L—FUZZY SEMI—OPEN SETS AND SEMI—CONTINUOUS MAPPINGS*

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Abstract In this paper, a new definition for L—fuzzy semi—open sets in L—fuzzy topological spaces is presented. The L—fuzzy semi—open sets possess almost all properties of usual semi—open sets in crisp topology. The notions of L—fuzzy semi—continuous mappings and L—fuzzy semi—open mappings and L—fuzzy semi—closed mappings are defined, and some characterizations for L—fuzzy semi—continuous mapping are obtained.

Keywords: L—fuzzy topology, L—fuzzy semi—open set, L—fuzzy semi—continuous mapping, Fuzzy lattice

1 Introduction

In 1981, K. K. Azad[1] first presented the concepts of fuzzy semi—open sets and fuzzy semi—continuous mappings in fuzzy topological spaces. Based on this, a series of works have been launched [2,3,9,10]. But just as Azad himself pointed out, Azad's fuzzy semi—open sets have some shortcomings: for example, the intersection of a fuzzy semi—open set with a fuzzy open set may fail to be a fuzzy semi—open set, which was quite different from the classical semi—open sets. Trace sth, to its source, Azad's definition is only the simple translation for crisp semi—open sets [4], which it was not considered that the stratification structures of fuzzy sets. Moreover, all works relate to fuzzy semi—open sets were done only for the case of fuzzy topological spaces but not the general L—fuzzy topological spaces.

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The main purpose of this paper is to define $L-$fuzzy semi$-$open sets in the general $L-$fuzzy topological spaces taking the stratification structures of $L-$fuzzy sets as the point of departure, and then proceed to build a mathematical sound foundation for to study so$-$called the theory of $L-$fuzzy semi$-$topological spaces.

2. Preliminaries

Throughout this paper, we assume that L is a fuzzy lattice, i. e., a complete, completely distributive lattice with an order$-$reversing involution" '", its largest and smallest element are deroted by 1 and 0 respectively. Let $M(L)$ denote the set of all the union$-$irreducible nonzero elements (i. e., $p \in L$ is irreducible if $p \leqslant a \vee b \Rightarrow p \leqslant a$ *or* $p \leqslant b$). Let $p(L)$ denote the set of all the prime elements (i. e., $p \in L$ is a prime element if $a \wedge b \leqslant p \Rightarrow a \leqslant p$ *or* $b \leqslant p$) of L which are different from 1. It is easy to see that $a \in M(L) \Leftrightarrow a' \in p(L)$. Let X be a nonempty crisp set. If A is a crisp subset of X, then we let χ_A denote the characteristic function of A which takes value on $\{0,1\} \subset L$, A mapping from X into L is said to be $L-$fuzzy set on X. The collection of all the $L-$fuzzy set on X, denoted by $LF(X)$, can be naturally seen as a fuzzy lattice $(LF(X), \leqslant, \vee, \wedge, ')$, its largest and smallest element are denoted by 1_X and 0_X respectively. For $A \in LF(X), \Psi \subset LF(X), r \in L$, we write $\iota_r(A) = \{x \in X : A(x) \leqslant r\}, \Psi' = \{B' : B \in \Psi\}$. Moreover, the empty crisp set is always denoted by φ.

2. 1. **Definition**[8]. A pair $(LF(X), \delta)$ is called an $L-$fuzzy topological space, or an $L-$fts for short, if $0_X, 1_X \in \delta$ and δ is closed for finite intersection and arbitrary union. Each member of δ is called an open set and its complementary set is called an closed set.

2. 2. **Definition**[8]. Let $(LF(X), \delta)$ be an $L-$fts and $x \in X, a \in L \backslash \{0\}$. we let x_a denote the $L-$fuzzy set

$$x_a(y) = \begin{cases} 0, y \in X \backslash \{x\}, \\ a, y = x, \end{cases}$$

We call x_a an $L-$fuzzy point, or point for short, and call x_a a molecule if $a \in$

$M(L)$. The set of all molecules of $LF(X)$ is denoted by $M(L,X)$.

A closd set P is called a R—neighorhood[11] of a point x_a, if $a \nleq P(x)$, $\eta(e)$ denotes the collection of all the R—neighborhood of the point e.

2.3. **Definition** [8]. Let $(LF(X),\delta)$ be an L—fts, and $A \in LF(X)$. The closure A^- and the interior A° of A are defined respectively, as

$$A^- = \wedge \{B : B \geqslant A, B' \in \delta\}, A^\circ = \vee \{B : B \leqslant A, B \in \delta\}.$$

2.4. **Remark.** For $A \in LF(X)$, A' will denote the complement of A

2.5. **Definition.** Let $f: X \longrightarrow Y$ be an ordinary mapping. We define the L—fuzzy mapping $f: LF(X) \longrightarrow LF(Y)$ and its inverse mapping $f^{-1}: LF(Y) \longrightarrow LF(X)$ as follows:

$f(A)(y) = \vee \{A(x) : x \in X, f(x) = y\}, \forall A \in LF(X), \forall y \in Y,$

$f^{-1}(B)(x) = B(f(x)), \forall B \in LF(Y), \forall x \in X.$

Notice that we only consider the L—fuzzy mappings which are induced by crisp mappings as above, and use the same symbol to denote the crisp mapping and its induced mapping. Moreover, in[5,7], the L—fuzzy mappings here are called Zadeh's type functions, hence from Theorem 1.1 and 1.2 in[7]we have.

2.6. **Theorem**[7]. Let $f: LF(X) \longrightarrow LF(Y)$ be an L—fuzzy mapping, then

(1) $f^{-1}(1_Y) = 1_X, f^{-1}(0_Y) = 0_X$; (2) $f^{-1}(\vee B_i) = \vee f^{-1}(B_i), f^{-1}(\wedge B_i) = \wedge f^{-1}(B_i)$;

(3)If $A \leqslant B$, then $f(A) \leqslant f(B)$. ; (4) $\forall A \in LF(X), A \leqslant f^{-1}(f(A))$;

(5) $\forall B \in LF(Y), B \geqslant f(f^{-1}(B)), f^{-1}(B') = (f^{-1}(B))'$;

(6) $f(A) \leqslant B$ iff $A \leqslant f^{-1}(B)$; (7) For point $x_a, f(x_a) = (f(x))_a$.

2.7. **Definition**[8] Let $(LF(X),\delta)$ be an L—fts, $\varphi \neq Y \subset X$.

(1) For $A \in LF(X)$, we define $A|Y \in LF(Y)$ as follows:

$$\forall y \in Y, (A|Y)(y) = A(y),$$

and call $A|Y$ a restriction of A to Y.

(2)$\delta|Y=\{A|Y:A\in\delta\}$ is an L—fuzzy topology on $LF(Y)$, and $(LF(Y),\delta|Y\}$ is called an L—fuzzy subspace of $(LF(X),\delta)$.

(3) For $B\in LF(Y)$, we define $B^*\in LF(X)$ as follows:

$$\forall\ x\in X, B^*(x)=\begin{cases} B(x), & x\in Y, \\ 0, & x\bar{\in} Y, \end{cases}$$

and call B^* a extension of B over X.

2.8. **Theorem** [8]. Let $(LF(X),\delta)$ be an L—fts, and $(LF(Y),\delta|Y)$ be an L—fuzzy subspace of $(LF(X),\delta)$, $B\in LF(Y)$, Then

(1)$B^*|Y=B$; (2) $B^\circ\geqslant(B^*)^\circ|Y$.

2.9. **Theorem**[8]. Let L be *a* fuzzy lattice, then $\forall\ a\in L$, a can be represented as the intersection of some prime element of L.

2.10. **Theorem Let** $f:LF(X)\longrightarrow LF(Y)$ be an L—fuzzy mapping, $A,B\in LF(X)$, $W\in LF(Y)$, $r\in p(L)$. Then

(1) $\iota_r(\wedge B)=\iota_r(A)\cap\iota_r(B)$; (2.1)

(2) $\iota_r(f(A))=f(\iota_r(A))$; (2.2)

(3) $\iota_r(f^{-1}(W))=f^{-1}(\iota_r(W))$; (2.3)

(4) $\iota_r(f^{-1}(W)\wedge f^{-1}(f(A))\neq\varphi\Rightarrow\iota_r(f^{-1}(W))\wedge A)\neq\varphi$. (2.4)

Proof. (1) $\forall\ x\in X.\ x\in\iota_r(A)\cap\iota_r(B)\Leftrightarrow x\in\iota_r(A)$ and $x\in\iota_r(B)\Leftrightarrow A(x)\leqslant r$ and $B(x)\nleqslant r\overset{r\in p(L)}{\Leftrightarrow}A(x)\wedge B(x)\nleqslant r\Leftrightarrow(A\wedge B)(x)\leqslant r\Leftrightarrow x\in\iota_r(A\wedge B)$.

(2) $\forall\ y\in Y, y\in\iota_r(f(A))\Leftrightarrow f(A)(y)=\vee\{A(x):x\in X, f(x)=y\}\nleqslant r\Leftrightarrow\exists\ x\in X$ such that $f(x)=y, A(x)\nleqslant r\Leftrightarrow\exists\ x\in X, x\in\iota_r(A), f(x)=y\Leftrightarrow y\in f(\iota_r(A))$.

(3) $\forall\ x\in X, x\in\iota_r(f^{-1}(W))\Leftrightarrow f^{-1}(W)(x)\nleqslant r$
$\Leftrightarrow W(f(x))\nleqslant r\Leftrightarrow f(x)\in\iota_r(W)\Leftrightarrow x\in f^{-1}(\iota_r(W))$.

(4)$\iota_r(f^{-1}(W)\wedge f^{-1}(f(A)))\neq\varphi$

$\Rightarrow\iota_r(f^{-1}(W\wedge f(A)))\neq\varphi$ (by Theorem 2.6(2))

$\Rightarrow f^{-1}(\iota_r(W\wedge f(A)))\neq\varphi$ (by Theorem 2.10(3))

$\Rightarrow\iota_r(W\wedge f(A))\neq\varphi$

$\Rightarrow\iota_r(W)\wedge\iota_r(f(A))\neq\varphi$ (by Theorem 2.10(1))

$\Rightarrow \exists\ y \in Y, W(y) \nleq r, f(A)(y) \nleq r$

$\Rightarrow \exists\ y \in Y, W(y) \nleq r, f(A)(y) = V\{A(x) : f(x) = y\} \nleq r$

$\Rightarrow \exists\ y \in Y, W(y) \nleq r, \exists\ x \in X, f(x) = y, A(x) \neq r$

$\Rightarrow \exists\ x \in X, f(x) = y, W(f(x)) = f^{-1}(W)(x) \nleq r, A(x) \nleq r$

$\Rightarrow x \in \iota_r(A) \cap \iota_r(f^{-1}(W) = \iota_r(f^{-1}(W) \wedge A)$

$\Rightarrow \iota_r(f^{-1}(W) \wedge A) \neq \varphi.$

3 *L*—fuzzy semi—open sets

3. 1 **Definition.** Let $(LF(X), \delta)$ be an L—fts, $A \in LF(X)$. A is called an L—fuzzy semi—open set ,if there exists an $B \in \delta$ such that $B \leqslant A$ and

$$\forall\ r \in p(L), \forall\ W \in \delta, \iota_r(W \wedge A) \neq \varphi \Rightarrow \iota_r(W \wedge B) \neq \varphi.$$

The set of all L—fuzzy semi—open sets in $(LF(X), \delta)$ is denoted by $SO(LF(X), \delta)$, or simply $SO(X)$.

3. 2. **Definition.** Let $(LF(X), \delta)$ be an L—fts. If $A \in SO(X)$, then A' is called an L—fuzzy semi—closed set. The set of all L—fuzzy semi—closed sets in $(LF(X), \delta)$is denoted by $SC(LF(X), \delta)$, or simply $SC(X)$

3. 3. **Theorem.** Let$(LF(X), \delta)$ be an L—fts. Then $A \in SO(X)$ iff there is an $B \in \delta$ such that $B \leqslant A$ and

$$\forall\ a(\neq 1) \in L, \forall\ W \in \delta, \iota_a(W \wedge A) \neq \varphi \Rightarrow \iota_a(W \wedge B) \neq \varphi.$$

Proof. Sufficiency. It is obvious.

Necessity. Suppose that $A \in SO(X)$. For any $a\ (\neq 1) \in L$, from Theorem 2. 9 we have $a = \bigwedge_{t \in T} r_t$. Where $r_t \in p(L)$. It follows from $A \in SO(X)$ that there exists $B \in \delta$ such that $B \leqslant A$. If $\iota_a(W \wedge A) \neq \varphi$, then there exist $x \in X$ such that $(W \wedge A) \nleq a$, and so there is $t \in T$ such that $(W \wedge A)(x) \nleq r_t$. From definition 3. 1 wesee that $(W \wedge B)(x) \nleq r_t$, and so $(W \wedge B)(x) \nleq a$, This shows that $\iota_a(W \wedge B) \neq \varphi$.

3. 4. **Theorem.** Let$(LF(X), \delta)$ be an L—fts. Then $A \in SO(X)$ iff $\forall\ r \in p(L), \forall\ W \in \delta, \iota_r(A \wedge W) \neq \varphi \Rightarrow \iota_r(A^\circ \wedge W) \neq \varphi$.

Proof. Necessity. Suppose that $A \in SO(X)$. Then there exists an $V \in \delta$ such that $V \leqslant A$ and

$\forall r \in p(L), \forall W \in \delta, \iota_r(W \wedge A) \neq \varphi \Rightarrow \iota_r(W \wedge V) \neq \varphi$.

But $V \leqslant A^\circ$, and so.

$\forall r \in p(L), \forall W \in \delta, \iota_r(W \wedge A) \neq \varphi \Rightarrow \iota_r(W \wedge A^\circ) \neq \varphi$

Sufficiency. It follows easily from $A \geqslant A^\circ \in \delta$.

3. 5. **Theorem.** Let $(LF(X), \delta)$ be an L-fts. $A \in SO(X), B \in \delta$, Then $A \wedge B \in SO(X)$.

Proof. $\forall r \in p(L), \forall V \in \delta$, if $\iota_r(V \wedge A \wedge B) \neq \varphi$, then from $V \wedge B \in \delta$ and $A \in SO(X)$ we have $\iota_r((V \wedge B) \wedge A^\circ) \neq \varphi$, hence $\iota_r(V \wedge A^\circ \wedge B^\circ) = \iota_r(V \wedge (A \wedge B)^\circ) \neq \varphi$, This shows that $A \wedge B \in SO(X)$.

3. 6. **Theorem** Let $(LF(X), \delta)$ be an L-fts. If $\{A_t : t \in T\} \subset SO(X)$, then $\bigvee_{t \in T} A_t \in SO(X)$.

Proof: $\forall r \in p(L), \forall V \in \delta$, if $\iota_r(V \wedge (\bigvee_{t \in T} A_t)) \neq \varphi$, then there exists an $t \in T$ such that $\iota_r(V \wedge A_t) \neq \varphi$. Because $A_t \in SO(X)$, $\iota_r(V \wedge A^\circ_t) \neq \varphi$, and then $\iota_r(W \wedge (\bigvee_{t \in T} A^\circ_t)) \neq \varphi$, But $(\bigvee_{t \in T} A_t)^\circ \geqslant \bigvee_{t \in T} A^\circ_t$, thus $\iota_r(V \wedge (\bigvee_{t \in T} A_t)^\circ) \neq \varphi$. This shows that $\bigvee_{t \in T} A_t \in SO(X)$.

3. 7. **Corollary.** Let $(LF(X), \delta)$ be an L-fts.

(1) If $A \in SC(X), B \in \delta'$, then $A \vee B \in SC(X)$;

(2) If $\{A_r : t \in T\} \subset SC(X)$, then $\bigwedge_{t \in T} A_t \in SC(X)$.

3. 8. **Theorem** Let $(LF(X), \delta)$ be an L-fts, and $(LF(Y), \delta | Y)$ an L-fuzzy subspace of $(LF(X), \delta)$, and $A \in LF(Y)$. If $A^* \in SO(LF(X), \delta)$, then $A \in SO(LF(Y), \delta | Y)$.

Proof. $\forall r \in p(L), \forall V \in \delta | Y$, we need to prove that $\iota_r(V \wedge A) \neq \varphi \Rightarrow \iota_r(V \wedge (A)^\circ_Y) \neq \varphi$, where $(A)^\circ_Y$ denotes the interior of A in $(LF(Y), \delta | Y)$. For $V \in \delta | Y, \exists\, U \in \delta$ such that $V = U | Y$, and then $\iota_r((U | Y) \wedge A) \neq \varphi$, and so $\iota_r(U \wedge A^*) \neq \varphi$. From $A^* \in AO(LF(X), \delta)$ we obtain that $\iota_r(U \wedge (A^*)^\circ_X) \neq \varphi$ and thus $\exists$ $y_0 \in X$ such that $U(y_0) \not\leqslant r, (A^*)^\circ_X(y_0) \not\leqslant r$. Clearly, $y_0 \in Y$ because $A^*(y_0)=0$ for $y_0 \in X \backslash Y$. Hence $(U|Y)(y_0)=U(y_0) \not\leqslant r$. $((A^*)^\circ_X|Y)(y_0)=(A^*)^\circ_X(y_0) \not\leqslant r$, and so $\iota_r((U|Y) \wedge ((A^*)^\circ_X|Y) \neq \varphi$, but $(A)^\circ_Y \geqslant (A^*)^\circ_X|Y)$, and therefore $\iota_r(V \wedge (A)^\circ_Y) \neq \varphi$.

The converse of Theorem 3. 8 is ,in general, false. However we have.

3. 9. **Theorem.** Let $(LF(X), \delta)$ be an $L-$fts, and $(LF(Y), \delta|Y)$ an $L-$fuzzy subspace of $(LF(X), \delta)$. If $A \in LF(Y)$, and $\chi_Y \in SO(LF(X), \delta)$, then $A^* \in SO(LF(X), \delta)$ iff $A \in SO(LF(Y), \delta|Y)$.

The proof of next Lemma is direct.

3. 10. **Lemma** Let $(LF(X), \delta)$ be an $L-$fts, then $\delta = \{A^\circ : A \in SO(LF(X), \delta)\}$.

From Lemma 3. 10 we have directly the following

3. 11. **Theorem.** Let $(LF(X), \delta)$ and $(LF(X), \tau)$ be $L-$fts's. If $SO(LF(X), \delta) \subset SO(LF(X), \tau)$, then $\delta \subset \tau$; If $SO(LF(X), \delta) = SO(LF(X), \tau)$, then $\delta = \tau$.

4 $L-$fuzzy semi$-$continuous mappings

6. 1 **Definition**[8,12]. Let $(LF(X), \delta)$ *and* $(LF(Y), \tau)$ be two $L-$fts's, and $f: (LF(X), \delta) \longrightarrow (LF(Y), \tau)$ an $L-$fuzzy mapping. Then

(1) f is called $L-$fuzzy semi$-$continuous (or LFSC, for short), if $f^{-1}(V) \in SO(LF(X), \delta)$ holds for each $V \in \tau$;

(2) f is called $L-$fuzzy semi$-$open, if $f(U) \in SO(LF(Y), \tau)$ holds for any $U \in \delta$;

(3) f is called $L-$fuzzy semi$-$closed, if $f(W) \in SC(LF(Y), \tau)$ holds for each $W \in \delta'$;

(4) f is called $L-$fuzzy semi$-$continuous at $e \in M(L, X)$, if $f^{-1}(P) \in s\eta(e)$ holds for each $P \in \eta(f(e))$;

(5) f is called $L-$fuzzy continuous, if $f^{-1}(V)\in\delta$ holds for each $V\in\tau$;

(6) f is called $L-$fuzzy open, if $f(V)\in\tau$ hold for each $V\in\delta$.

4.2. **Theorem.** Let $f:(LF(X),\delta)\longrightarrow(LF(Y),\tau)$ be an $L-$fuzzy continuous mapping and an $L-$fuzzy semi$-$open mapping, then $f(A)\in SO(LF(Y),\tau)$ holds for each $A\in SO(LF(X),\delta)$.

Proof. $\forall A\in SO(LF(X),\delta)$ we need to prove that

$$\forall r\in p(L),\forall W\in\tau,\iota_r(W\wedge f(A))\neq\varphi\Rightarrow\iota_r(W\wedge(f(A))^\circ)\neq\varphi,$$

$$\begin{aligned}
&\iota_r(W\wedge f(A))\neq\varphi\\
&\Rightarrow f^{-1}(\iota_r(W\wedge f(A))\neq\varphi\\
&\Rightarrow \iota_r(f^{-1}(W)\wedge f^{-1}(f(A)))\neq\varphi && (\text{by}(2.1))\\
&\Rightarrow \iota_r(f^{-1}(W)\wedge A)\neq\varphi && (\text{by}(2.4))\\
&\Rightarrow \iota_r(f^{-1}(W)\wedge A^\circ)\neq\varphi && (f^{-1}(W)\in\delta,A\in SO(X))\\
&\Rightarrow f(\iota_r(f^{-1}(W)\wedge A^\circ))\neq\varphi\\
&\Rightarrow \iota_r(f(f^{-1}(W)\wedge A^\circ))\neq\varphi && (\text{by}(2.2))\\
&\Rightarrow \iota_r(f(f^{-1}(W))\wedge f(A^\circ))\neq\varphi\\
&\Rightarrow \iota_r(W\wedge f(A^\circ))\neq\varphi\\
&\Rightarrow \iota_r(W\wedge(f(A^\circ))^\circ)\neq\varphi && (f(A^\circ)\in SO(Y))\\
&\Rightarrow \iota_r(W\wedge(f(A))^\circ)\neq\varphi.
\end{aligned}$$

4.3. **Theorem.** Let $f:(LF(X),\delta)\longrightarrow(LF(Y),\tau)$ be an $L-$fuzzy open mapping and an LFSC. Then $f^{-1}(V)\in SO(LF(X),\delta)$ holds for each $V\in SO(LF(Y),\tau)$.

Proof. $\forall V\in SO(LF(Y),\tau)$, we need to prove that

$$\forall r\in p(L),\forall W\in\delta,\iota_r(W\wedge f^{-1}(V))\neq\varphi\Rightarrow\iota_r(W\wedge(f^{-1}(V))^\circ)\neq\varphi$$

$$\begin{aligned}
&\iota_r(W\wedge f^{-1}(V))\neq\varphi\\
&\Rightarrow f(\iota_r(W\wedge f^{-1}(V)))\neq\varphi\\
&\Rightarrow \iota_r(f(W\wedge f^{-1}(V))\neq\varphi && (\text{by}(2.2))\\
&\Rightarrow \iota_r(f(W)\wedge f(f^{-1}(V)))\neq\varphi\\
&\Rightarrow \iota_r(f(W)\wedge V)\neq\varphi\\
&\Rightarrow \iota_r(f(W)\wedge V^\circ)\neq\varphi && (f(W)\in\tau,V\in SO(Y))
\end{aligned}$$

$f(W) \wedge V^{\circ}))\neq\varphi$

$(f(W) \wedge V^{\circ}))\neq\varphi$ (by(2.3))

$(f(W) \wedge f^{-1}(V^{\circ}))\neq\varphi$

$\wedge f^{-1}(V^{\circ}))\neq\varphi$ (by(2.4))

$\wedge (f^{-1}(V^{\circ}))^{\circ})\neq\varphi$ $(f^{-1}(V^{\circ})\in SO(X))$

$\wedge (f^{-1}(V))^{\circ})\neq\varphi.$

4.4. **Theorem.** Let $f:(LF(X),\delta)\longrightarrow(LF(Y),\tau)$ be an L—fuzzy mapping, then f is L—fuzzy semi—closed iff for each $B\in LF(Y$ and any $A\in SO(X)$ with $A\geqslant f^{-1}(B)$ there exists $C\in SO(Y)$ such that $C\geqslant B$ and $A\geqslant f^{-1}(C)$.

Proof. Suppose that f is L—fuzzy semi—closed and $A\in SO(X), A\geqslant f^{-1}(B)$. Now $C=(f(A'))'\in SO(Y)$, and from $A'\leqslant(f^{-1}(B))'=f^{-1}(B')$ we have $f(A')\leqslant f(f^{-1}(B'))\leqslant B'$, hence $B\leqslant(f(A'))'=C$ and $f^{-1}(C)=f^{-1}((f(A'))')=(f^{-1}(f(A')))'\leqslant(A')'=A$

Conversely, suppose that the condition of the theorem is satisfied and $D\in SC(LF(X),\delta)$, Put $B=(f(D))'$, then

$$f^{-1}(B)=f^{-1}((f(D))')=(f^{-1}(f(D)))'\leqslant D'\triangleq A\in SO(LF(X),\delta)$$

Hence there exists $C\in SO(LF(Y),\tau)$ such that $C\geqslant B=(f(D))'$ and $f^{-1}(C)\leqslant A=D'$, i.e., $f^{-1}((C')\geqslant D$, and thus $f(D)\leqslant f(f^{-1}(C'))\leqslant C'$. Hence $f(D)=C'\in SC(LF(Y),\tau)$. This shows that f is L—fuzzy semi—closed.

Similarly, we can prove

4.5. **Theorem.** Suppose that $f:(LF(X),\delta)\longrightarrow(LF(Y),\tau)$ is an L—fuzzy mapping, then f is L—fuzzy semi—open iff for each $B\in LF(Y)$ and any $A\in SC(LF(X),\delta)$ with $A\geqslant f^{-1}(B)$ there exists $C\in SC(LF(Y),\tau)$ such that $C\geqslant B$ and $A\geqslant f^{-1}(C)$.

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