# Intuitionistic gradation of openness : intuitionistic fuzzy topology

by

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*Abstract* : In this paper we introduce a concept of intuitionistic gradation of openness on fuzzy sets of a nonempty set $X$ and define an intuitionistic fuzzy topological space. We prove that the category of intuitionistic fuzzy topological spaces and gradation preserving mappings is a topological category. We study compactness of intuitionistic fuzzy topological spaces and prove Tychonoff theorem.

*Keywords* : fuzzy set, intuitionistic fuzzy set, fuzzy topology, gradation of openness, intuitionistic gradation of openness, intuitionistic fuzzy topological space.

## 0. Introduction

After the introduction of fuzzy sets by Zadeh [19], Chang [8] firstly introduced the concept of fuzzy topology on a set $X$ by axiomatizing a colletion $T$ of fuzzy subsets of $X$ (we call fuzzy sets of $X$), where he referred to each member of $T$ as an open set. In his definition of fuzzy topology, fuzziness in the concept of openness of a fuzzy set was absent. In [13] and [9], Samanta et al. introduced a concept of gradation of openness (closedness) of fuzzy sets of $X$ in two different ways, and gave definitions of fuzzy topology on $X$. They referred to the fuzzy topology in the sense of Chang as topology of fuzzy sets (briefly TFS).

The gradation of openness on $X$ as introduced in [9], is a mapping $\tau : I^X \to I$, satisfying the following conditions :
(i) $\tau(\tilde{0}) = \tau(\tilde{1}) = 1$,
(ii) $\tau(\lambda_1 \bigcap \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$,
(iii) $\tau(\bigcup_{i\in\Delta} \lambda_i) \geq \wedge_{i\in\Delta} \tau(\lambda_i)$.
The pair $(X, \tau)$ is called a fuzzy topological space (briefly FTS).

On the otherhand various generalisations of the concept of fuzzy sets have been done by many authors. Among others, L.A.Zadeh [20] introduced the idea of interval-valued fuzzy sets. Later in [2], K.T.Atanassov introduced the idea of intuitionistic fuzzy sets. Recently much work have been done with these concepts. For reference see Atanassov [3], [5] and Bustince [6], [7], who have worked mainly on operators and relations on intuitionistic/interval-valued intuitionistic fuzzy sets. D.Coker in [11], introduced the idea of topology of intuitionistic fuzzy sets. In [17] and [18] we introduced the definitions of topology of interval-valued fuzzy sets and topology of interval-valued intuitionistic fuzzy sets respectively.

The purpose of this paper is to introduce some sort of gradation of openness ; in fact we are able to define intuitionistic gradation of openness of fuzzy sets and thereby give the definition of intuitionistic fuzzy topology.

The novelty of our definition is that corresponding to an intuitionistic fuzzy topology in our sense, each fuzzy subset enjoys a definite grade of openness as well as a definite grade of nonopenness.

In section 2, we give the definition of intuitionistic gradation of openness of fuzzy sets and obtain some basic results. We prove the decomposition theorem of intuitionistic gradation in the sense that an intuitionistic gradation is shown to generate as well as is generated by a pair of descending families of topologies of fuzzy sets.
In section 3, definition of a subspace of an intuitionistic fuzzy topological space is given and some properties are studied.
In section 4, some properties of gradation preserving mapping are studied.
In section 5, it is observed that the category of intuitionistic fuzzy topological spaces and gradation preserving maps forms a topological category.
In section 6, we study compactness and prove Tychonoff theorem in this setting.

## 1. Notations and Preliminaries

In this paper $X$ will denote a nonempty set ; $I = [0, 1]$, the closed unit interval of the real line ; $I_o = (0, 1]$ ; $I_1 = [0, 1)$ ; $I^X$ = the set of all fuzzy sets of $X$. We shall denote fuzzy sets by Greek lower case letters, such as $\lambda, \mu, \nu, \eta$ etc. By $\tilde{0}_X$ and $\tilde{1}_X$ we denote the constant fuzzy sets of $X$ taking values 0 and 1 respectively ; when $X$ is understood, we shall omit the suffix $X$ and denote them by

$\tilde{0}$ and $\tilde{1}$ respectively. For briefing notations, we write IGO for intuitionistic gradation of openness ; IFTS for intuitionistic fuzzy topological space ; IFT for intuitionistic fuzzy topology ; BTFS for bitopology of fuzzy sets ; gp for gradation preserving. All other notations are standard notations of fuzzy set theory.

**Definition 1.1** **[13]** Let $Y$ be a subset of $X$ and $\mu \in I^X$; the restriction of $\mu$ on $Y$ is denoted by $\mu/Y$. For each $\mu \in I^Y$ the extension of $\mu$ on $X$, denoted by $\mu_X$, is defined by

$\mu_X = \mu$ on $Y$

$= \tilde{0}$ on $X - Y$.

**Definition 1.2** A bitopological space of fuzzy sets $(X, T, T^*)$ is said to be inclusive if $T \subset T^*$.

## 2. Intuitionistic gradation of openness

**Definition 2.1** Let $X$ be a nonempty set. An intuitionistic gradation of openness of fuzzy sets of $X$ (when it is understood that we are considering fuzzy sets only we call it, for simplicity, intuitionistic gradation of openness on $X$, briefly IGO on $X$), is an ordered pair $(\tau, \tau^*)$ of functions from $I^X$ to $I$ such that

(IGO1) $\tau(\lambda) + \tau^*(\lambda) \leq 1, \quad \forall\, \lambda \in I^X$,

(IGO2) $\tau(\tilde{0}) = \tau(\tilde{1}) = 1$

$\tau^*(\tilde{0}) = \tau^*(\tilde{1}) = 0$,

(IGO3) $\tau(\lambda_1 \bigcap \lambda_2) \geq \tau(\lambda_1) \wedge \tau(\lambda_2)$

and

$\tau^*(\lambda_1 \bigcap \lambda_2) \leq \tau^*(\lambda_1) \vee \tau^*(\lambda_2), \quad \lambda_i \in I^X, i = 1, 2$,

(IGO4) $\tau(\bigcup_{i\in\Delta} \lambda_i) \geq \wedge_{i\in\Delta}\tau(\lambda_i)$

and

$\tau^*(\bigcup_{i\in\Delta} \lambda_i) \leq \vee_{i\in\Delta}\tau^*(\lambda_i), \quad \lambda_i \in I^X,\ i \in \Delta$.

The triplet $(X, \tau, \tau^*)$ is called an intuitionistic fuzzy topological space. $\tau$ and $\tau^*$ may be interpreted as gradation of openness and gradation of nonopenness respectively.

**Definition 2.2** Let $X$ be a nonempty set and $\mathcal{F}, \mathcal{F}^* : I^X \to I$ be two mappings satisfying :

(IGC1) $\mathcal{F}(\lambda) + \mathcal{F}^*(\lambda) \leq 1, \ \forall\, \lambda \in I^X$,

(IGC2) $\mathcal{F}(\tilde{0}) = \mathcal{F}(\tilde{1}) = 1$

$\mathcal{F}^*(\tilde{0}) = \mathcal{F}^*(\tilde{1}) = 0$,

(IGC3) $\mathcal{F}(\lambda_1 \bigcup \lambda_2) \geq \mathcal{F}(\lambda_1) \wedge \mathcal{F}(\lambda_2)$

and

$\mathcal{F}^*(\lambda_1 \bigcup \lambda_2) \leq \mathcal{F}^*(\lambda_1) \vee \mathcal{F}^*(\lambda_2), \quad \lambda_i \in I^X, i = 1, 2$,

(IGO4) $\mathcal{F}(\bigcap_{i\in\Delta} \lambda_i) \geq \wedge_{i\in\Delta}\mathcal{F}^*(\lambda_i)$

and

$$\mathcal{F}^*(\bigcap_{i\in\Delta}\lambda_i) \leq \vee_{i\in\Delta}\mathcal{F}^*(\lambda_i), \quad \lambda_i \in I^X,\ i \in \Delta.$$

Then the pair $(\mathcal{F}, \mathcal{F}^*)$ is an intuitionistic gradation of closedness on $X$ (briefly IGC on $X$).

**Definition 2.3** For two pairs of mappings $(\tau, \tau^*)$ and $(\mathcal{F}, \mathcal{F}^*)$ from $I^X \to I$ define
$\tau_{\mathcal{F}}(\lambda) = \mathcal{F}(\lambda^c),\ \ \tau^*_{\mathcal{F}^*}(\lambda) = \mathcal{F}^*(\lambda^c)$
$\mathcal{F}_\tau(\lambda) = \tau(\lambda^c),\ \ \mathcal{F}^*{}_{\tau^*}(\lambda) = \tau^*(\lambda^c).$

**Theorem 2.4**
*(a)* $(\tau, \tau^*)$ *is an IGO on* $X$ *iff* $(\mathcal{F}_\tau, \mathcal{F}^*{}_{\tau^*})$ *is an IGC on* $X$,
*(b)* $(\mathcal{F}, \mathcal{F}^*)$ *is an IGC on* $X$ *iff* $(\tau_{\mathcal{F}}, \tau^*_{\mathcal{F}^*})$ *is an IGO on* $X$,
*(c)* $\tau_{\mathcal{F}_\tau} = \tau,\ \tau^*_{\mathcal{F}^*{}_{\tau^*}} = \tau^*,\ \mathcal{F}_{\tau_{\mathcal{F}}} = \mathcal{F},\ \mathcal{F}^*{}_{\tau^*{}_{\mathcal{F}^*}} = \mathcal{F}^*.$

**Definition 2.5** Let $\{(\tau_i, \tau_i^*)\}_{i\in\Delta}$ be a family of IGOs on $X$. Then their intersection is defined by

$$\bigcap_{i\in\Delta}(\tau_i, \tau_i^*) = (\wedge_{i\in\Delta}\tau_i, \vee_{i\in\Delta}\tau_i^*).$$

**Theorem 2.6** *An arbitrary intersection of IGOs is an IGO.*

**Definition 2.7** Let $(\tau, \tau^*)$ and $(\mathcal{U}, \mathcal{U}^*)$ be two IGOs on $X$. Then we define a relation '$\lesssim$' by

$$(\tau, \tau^*) \lesssim (\mathcal{U}, \mathcal{U}^*) \Leftrightarrow \tau \leq \mathcal{U} \text{ and } \tau^* \leq \mathcal{U}^*.$$

**Remark 2.8** Let $X$ be a nonempty set. Define
$\tau_w, \tau_w^*, \tau_s, \tau_s^* : I^X \to I$ by the rule :
$\tau_w(\tilde{0}) = \tau_w(\tilde{1}) = 1, \tau_w^*(\tilde{0}) = \tau_w^*(\tilde{1}) = 0$
$\tau_w(\mu) = 0, \tau_w^*(\mu) = 1,\ \forall\mu \in I^X - \{\tilde{0}, \tilde{1}\}$
and
$\tau_s(\mu) = 1, \tau_s^*(\mu) = 0,\ \forall\mu \in I^X.$
Then, $(\tau_w, \tau_w^*)$ and $(\tau_s, \tau_s^*)$ are two IGOs on $X$ such that for any IGO $(\tau, \tau^*)$ on $X$,

$$(\tau_w, \tau_w^*) \lesssim (\tau, \tau^*) \lesssim (\tau_s, \tau_s^*).$$

From Theorem 2.6 and Remark 2.8, we have the following :

**Theorem 2.9** *The collection* $\mathcal{G}$ *of all IGOs on* $X$ *forms a complete lattice w.r.t. '*$\lesssim$*' of which* $(\tau_w, \tau_w^*)$ *and* $(\tau_s, \tau_s^*)$ *are respectively the smallest and largest element.*

**Notation 2.10** Let $(\tau, \tau^*)$ be an IGO on $X$. For $r \in I_o$, denote

$$\tau_r = \tau^{-1}[r, 1]$$
$$\tau_r^* = (\tau^*)^{-1}[0, 1-r].$$

**Theorem 2.11** *Let $(X, \tau, \tau^*)$ be an IFTS. Then $\{\tau_r\}_{r \in I_o}$ and $\{\tau_r^*\}_{r \in I_o}$ are two descending families of topologies of fuzzy sets on $X$ such that*
*(a)* $\tau_r \subset \tau_r^*$,
*(b)* $\tau_r = \bigcap_{s<r} \tau_s$ ; $\tau_r^* = \bigcap_{s<r} \tau_s^*$.

**Note 2.12** If $(\tau, \tau^*)$ is an IGO on $X$, then for $r \in I_o$, $(\tau_r, \tau_r^*)$ is an inclusive BTFS on $X$, which will be called $r$-level inclusive BTFS.

**Theorem 2.13** *Let $\{T_r\}_{r \in I_o}$ and $\{T_r^*\}_{r \in I_o}$ be two descending families of topologies of fuzzy sets on $X$ such that $T_r \subset T_r^*$, $\forall\, r \in I_o$. Define $\tau, \tau^* : I^X \to I$ by*
$\tau(\lambda) = \vee\{r \; ; \; \lambda \in T_r\}$
*and*
$\tau^*(\lambda) = \wedge\{1 - r \; ; \; \lambda \in T_r^*\}$.
*Then,*
*(a)* $(\tau, \tau^*)$ *is an IGO on $X$,*
*(b)* $\tau_r = T_r$ *iff* $\bigcap_{s<r} T_s = T_r$,
*(c)* $\tau_r^* = T_r^*$ *iff* $\bigcap_{s<r} T_s^* = T_r^*$.

**Definition 2.14** Let $(T, T^*)$ be a BTFS on a nonempty set $X$. Define for each $r \in I_o$, two mappings $T^r$, $(T^*)^r : I^X \to I$ by the rule :
$T^r(\tilde{0}) = T^r(\tilde{1}) = 1$, $(T^*)^r(\tilde{0}) = (T^*)^r(\tilde{1}) = 0$,
$T^r(\mu) = r$, if $\mu \in T - \{\tilde{0}, \tilde{1}\}$
=0, otherewise
and
$(T^*)^r(\mu) = 1 - r$, if $\mu \in T^* - \{\tilde{0}, \tilde{1}\}$
=1, otherewise.

**Theorem 2.15** *Let $(T, T^*)$ be an inclusive BTFS on $X$. Then $(T^r, (T^*)^r)$ is an IGO on $X$ such that $(T^r)_r = T$ and $((T^*)^r)_r = T^*$.*

**Definition 2.16** If $(T, T^*)$ is an inclusive BTFS on $X$, then $(T^r, (T^*)^r)$ is called r-th intuitionistic gradation of openness (briefly r-th IGO) on $X$ and $(X, T^r, (T^*)^r)$ is called an r-th graded intuitionistic fuzzy topological space (briefly r-th graded IFTS).

**Theorem 2.17** *If $(\tau, \tau^*)$ is an r-th IGO on $X$ then* $(\tau_r)^r = \tau$, $(\tau_r^*)^r = \tau^*$.

## 3. Fuzzy subspaces

**Theorem 3.1** *Let $(X, \tau, \tau^*)$ be an IFTS and $Y \subset X$. Define two mappings $\tau_Y, \tau_Y^* : I^Y \to I$ by the* *rule :*

$$\begin{aligned}\tau_Y(\mu) &= \vee\{\tau(\lambda)\ ;\ \lambda \in I^X,\ \lambda/Y = \mu\}\\ \tau^*_Y(\mu) &= \wedge\{\tau^*(\lambda)\ ;\ \lambda \in I^X,\ \lambda/Y = \mu\}\end{aligned}$$

*Then* $(\tau_Y, \tau^*_Y)$ *is an IGO on* $Y$ *and* $\tau_Y(\mu) \geq \tau(\mu_X)$, $\tau^*_Y(\mu) \leq \tau^*(\mu_X)$.

**Definition 3.2** The IFTS $(Y, \tau_Y, \tau^*_Y)$, so determined, is called a subspace of the IFTS $(X, \tau, \tau^*)$ and $(\tau_Y, \tau^*_Y)$ is called the induced IGO on $Y$ from $(X, \tau, \tau^*)$.

**Theorem 3.3** *Let* $(Y, \tau_Y, \tau^*_Y)$ *be a fuzzy subspace of the IFTS* $(X, \tau, \tau^*)$ *and* $\mu \in I^Y$. *Then*
*(i)* $\mathcal{F}_{\tau_Y}(\mu) = \vee\{\mathcal{F}_\tau(\eta) :\ \eta \in I^X, \eta/Y = \mu\}$
$\mathcal{F}^*{}_{\tau^*_Y}(\mu) = \wedge\{\mathcal{F}^*{}_{\tau^*}(\eta) :\ \eta \in I^X, \eta/Y = \mu\}$.
*(ii) If* $Z \subset Y \subset X$, *then* $\tau_Z = (\tau_Y)_Z, \tau^*_Z = (\tau^*_Y)_Z$.

## 4. Gradation preserving maps

**Definition 4.1** Let $(X, \tau, \tau^*)$ and $(Y, \mathcal{U}, \mathcal{U}^*)$ be two IFTSs and $f : X \to Y$ be a mapping. Then $f$ is called a gradation preserving map (briefly gp-map) if for each $\mu \in I^Y$,
$\mathcal{U}(\mu) \leq \tau(f^{-1}(\mu))$
and
$\mathcal{U}^*(\mu) \geq \tau^*(f^{-1}(\mu))$.

**Definition 4.2** [1] Let $f : (X, T, T^*) \to (Y, S, S^*)$ be a mapping, where $(X, T, T^*)$ and $(Y, S, S^*)$ are two bitopological spaces of fuzzy sets. Then $f$ is said to be continuous if
$f : (X, T) \to (Y, S)$
and
$f : (X, T^*) \to (Y, S^*)$
are continuous.

**Theorem 4.3** *Let* $(X, \tau, \tau^*)$ *and* $(Y, \mathcal{U}, \mathcal{U}^*)$ *be two IFTSs and* $f : X \to Y$ *be a mapping. Then*
*(1)* $f : (X, \tau, \tau^*) \to (Y, \mathcal{U}, \mathcal{U}^*)$ *is a gp-map*
*iff*
*(2)* $f : (X, \tau_r, \tau^*_r) \to (Y, \mathcal{U}_r, \mathcal{U}^*_r)$ *is continuous,* $\forall\ r \in I_o$.

**Theorem 4.4** *Let* $(X, \tau, \tau^*), (Y, \mathcal{U}, \mathcal{U}^*)$ *and* $(Z, \mathcal{W}, \mathcal{W}^*)$ *be three IFTSs. If* $f : (X, \tau, \tau^*) \to (Y, \mathcal{U}, \mathcal{U}^*)$ *and* $g : (Y, \mathcal{U}, \mathcal{U}^*) \to (Z, \mathcal{W}, \mathcal{W}^*)$ *be gp-maps, then* $gof : (X, \tau, \tau^*) \to (Z, \mathcal{W}, \mathcal{W}^*)$ *is a gp-map.*

**Definition 4.5** Let $(T, T^*)$ be a BTFS on $X$. Then $(\mathcal{B}, \mathcal{B}^*)$ is said to form a base of $(T, T^*)$ if $\mathcal{B}$ and $\mathcal{B}^*$ are bases of $T$ and $T^*$ respectively.

**Definition 4.6** Let $(T, T^*)$ be a BTFS on $X$. Then $(\mathcal{S}, \mathcal{S}^*)$ is said to form a subbase of $(T, T^*)$ if $\mathcal{S}$ and $\mathcal{S}^*$ are subbases of $T$ and $T^*$ respectively.

**Theorem 4.7** *Let $(X, \tau, \tau^*)$ be an IFTS and $f : X \to Y$ be a mapping. Let $\{(T_r, T_r^*)\ ;\ r \in I_o\}$ be descending families of inclusive BTFS on $Y$, $\forall\, r \in I_o$. Let $(\mathcal{U}, \mathcal{U}^*)$ be the IGO on $Y$ generated by this family. Further suppose, for each $r \in I_o$, $(\mathcal{B}_r, \mathcal{B}_r^*)$ and $(\mathcal{S}_r, \mathcal{S}_r^*)$ are respective base and subbase of $(T_r, T_r^*)$. Then the following statements are equivalent :*
*(a)* $f : (X, \tau, \tau^*) \to (Y, \mathcal{U}, \mathcal{U}^*)$ *is a gp-map.*
*(b)* $\tau(f^{-1}(\mu)) \geq r,\ \forall\, \mu \in T_r,\ \forall\, r \in I_o$ and $\tau^*(f^{-1}(\mu)) \leq 1 - r,\ \forall\, \mu \in T_r^*,\ \forall\, r \in I_o$.
*(c)* $\tau(f^{-1}(\mu)) \geq r,\ \forall\, \mu \in \mathcal{B}_r,\ \forall\, r \in I_o$ and $\tau^*(f^{-1}(\mu)) \leq 1 - r,\ \forall\, \mu \in \mathcal{B}_r^*,\ \forall\, r \in I_o$.
*(d)* $\tau(f^{-1}(\mu)) \geq r,\ \forall\, \mu \in \mathcal{S}_r,\ \forall\, r \in I_o$ and $\tau^*(f^{-1}(\mu)) \leq 1 - r,\ \forall\, \mu \in \mathcal{S}_r^*,\ \forall\, r \in I_o$.

**Theorem 4.8** *Let $(X, T, T^*)$ and $(Y, S, S^*)$ be two inclusive bitopological spaces of fuzzy sets and $f : X \to Y$. Then,*
*(1)* $f : (X, T, T^*) \to (Y, S, S^*)$ *is continuous*
*iff*
*(2)* $f : (X, T^r, (T^*)^r) \to (Y, S^r, (S^*)^r)$ *is a gp-map,* $\forall\, r \in I_o$.

## 5. Category of Intuitionistic fuzzy topological spaces

Let BF denote the category of all inclusive bitopological spaces of fuzzy sets and continuous functions. IF-top denote the category of all intuitionistic fuzzy topological spaces and gp-maps. For each $r \in I_o$, IF$_r$-top denote the category of r-rh graded IFTSs and gp-maps.

**Theorem 5.1**
*(a) IF$_r$-top is a full subcategory of IF-top.*
*(b) For each $r \in I_o$, BF and IF$_r$-top are isometric.*
*(c) IF$_r$-top is a bireflective full subcategory of IF-top, for all $r \in I_o$.*

**Remark 5.2** Because of (b) and (c), henceforth BF may be called a bireflective full subcategory of IF-top.

**Theorem 5.3** *Let $\{(X_i, \mathcal{U}_i, \mathcal{U}_i^*) : i \in \Delta\}$ be a class of IFTSs and $X$ be a nonempty set. Let for each $i \in \Delta$, $f_i : X \to X_i$ be a map. Then there exixts an IGO $(\tau, \tau^*)$ on $X$ such that the following conditions hold :*
*(a) for each $i \in \Delta$, $f_i : (X, \tau, \tau^*) \to (X_i, \mathcal{U}_i, \mathcal{U}_i^*)$ is a gp-map.*
*(b) if $(Y, \mathcal{V}, \mathcal{V}^*)$ is an IFTS then $g : (Y, \mathcal{V}, \mathcal{V}^*) \to (X, \tau, \tau^*)$ is a gp-map iff $f_i o g$ is a gp-map for each $i \in \Delta$.*

**Definition 5.4** The IGO $(\tau, \tau^*)$ on $X$ so defined in the above Theorem 5.3 generated by the IFTSs

$\{(X_i, \mathcal{U}_i, \mathcal{U}_i^*)\}_{i\in\Delta}$ and the family of functions $\{f_i : X \to X_i\}_{i\in\Delta}$ is called the initial IGO on $X$ generated by the family $\{f_i\}_{i\in\Delta}$.

**Definition 5.5** Let $\{(X_i, \tau_i, \tau_i^*)\}_{i\in\Delta}$ be a family of IFTSs and $X = \pi_{i\in\Delta} X_i$ and $p_i : X \to X_i$, $i \in \Delta$ be the projection mapping. Then the initial IGO on $X$ generated by the family $\{p_i : X \to (X_i, \tau_i, \tau_i^*)\}_{i\in\Delta}$ is called the product IGO on $X$ and is denoted by $(\pi_{i\in\Delta}\tau_i\ , \pi_{i\in\Delta}\tau_i^*)$. The triplet $(X, \pi_{i\in\Delta}\tau_i, \pi_{i\in\Delta}\tau_i^*)$ is called the product IFTS of the family $\{(X_i, \tau_i, \tau_i^*)\}_{i\in\Delta}$.

## 6. Compactness

Let $(X, T, T^*)$ be a BTFS and let $\mathcal{C}, \mathcal{C}^* \subset I^X$. Then the pair $(\mathcal{C}, \mathcal{C}^*)$ is said to be an $(\alpha, \beta)$-shading family $(\alpha, \beta \in I_1)$ if for any $x \in X$, $\exists\ \mu \in \mathcal{C}$, $\mu^* \in \mathcal{C}^*$ such that $\mu(x) > \alpha$ and $\mu^*(x) > \beta$.
An $(\alpha, \beta)$-shading family $(\mathcal{D}, \mathcal{D}^*)$ is said to be a subfamily of the $(\alpha, \beta)$-shading family $(\mathcal{C}, \mathcal{C}^*)$ if $\mathcal{D} \subset \mathcal{C}$ and $\mathcal{D}^* \subset \mathcal{C}^*$.
An $(\alpha, \beta)$-shading family $(\mathcal{C}, \mathcal{C}^*)$ is said to be finite if $\mathcal{C}$ and $\mathcal{C}^*$ are finite collection.
For any set $A$, let $2^{(A)}$ denote the family of all finite subsets of $A$. Let,
$i_\alpha(T) = \{\lambda^{-1}(\alpha, 1]\ ;\ \lambda \in T\}$, $\alpha \in I_1$,
$i_\beta(T^*) = \{\nu^{-1}(\beta, 1]\ ;\ \nu \in T^*\}$, $\nu \in I_1$
and $i(T)$, $i(T^*)$ be the topologies generated by $\bigcup_{\alpha\in I_1} i_\alpha(T)$ and $\bigcup_{\beta\in I_1} i_\beta(T^*)$ respectively as sub-bases.

**Definition 6.1** For $\alpha, \beta \in I_1$, $(X, T, T^*)$ is said to be $(\alpha, \beta)$-compact if each $(\alpha, \beta)$-shading family in $(T, T^*)$ has a finite $(\alpha, \beta)$-shading subfamily.

**Definition 6.2** $(X, T, T^*)$ is said to be strong fuzzy compact if it is $(\alpha, \beta)$-compact, $\forall\ \alpha, \beta \in I_1$.

**Definition 6.3** $(X, T, T^*)$ is said to be ultra-fuzzy compact if $(X, i(T))$ and $(X, i(T^*))$ are compact.

**Definition 6.4** $(X, T, T^*)$ is said to be fuzzy compact if for two families $\mathcal{C} \subset T$, $\mathcal{C}^* \subset T^*$ and for $\alpha$, $\beta \in I$ such that $\sup_{\mu\in\mathcal{C}}\mu \geq \alpha$, $\sup_{\nu\in\mathcal{C}^*}\nu \geq \beta$ and for $\epsilon \in (0, \alpha]$, $\epsilon^* \in (0, \beta]$, $\exists\ \mathcal{C}_o \in 2^{(\mathcal{C})}$, $\mathcal{C}_o^* \in 2^{(\mathcal{C}^*)}$ such that $\sup_{\mu\in\mathcal{C}_o}\mu \geq \alpha - \epsilon$, $\sup_{\nu\in\mathcal{C}_o^*}\nu \geq \beta - \epsilon^*$.

**Definition 6.5** Let $(X, \tau, \tau^*)$ be an IFTS. Then $(X, \tau, \tau^*)$ is said to be $(\alpha, \beta)$-compact, strong fuzzy compact, ultra-fuzzy compact and fuzzy compact if $(X, \tau_r, \tau_r^*)$ is $(\alpha, \beta)$-compact, strong fuzzy compact, ultra-fuzzy compact and fuzzy compact respectively, $\forall\ r\ \in I_o$.

**Theorem 6.6** *If $f : (X, \tau, \tau^*) \to (Y, \mathcal{U}, \mathcal{U}^*)$ is a surjective gp-map and if $(X, \tau, \tau^*)$ is fuzzy compact in any of the above senses then $(Y, \mathcal{U}, \mathcal{U}^*)$ is fuzzy compact.*

**Theorem 6.7** *Each IFTSs of a family* $\{(X_i, \tau_i, \tau_i^*)\}_{i\in\Delta}$ *is fuzzy compact in any one of the above senses iff* $(\pi_{i\in\Delta}X_i, \pi_{i\in\Delta}\tau_i, \pi_{i\in\Delta}\tau_i^*)$ *is so* ($\pi_{i\in\Delta}X_i$ *is assumed to be nonvoid).*

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