ON THE LAW OF LARGE NUMBERS FOR FUZZY NUMBERS

R.Z. Salakhutdinov
Scientific Industrial Union "Cybernetics"
Uzbekistan, Tashkent

This paper deals with some new rezult about the law of large numbers for fuzzy numbers in the framework of theory possibility

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1. Introduction. In the framework of theory possibility Zadeh, R.Fuller [3] is shown that if ξ_1, $\xi_2, \ldots$ are fuzzy number of triangular form with common width α and t-norm is weaker than the Hamacher's operator with zero parameter, then this sequense obeys the law of large numbers for fuzzy numbers (see further theorem 1).

Note that a triangular fuzzy number ξ denoted by (m,α), and its membership function defined as $\xi(x) = 1 - |x-m|/\alpha$, if $m-\alpha \leq x \leq m + \alpha$; otherwise $\xi(x) = 0$. Here α - is its width; m - is its modal values; $(\alpha>0, -\infty < m < \infty)$. Now, the grade possibilty of the statement: " [a,b] contains the value of ξ" is defined as $Pos(a \leq \xi \leq b) = \sup_{a<x<b} \xi(x)$; Necessity: as $Nes(a \leq \xi \leq b) = 1 - Pos(\xi<a, \xi>b)$.

Function T: $[0;1] \times [0;1] \to [0;1]$ is t-norm, if T is commutative, associative, non decreasing and $T(0,1)= 0$, $T(1,1)=1$. As a examples of t-norms are Hamacher's (H_r) and Dombi's (D_q) operators [2]:

$$H_r(u,v) = \frac{uv}{r+(1-r)(u+v-uv)}, \quad r>0; \qquad D_q(u,v)= \left\{1+ \left[\left(\frac{1-u}{u} \right)^q + \left(\frac{1-v}{v} \right)^q \right]^{1/q}\right\}^{-1}, \quad q>0$$

Let ξ_1, ξ_2 are fuzzy numbers. Then their T-sum denoted by $(\xi_1+\xi_2)_T$ and its membership function defined by:

$$(\xi_1+\xi_2)_T(z) = \sup_{x+y=z} T(\xi_1(x), \xi_2(y))$$

Obviously, that for a tasks of applicable character it is interesting to study the behavior of the T-sum of fuzzy nambers $S_n = ((\xi_1+\xi_2+\ldots+\xi_n)/n)_T$ when $n \to \infty$.

Theorem 1. [3] If $T \ll H_0$, $\xi_i = (M_i,\alpha)$, then for any $\beta>0$

$$\lim_{n\to\infty} \mathrm{Nes}\left[M_n-\beta \ll \left(\frac{\xi_1+\xi_2+\ldots+\xi_n}{n}\right)_T \ll M_n+\beta\right] = 1, \quad M_n = \frac{m_1+m_2+\ldots+m_n}{n}.$$

Because $T(u,v) < \min(u,v)$, then the question about acting the low of large numbers for $T < \min$ has been steel opened [3].

Results.

Theorem 2. If $T < \min$, $\xi_i = (M_i,\alpha)$, then for any $\beta>0$

$$\lim_{n\to\infty} \mathrm{Nes}\left[M_n-\beta \ll \left(\frac{\xi_1+\xi_2+\ldots+\xi_n}{n}\right)_T \ll M_n+\beta\right] = 1$$

Proof. Let $0<\beta<\alpha$, else when $\beta>\alpha$ then we get trivial case. Let $T=D_q$. After correspoding calculation we established that membership function of the T-sum S_n will be following:

$$S_n(z) = \frac{1 - |z-M_n|/\alpha}{1 + (n^{1/q} - 1)|z-M_n|/\alpha}, \text{ if } |z-M_n| \ll \alpha$$

$S_n(z) = 0$, if $|z-M_n| \gg \alpha$

From this $\mathrm{Nes}(|S_n-M_n| \ll \beta) = 1 - \mathrm{Pos}(|S_n-M_n| > \beta) =$

$$= 1 - \sup_{\substack{z \\ |z-M_n|>\beta}} \left(\frac{\xi_1+\xi_2+\ldots+\xi_n}{n} \right)_T (z) = 1 - \frac{1 - \beta/\alpha}{1+(n^{1/q}-1)\beta/\alpha}$$

Thus we get $\lim_{n \to \infty} \mathrm{Nes}(|S_n-M_n| \ll \beta) = 1$

Now, taking into consideration that $H_{oo} \ll H_1 \ll \ldots \ll H_0 = D_1 \ll D_2 \ll \ldots \ll D_{oo} = \min$, we convincing in true of affirmation this theorem.

Extrat from Theorem 2 the most interesting corollaries.

Corollary 1. When $q = 1$, then from theorem 2 as a corollary follows the proposition of theorem 1.

Corolarry 2. If $T(u,v) = \min(u,v)$, $\xi_i = (M_i, \alpha)$, then for any $0 < \beta < \alpha$ the law of large numbers is not true.

Proof. If $q \to \infty$, then $T(u,v) = D_{oo}(u,v) = \min(u,,v)$, and concequently $\mathrm{Nes}(|S_n-M_n| \ll \beta) = \beta/\alpha$.

References

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