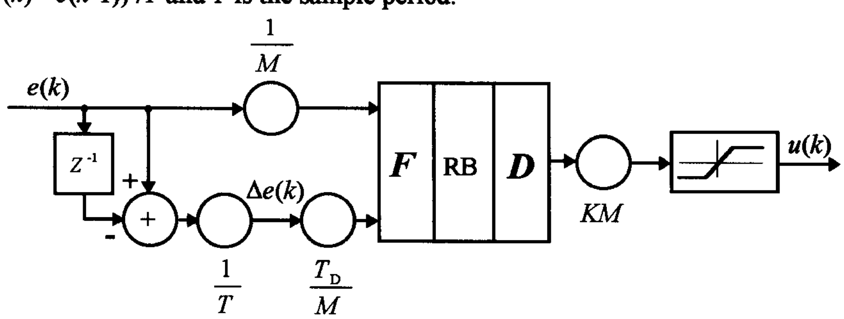

Fig. 5 Structure of the fuzzy PD controller with the normalised universe range

In the next step we map the rule base to the discrete state space $\Delta e(k)$, $e(k)$. We initiate the scale M for the universe range, $M > 0$. This scale sets ranges for the error and the change-in-error. After extending the equation (15) we get

$$u(k) = KM\,(\frac{1}{M}e(k) + \frac{T_D}{M}\,\Delta e(k)) \tag{16}$$

We apply fuzzification at input variables and after defuzzification we get the equation

$$u(k) = KM\,\boldsymbol{D}\{\,\boldsymbol{F}\{\frac{1}{M}e(k) + \frac{T_D}{M}\Delta e(k)\}\} \tag{17}$$

The resultant action value of the fuzzy PD controller is given by the equation (17). The fuzzy PD controller realisation according this equation is in *Fig. 5*. A fuzzy PID controller can have got many variants. From a practise point of view there are the most frequent versions: a parallel combinations of PD+PI controllers.

4. FUZZY PD+PI CONTROLLER DESIGN

The controller have three common constants - the gain K, the scale M and the sample period T. It is advantageous if we establish the gain and the scale extra for every controller even in this case. So the fuzzy PI controller have the gain K_I and the scale M_I. The fuzzy PD controller have the gain K_D and the scale M_D. The period can be the same or different.

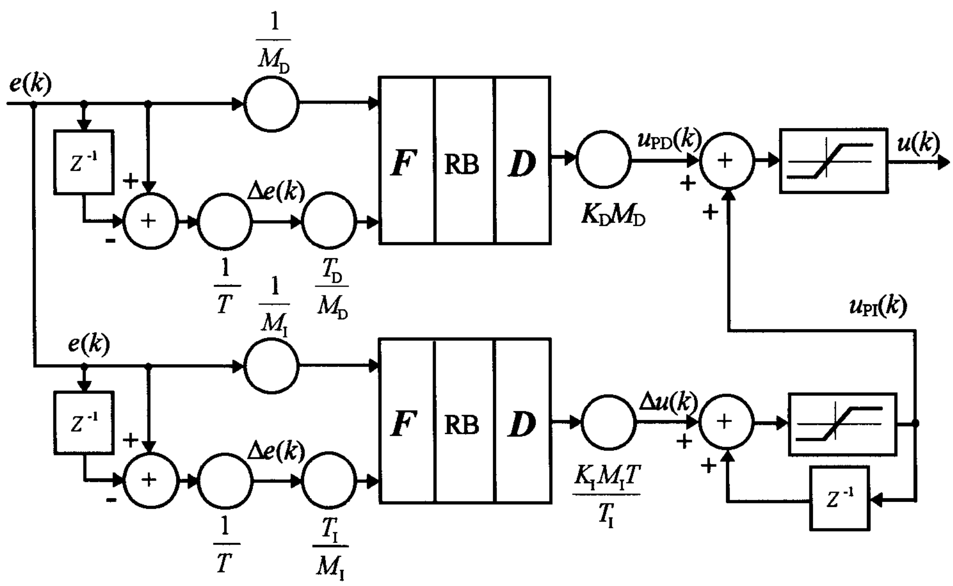

Fig. 6 Structure of the fuzzy PD+PI controller with the normalised universe range

5. EXAMPLES

On the transfer function

$$F(s) = \frac{2}{(10s+1)(s+1)^2} \tag{18}$$

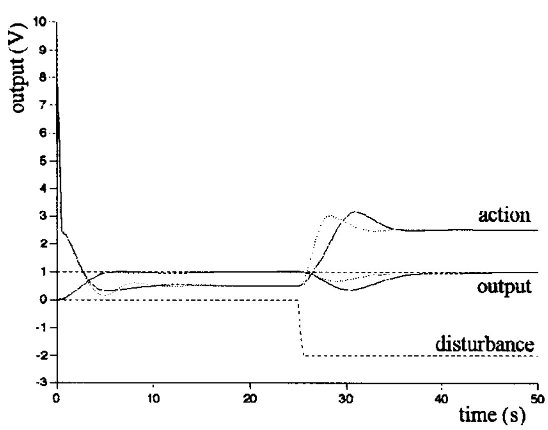

Fig. 7 Responses in the system with the fuzzy PD+PI controller. Dot curves are for the Min-Max inference and solid curves are for the Prod-Max inference

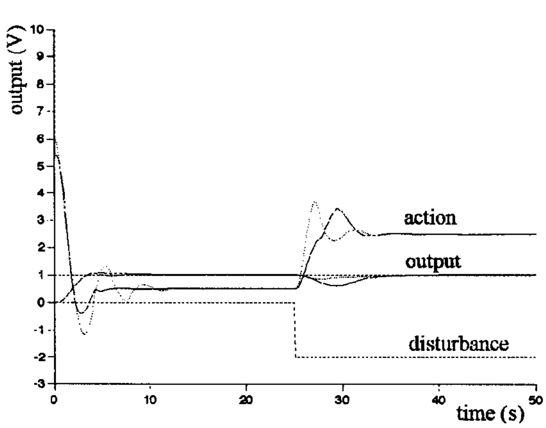

Fig. 8 Responses in the system with the fuzzy PD+PI controller and with the setting for the faster response on the load change

was designed PID controller. The controller have three common constants - the gain K, the scale M and the sample period T. It is advantageous if we establish the gain and the scale extra for every controller even in this case. So the fuzzy PI controller have the gain K_I and the scale M_I. The fuzzy PD controller have the gain K_D and the scale M_D. The period can be the same or different. If we optimise the fuzzy controller parameters setting with regard to a very little oscillatory action on the change in desired value, we can set parameters on $K_I = K_D = 2$, $T_I = 3$, $T_D = 1.5$, $M_I = M_D = 10$, $T = 0.1$ *s*. Simulation results with the transfer function (18) are in *Fig. 7*. If we will optimise the setting of the fuzzy PD+PI controller with regard to the fast response of the load change, works on the input of the transfer function (18) we can set parameters of the controller for instance: $K_I = K_D = 4$, $T_I = 2.2$, $T_D = 2$, $M_I = M_D = 10$, $T = 0.1$ *s* (*Fig. 8*). If a user is familiarised with basic ideas of control theory, he can relatively fast set the fuzzy controller even it have the non-linear membership function lay-out. When the same setting of all parameters of the fuzzy controller is used like in *Fig. 9* except variances in the normalised membership function lay-out for the fuzzy PI controller change-in-action.

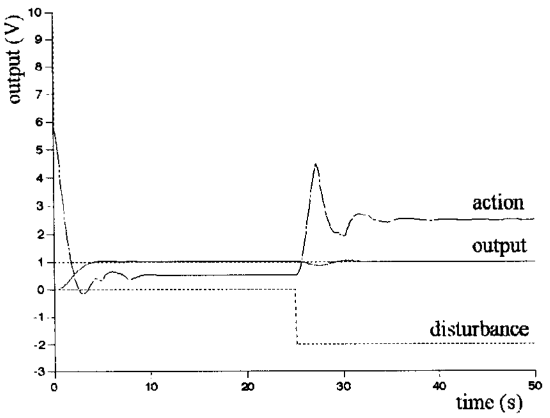

Fig. 9 Non-linear membership function lay-out in the calculation of the change-in-action (for PI controller). A used inference method is Min-Max.

Fig. 10 Responses in the system with the fuzzy PI+PD controller and the time transformation to slow. $K_I = K_D = 4$, $M_I = M_D = 10$, $T_I = 8*2.2 = 17.6$ *s*, $T_D = 2*8 = 16$ *s*

The fuzzy PI+PD controller realising according *Fig. 6* have even opinion to change time scale. In *Fig. 10* are responses of the control system with the transfer function $F(s) = 2/((80s+1)(8s+1)^2)$ (the multiplying coefficient is 8x).

6. CONCLUSION

For the fuzzy PID controller setting it is necessary to determine universe ranges and perform tens or hundreds simulation experiments until we find acceptable values. A retrieval of optimal parameters is very difficult, because the setting is dependent on a lot of other parameters. In addition optimal parameters could be dependent on the error value. The new method with the unified universe range, stated in this article, considerably simplify setting of fuzzy PI/PD/PID controllers. The fuzzy PID controller can be programmed like a unified block in a controller and therefore work consumed on a implementation to the particular control system can be cut short.

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