FUZZY LINEAR PROGRAMMING PROBLEMS WITH FUZZY VARIABLES AND FUZZY COEFFICIENTS

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Abstract Fuzzy linear programming problem with fuzzy variables and fuzzy coefficients(FLPV) is discussed in this paper and is turned to two- stage multiobjective linear programming problem by applying a partial order method defining the order of bounded closed fuzzy numbers. Solution to FLPV is given.

Keywords Fuzzy variable, fuzzy linear programming, triangular fuzzy number, quasi-triangular fuzzy number

1. Introduction

Fuzzy linear programming(FLP) was first discussed by Tanaka et al and Zimmermann. So far several kinds of FLP problems and corresponding approaches of solving them have appeared in the literature. According to solving method,FLP can be divided into two categories: FLP with fuzzy constraints and FLP with fuzzy coefficients. But variables in these models are regarded as crisp.However, in engineering problem, we often met such problems that how many optimal solution of FLP approximately is and how many fairly superior solution is. Now variables of FLP are fuzzy ones and corresponding FLP problem are ones with fuzzy variables. [1,2] studied one objective and multiobjective FLP problems with fuzzy variables respectively while coefficients of objective function and constraint matrix are regarded as crisp numbers . This restricts the range of application of the model greatly. On the basis of [1,2],this paper study FLP problems with both fuzzy variables and fuzzy coefficients.

2. Order and operations of fuzzy number

Definition 1. Fuzzy number $\tilde{u}$ is called a bounded closed fuzzy number if and only if it has following membership function:

$$\mu_{\tilde{u}}(x)=\begin{cases}1 & x\in[m,n]\neq\varnothing\\ L_u(x) & x<m\\ R_u(x) & x>n\end{cases}$$

where $L_u(x)$ is right continuos increasing function, $0\leqslant L_u(x)\leqslant 1$ and $\lim_{x\to-\infty}L_u(x)=0$; $R_u(x)$ is left continuos decreasing function, $0\leqslant R_u(x)\leqslant 1$, and $\lim_{x\to+\infty}R_u(x)=0$. It is denoted as $\tilde{u}$=([m,n], $L_u(x), R_u(x)$).

Theorem 1. Let $\tilde{u}$, $\tilde{v}$ be two bounded closed fuzzy numbers. Then the relation ⓢ defined by the following

$$\tilde{u}\ ⓢ\ \tilde{v} \Leftrightarrow \tilde{u}\vee\tilde{v}=\tilde{v} \Leftrightarrow \tilde{u}\wedge\tilde{v}=\tilde{u}$$

is a partial order where

$$\tilde{u}\vee\tilde{v} = \bigcup_{\lambda\in[0,1]} \lambda\,(u_\lambda\vee v_\lambda),\quad \tilde{u}\wedge\tilde{v} = \bigcup_{\lambda\in[0,1]} \lambda\,(u_\lambda\wedge v_\lambda)$$

$u_\lambda=[u_\lambda^-,\ u_\lambda^+]$ and $v_\lambda=[v_\lambda^-,\ v_\lambda^+]$ are λ-cut sets of $\tilde{u}$ and $\tilde{v}$ respectively. " $\wedge$ " and " $\vee$ " represent "get small" and "get big" operators.

Corollary. $\tilde{u}$ ⓢ $\tilde{v}$ $\Leftrightarrow u_\lambda^-\leqslant v_\lambda^-,\ u_\lambda^+\leqslant v_\lambda^+,\ \forall\ \lambda\in[0,1]$.

Definition 2. For a bounded closed fuzzy number $\tilde{u}$, if m=n=u and $L_u(x)$ is a linear function on support set $(\underline{u},u)$ and $R_u(x)$ is a linear function on support set $(u,\ \overline{u})$, then $\tilde{u}$ is called a triangular fuzzy number denoted as $\tilde{u}=(u,\underline{u},\overline{u})_\Delta$.

For a bounded closed fuzzy number $\tilde{u}$, if m=n=u and $L_u(x)$ is a linear function or strictly increasing convex function on support set $(\underline{u},u)$ and $R_u(x)$ is a linear function or strictly decreasing concave function on support set $(u,\ \overline{u})$, then $\tilde{u}$ is called a quasi-triangular fuzzy number denoted by

$\tilde{u} = (u, L_u(x), R_u(x))_\nabla$.

It is easy to know that a triangular fuzzy number is a quasi- triangular fuzzy number.

In the following discussion, we don't involve concrete representation of $L_u(x)$ and $R_u(x)$. Thus quasi-triangular fuzzy number $\tilde{u} = (u, L_u(x), R_u(x))_\nabla$ is denoted by $\tilde{u} = (u, \underline{u}, \overline{u})_\nabla$ shortly.

Theorem 2. Let $\tilde{u} = (u, \underline{u}, \overline{u})_\nabla$ be a quasi-triangular fuzzy number and $\tilde{v} = (v, \underline{v}, \overline{v})_\Delta$ be a triangular fuzzy number. Then

$$\tilde{u} \circledS \tilde{v} \Leftrightarrow u \le v,\ \underline{u} \le \underline{v},\ \overline{u} \le \overline{v}.$$

Theorem 3. If $\tilde{u}$, $\tilde{v}$ are quasi-triangular fuzzy numbers and $\tilde{u} = (u, \underline{u}, \overline{u})_\nabla$, $\tilde{v} = (v, \underline{v}, \overline{v})_\nabla$, then

$$\tilde{u} + \tilde{v} = (u + v, \underline{u} + \underline{v}, \overline{u} + \overline{v})_\nabla, \quad \tilde{u} - \tilde{v} = (u - v, \underline{u} - \underline{v}, \overline{u} - \overline{v})_\nabla.$$

Theorem 1-3 are easy to prove.

Theorem 4. If $\tilde{u}$, $\tilde{v}$ are two triangular fuzzy numbers and $\tilde{u} = (u, \underline{u}, \overline{u})_\Delta$, $\tilde{v} = (v, \underline{v}, \overline{v})_\Delta$ and $\tilde{u}$, $\tilde{v}$ >0, then product $\tilde{w}$ of $\tilde{u}$ and $\tilde{v}$ is a quasi-triangular fuzzy number and $\tilde{w} = (uv, \underline{uv}, \overline{uv})_\nabla$ denoted by $\tilde{w} = \tilde{u}\tilde{v}$.

Proof. If $\tilde{u}$, $\tilde{v}$ are two triangular fuzzy numbers and $\tilde{u} = (u, \underline{u}, \overline{u})_\Delta$, $\tilde{v} = (v, \underline{v}, \overline{v})_\Delta$, then from [5] we can obtain $L_w(x)$ and $R_w(x)$. It is easy to verify that $L_w(x)$ is strictly increasing convex on support set $(\underline{uv}, uv)$ and $R_w(x)$ is strictly decreasing concave on support set $(uv, \overline{uv})$. Thus, $\tilde{w}$ is a quasi-triangular fuzzy number and $\tilde{w} = \tilde{u}\tilde{v} = (uv, \underline{uv}, \overline{uv})_\nabla$.

3. Solutions to FLPV problems

Definition 3. Let $\tilde{A} = (a_{ij})_{m \times n}$, $\tilde{c} = (c_j)_{1 \times n}$, $\tilde{b} = (b_i)_{m \times 1}$, $\tilde{x} = (x_j)_{n \times 1}$, where $a_{ij}, c_j, b_i (i = 1,2,\cdots,m, j = 1,2,\cdots,n)$ are triangular fuzzy constants and $x_j (j = 1,2,\cdots,n)$ are triangular fuzzy variables(a_{ij}, c_j, b_i >0). Then

$$\max \quad \tilde{c}\tilde{x}$$

$$\text{s.t.} \quad \tilde{A}\tilde{x} \le \tilde{b} \qquad (1)$$
$$\tilde{x} \ge 0$$

is said to be a FLP problem with fuzzy variables denoted by FLPV shortly.

From Theorem 3-4, we obtain equivalent description of (1)

$$\max \quad (cx, \underline{cx}, \overline{cx})_{\nabla}$$
$$\text{s.t.} \quad (Ax, \underline{Ax}, \overline{Ax})_{\nabla} \le (b, \underline{b}, \overline{b})_{\Delta} \qquad (2)$$
$$(x, \underline{x}, \overline{x})_{\Delta} \ge 0$$

We apply results of Theorem 2 in the problem (2) and have

$$\max \begin{bmatrix} \underline{cx} \\ cx \\ \overline{cx} \end{bmatrix}$$
$$\text{s.t.} \quad Ax \le b$$
$$\underline{Ax} \le \underline{b}$$
$$\overline{Ax} \le \overline{b}$$
$$x - \underline{x} \ge 0 \qquad (3)$$
$$\overline{x} - x \ge 0$$
$$x, \underline{x}, \overline{x} \ge 0$$

It is easy to know that this is a crisp multiobjective linear programming problem with 3 objectives and we can solve it with usual multiobjective programming method[3].

For solving (3), we can consider simplified method. From order of fuzzy numbers[4], we know that the key causing quasi-triangular fuzzy numbers $(cx, \underline{cx}, \overline{cx})_{\nabla}$ to be the biggest lies in causing cx to be big as soon as possible and then $\underline{cx}, \overline{cx}$ to be big as soon as possible. In other words, objective cx is more prior than $\underline{cx}$ and $\overline{cx}$ which have same priority. Thus, the problem (3) can be regarded as two- level multiobjective programming problem. Objectives with first priority and second priority are $F_1(x)$ and $F_2(x)$ respectively where $F_1(x) = cx$ and $F_2(x) = (\underline{cx}, \overline{cx})$. Assume that their optimal solutions are $x^*, \underline{x}^*$ and $\overline{x}^*$, then the fuzzy optimal solution of FLPV problem (1) is $(x^*, \underline{x}^*, \overline{x}^*)$.

4. Numerical example

We consider following FLPV problem

$$\max \quad \tilde{3}\tilde{x}_1 + \tilde{5}\tilde{x}_2$$
$$\text{s.t.} \quad \tilde{7}\tilde{x}_1 + \tilde{3}\tilde{x}_2 \le 2\tilde{3} \qquad (4)$$
$$\tilde{3}\tilde{x}_1 + \tilde{8}\tilde{x}_2 \le 3\tilde{4}$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0$$

where, $\tilde{x}_1 = (x_1, \underline{x}_1, \overline{x}_1)_\Delta$, $\tilde{x}_2 = (x_2, \underline{x}_2, \overline{x}_2)_\Delta$, $\tilde{3} = (3,2,4.5)_\Delta$, $\tilde{5} = (5,4,6)_\Delta$, $\tilde{7} = (7,6.5,7.5)_\Delta$, $\tilde{8} = (8,7.5,9)_\Delta$, $2\tilde{3} = (23,16,32)_\Delta$, $3\tilde{4} = (34,28,42)_\Delta$.

Thus, $c = (3,5)$, $\underline{c} = (2,4)$, $\overline{c} = (4.5,6)$, $A = \begin{pmatrix} 7 & 3 \\ 3 & 8 \end{pmatrix}$, $\underline{A} = \begin{pmatrix} 6 & 2 \\ 2 & 7 \end{pmatrix}$,

$\overline{A} = \begin{pmatrix} 7.5 & 4.5 \\ 4.5 & 9 \end{pmatrix}$, $b = \begin{pmatrix} 23 \\ 34 \end{pmatrix}$; $\underline{b} = \begin{pmatrix} 16 \\ 28 \end{pmatrix}$, $\overline{b} = \begin{pmatrix} 32 \\ 42 \end{pmatrix}$.

From section 3, we obtain the fuzzy optimal solution of problem (4)

$$\tilde{x}_1{*} = (1.74, 1.34, 2.095)_\Delta, \quad \tilde{x}_2{*} = (3.52, 3.382, 3.619)_\Delta.$$

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