A NEW METHOD OF ALIGNMENT OF ECG CYCLES USING ENERGY MEASURE OF FUZZINESS

J. Łęski, E. Czogała
Technical University of Silesia
Akademicka 16, 44-101 Gliwice, Poland

Abstract. A way of the application of energy measure of fuzziness to the alignment of QRS complexes of ECG signal for selected averaging of this signal has been presented. The formulation of the problem and an idea of energy measure of fuzziness has been presented. A fuzzy signal has been created on the basis of the original signal. For such a signal an energy measure of fuzziness was computed. Next it was filtered by means of a lowpass filter. Such a processed measure was applied to the alignment of QRS complexes. The theoretical considerations were illustrated by means of trade-off studies of the new method with the method known from literature as Fourier Shift Method (FSM).

1. Introduction

Taking into account a quasi-periodicity of ECG signal the most common method of noise suppression in such a signal is averaging. To make it possible, determination of fiducial points (FP), for which the location is constant with respect to averaged signal periods is necessary [1]. Fiducial points of ECG signals are determined from averaged signals which are noisy. There are also situations where fiducial points are determined from other signals, not from averaged signals. In case of EEG signal it may be flash or a sound signal. Determination of fiducial points is also of essential meaning for heart rate variability signals.

The most important feature of FPs determining method is their reproducible location with respect to successive signal periods even for low signal-to-noise ratio (SNR). Errors of FP locations are composed of errors caused by quantization of signal in time domain and noises [16]. The first type of errors can be eliminated applying methods of FP determining with precision greater than sampling period. The minimization of second type of errors is a much greater problem.

2. FPs determination errors and averaging

It can be proved that aligning errors of averaging cause distortion of averaged signal (limiting signal bandwidth). It can be shown that for normal distribution of determining of FP errors the distortions of averaged signal correspond to the filtration of such a signal by means of the filter of the following frequency characteristic [7]:

$$|K(f)| = e^{-2\pi^2 f^2 \sigma^2} \qquad (1)$$

$$\Phi(f) = -2\pi m f \qquad (2)$$

where: |K| - amplitude characteristic,
Φ - phase characteristic.

It is a low-pass filter with cut-off frequency:

$$f_g = \frac{132.5}{\sigma\ [ms]}\quad [Hz] \tag{3}$$

If standard deviation errors of alignment increases the distortions caused by averaging also increase for such a filter (limiting signal bandwidth). Hence, the greatest precision of alignment is very important.

3. Applied methods of determining fiducial points

Some of the methods of FPs determining allow the calculation of absolute location (on time axis) of FPs on averaged periods of signal (e.g. double-level method, DL). However, other methods determine a relative displacement of two arbitrary periods of signal (e.g. correlation method). Both types of methods are equally useful for averaging of signals. In case of variability analysis of signals the following approach can be applied: first we apply a preliminary alignment by means of a method allowing to determine an absolute location of FPs together with measuring of interesting time periods, next a method determining relative location of FPs is applied. Finally we modify previously measured time periods using a difference between FPs determined by means of both methods. Of course, this approach makes sense when the second method is burdened with smaller errors than the first one.

As the simplest methods of determining the location of FP we can consider: the determining of a location of the greatest amplitude sample, point where first derivative of signal is passing zero-line or point where signal is passing a respective threshold (single level) together with the best method from simplest, double-level method [19]. This method relies on determining of arithmetic mean of locations of two intersections: on increasing and decreasing parts of considered signal wave with a certain level. This level is usually taken as a percentage portion of the maximal amplitude of aligning signal. Usually this percentage portion equals 50 %. This method determining the absolute location of FP seems to be attractive from the computational point of view.

One of the most precise methods is a method based on classical theory of detection i.e. a matched filtering method (MF) [6]. An FP is determined as a location of maximum of linear filter output signal. An impulse response of such a filter is an unnoised useful (detected) signal which is in time inversion. However, this method allowing absolute locations of FPs carries a big computational burden. The most important disadvantage of that method is the lack of a priori information about the useful signal. Usually the following solution is applied [7]: (i) a preliminary aligning of signal periods using e.g. double-level method, (ii) an averaging of periods of signal aligned in such a way, (iii) an averaged signal is taken for a useful signal.

A correlation method is conceptually similar [11]. It determines mutual displacement of FPs in arbitrary periods of signals. It seeks such mutual location of aligned periods of signal in order to maximize the coefficient of correlation. Most frequently a template is modified after determining each FP [2,3].

One of the methods allowing the determining of the FPs with resolution greater than sampling period, is a method of normalized integral (NI) [8,18]. The above mentioned method is computationally very efficient.

The most precise method of determining of FPs for noised ECG signals is Fourier shift method (FSM) described in [13]. The comparison investigations of the proposed new method of determining the FPs with the FSM method will be shown in this paper as well.

4. Energy measure of fuzziness

To assess fuzzy uncertainty in signals quantitatively, measures of fuzziness [4, 5, 9, 12, 14, 17, 21] used in the fuzzy set theory can be applied. There are several types of measures, such as entropy and energy measures, negation-distance measures and others. Contrary to the entropy type of measures of fuzziness introduced in [5] and recalled in [4] where the evaluation of the difference between a fuzzy set and a crisp set is expected, we will now introduce the energy type of measure of fuzziness connected with the evaluation of the difference between a fuzzy set and a crisp singleton (energies and energy type of measures).

Of course the entropy measures cannot be used to get the result in a situation where we deal with the last case. For illustration let us consider the well known interval numbers of interval mathematics which describe very inexactly known real numbers, but every crisp fuzzy number has the entropy-value zero for every entropy measure, like any crisp singleton. Other types of measure of fuzziness should also be considered with the uncertainty of a decision-maker who has to choose one alternative out of alternatives described by means of a membership function [4]. This case can also be covered by applying the so called energy type of measure of fuzziness. However, the definition of energy measure of fuzziness sounds like that of entropy measure but because of the different properties of both measures the energy measure is totally different from entropy measure. Below we will recall a definition of the energy measure of fuzziness [4,5].

An energy measure of fuzziness E is a mapping from the set of all fuzzy subsets of a base set into the nonnegative reals, i.e.:

$$E\colon\ Fz(X) \rightarrow [0,\infty) \tag{4}$$

It is postulated that this function has the following properties:

1. The energy measure of fuzziness $E(A) = 0$, iff $A(x) = 0$ for all x
2. $E(A)$ is maximal, if $A\colon X \rightarrow \{1\}$
3. If $A \subseteq B$ that is, if $A(x) \leq B(x)$, for all x, then $E(A) \leq E(B)$.

It can be inferred from the above mentioned properties that

- if A is a singleton then $E(A) = 1$,
- if A is a normal fuzzy set then $E(A) \geq 1$.

A wide class of energy measure of fuzziness can be obtained assuming a measure υ for space X. The class can be expressed in the formula:

$$E(A) = F_2\left(\int_X f(A(x))\, d\upsilon\right) \tag{5}$$

where $A\colon X \rightarrow [0,1]$ is a υ-measurable function,
$f\colon [0,1] \rightarrow \mathbf{R}_+$ is an increasing function in interval $[0,1]$ for $f(0) = 0$,
$F_2\colon \mathbf{R}_+ \rightarrow \mathbf{R}_+$ is an increasing function for $F_2(z) = 0$ if and only if $z = 0$.

5. The idea of a fuzzy signal and its energy measure of fuzziness

Let us consider the original, uniformly sampled discrete signal with period T in the time interval from 0 to t. The value of the n-th sample of this signal is denoted as x(n) as well as 2k+1 window of the original samples: x(n-k),...x(n),...,x(n+k) centered at x(n).

The idea of the construction of a fuzzy signal denoted X(n,k) from the original signal is based on two postulates:

1. It is assumed that the constant signal contains no fuzzy uncertainty. The fuzzy value X(n,k) which represents the respective fuzzy number of the n-th sample is reduced to the real number x(n) (singleton). The measure of fuzziness will take the value of zero and this means that the information conveyed by such a signal is maximal.

2. The fuzziness of the variable signal X(n,k) is higher if the dynamics of the changes of the original signal is on the increase. It is assumed that if a signal contains fuzziness it means that the measures of fuzziness discussed above take values that are higher than zero. The amount of information is smaller for the variable signal than for the constant signal.

To construct a fuzzy signal from the crisp signal, let us consider a symmetric window consisting of 2k+1 original samples for each sampling point n, i.e. we will consider the following set of samples:

$$\{x(n-k),\ x(n-k+1),\dots,x(n),\dots,x(n+k-1),\ x(n+k)\} \tag{6}$$

Next let us sort these 2k+1 values starting with the minimal value, i.e. $x_{min}(n) = x_{(1)}(n)$ and ending with the maximal value i.e. $x_{max}(n) = x_{(2k+1)}(n)$. So the following relations are true:

$$x_{(1)}(n) \le x_{(2)}(n) \le \dots \le x_{(2k+1)}(n) \tag{7}$$

It is easy to verify that a median is expressed as

$$x_M(n,k) = x_{(k+1)}(n) \tag{8}$$

Allowing for the median and the set of samples mentioned above we will construct the membership function for each point n in the following way. First let us assume that $A_{n,k}(x_{min}(n,k)) = 0$, $A_{n,k}(x_{max}(n,k)) = 0$ and $A_{n,k}(x_M(n,k)) = 1$ and then let us build an upper semicontinuous step-wise membership function according to the formula

$$A_{n,k}(x) = \begin{cases} \frac{p}{k}; & \text{where p is the number of } x_{(i)} < x \text{ for } x \le x_M(n) \\ \frac{2k+1-p}{k} & \text{for } x > x_M \end{cases} \tag{9}$$

We can also create a parametrized version of our membership function that possesses the ability to discriminate between certain levels of membership, i.e.

$$A^{\lambda}_{n,k}(x) = A_{n,k}(x)\ \mathbb{I}(A_{n,k}(x) - \lambda) \tag{10}$$

where $A_{n,k}(x)$ denotes the membership function of a fuzzy number describing n-th sample, $\lambda \in [0,1]$ and $\mathbb{I}$ is the Heaviside pseudofunction.

In opposition to the parameterized entropy measure, the parameterized energy measure is exactly the same version as encountered in literature, i.e.:

$$E(A_{n,k},\lambda) = \int f_{\lambda}(A_{n,k}(x))\,d\upsilon \tag{11}$$

where

$$f_{\lambda}(z) = \begin{cases} f(z) & if\ z \in (\lambda,1) \\ 0 & otherwise \end{cases} \tag{12}$$

The role of the threshold λ is to filter out the insignificant degrees of membership that might otherwise have an undesirable impact on the values of the measure of fuzziness. If for the entropy measure of fuzziness these are the values of membership close to 0 and 1, for the energy measure of fuzziness the lowest grades of membership ($< \lambda$) are negligible.

The final formula for the energy measure of n-th sample may also be written in the form of the sum of respective rectangles, i.e.:

$$E(A^{\lambda}_{n,k}) = F_2\left(\sum_{i=1}^{2k} f(A^{\lambda}_{n,k}(x_{(i)}(n)))\,\Delta x_{(i)}(n)\right) \tag{13}$$

where $\Delta x_{(i)}(n) = x_{(i+1)}(n) - x_{(i)}(n)$.

Fiducial points are determined on the basis of a smoothed course of energy measure of fuzziness. The smoothness is obtained using a moving average filter:

$$O(n) = \frac{1}{N}\sum_{i=0}^{N} E(A^{\lambda}_{n-i,k}) \tag{14}$$

Additionally, in order to increase the method resolution the smoothed wave of energy measure of fuzziness in the neighborhood of maximum is interpolated. The following notation is introduced: O_{max} - the maximum value of signal O (in the neighbourhood of aligned QRS complex). It is assumed that the maximum occurs in a discrete time instant indexed by τ. The signal values in neighborhood samples τ-T, τ+T are denoted by O_{max-1}, O_{max+1}. Taking into account an interpolating square function, we get:

$$\begin{cases} O_{max-1} = a(\tau-T)^2 + b(\tau-T) + c \\ O_{max} = a\tau^2 + b\tau + c \\ O_{max+1} = a(\tau+T)^2 + b(\tau+T) + c \end{cases} \tag{15}$$

The maximum of the interpolating function is determined by means of equalling to zero a time derivative of interpolating function: FP = -b/2a. Parameters a, b are obtained by solving equation systems (15). After simple transformations we get:

$$FP = \tau + \frac{T}{2}\left[\frac{O_{max+1} - O_{max-1}}{2O_{max} - O_{max-1} - O_{max+1}}\right] \tag{16}$$

where T is a sampling period.

5. Comparison investigations of the methods of determining the FPs

The methodology for testing the method of determining FPs described in section 4 and FSM method are as follows. Ten QRS complexes (5 from healthy subjects and 5 from ill patients) were used for testing. The signals were sampled with frequency 1000 Hz and quantized by means of analogue-to-digital converter with resolution 16 bits. All these signals had high SNR, the worst SNR was equal to 40 dB. The low-frequency and main noises were suppressed according to the method described in [13]. Next, such obtained signals were modeled by means of linear combination of three first Hermit functions described in [10,15]. Thanks to the modeling of original signals it was possible to generate the signals shifted with arbitrary time value.

The muscle distortions modeled by means of Gaussian white noise [20] were added to such obtained signals. The Box-Müller generator was applied. Both methods were tested for the following SNR: 20, 10, 5, 0, -5, -10, -15 dB, where the signal-to-noise ratio (SNR) was defined as:

$$SNR\ [dB] := 10\ Log\frac{P_s}{P_n} \tag{17}$$

where: P_s - signal power,
P_n - noise power.

For each QRS complex 200 signals were generated and shifted on time axis with value a_i. In order to model the lack of sampling synchronization with the occurence of QRS complex the a_i values were the realizations of random variable with uniform probability density function in $<-10ms,+10ms>$. These values belonged to real numbers. Each of such obtained signals was distorted with a realization of modeled noise. Next, the known shifts a_i were determined using the described in paper and FSM methods. As a result, shifts d_i were obtained. We define the FP determination errors:

$$e_i = a_i - d_i \tag{18}$$

and the mean and the standard deviation of FPs determination errors:

$$\bar{e} = \frac{1}{2000}\sum_{i=1}^{2000} e_i \tag{19}$$

$$\sigma = \sqrt{\frac{1}{2000}\sum_{i=1}^{2000} (e_i - \bar{e})^2} \tag{20}$$

The investigations of parameter k and N influencing the value of standard deviation of FPs determination errors were conducted for SNR = -5 dB and illustrated graphically in Fig. 1.

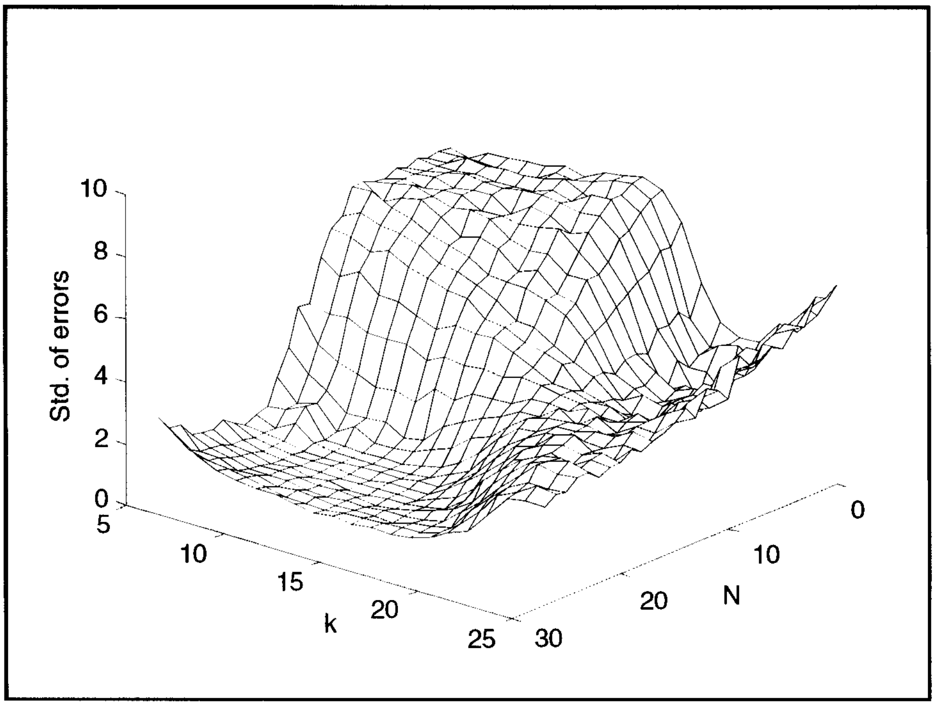

Fig. 1: Standard deviation of FPs determination errors as function of parameters k and N.

The minimal value of standard deviation was obtained for k=12, N=25. For these parameters the influence of parameter λ from 0 to 0.5 and SNR = -5dB is shown in table:

λ	0	0.1	0.2	0.3	0.4	0.5
σ [ms]	0.6213	0.5635	0.6472	0.6138	0.5914	0.6635

The minimal value of standard deviation occurs for λ=0.1. For such chosen parameters the comparison tests for the proposed method and FSM were conducted. The results are shown in the table:

σ [ms]\SNR	20 dB	10 dB	5 dB	0 dB	-5 dB	-10 dB	-15 dB
FUZZ	0.1672	0.2861	0.3656	0.4816	0.5635	0.9068	1.1801
FSM	0.1007	0.1708	0.2201	0.2867	0.3612	0.4603	0.5956

In this table the following abbreviations are applied: FSM - the fourier shift method, FUZZ - the method described in this paper.

From this table it can be seen that the method proposed in this paper leads to greater aligning errors than the comparison method FSM. However, it should be pointed out that the FSM method is the most precise method. For example for SNR = 10 dB the MF method has 2, the correlation method 3.3, normalized integral 6, double-level 4 times greater standard deviation of errors with respect to FSM method [13]. The advantage of the proposed method is its computing efficiency. The FSM method requires an application of nonlinear optimization which needs the execution of unknown number of iterations. Hence, this method generates problems in on-line applications. Let us notice that in the ECG stress test the signal has been processed in on-line mode, taking into account the safety of the patient.

7. Concluding remarks

A method of alignment of the QRS complex of the ECG signal for the averaging of signal in time domain on the basis of detection function obtained from the filtered course of energy measure of fuzziness has been presented. To increase the resolution of the method of determining the FPs the square interpolation function has been applied. Unknown parameters (width of signal window, length of impulse response of smoothing filter and cut level) are determined by means of minimization of standard deviation of FPs errors determination. The investigations of the proposed method were conducted for SNR from -15 to 20 dB. The described method can be used to construct the computer analysis systems of ECG signals, eg. exercise, monitoring and Holter systems.

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