LOCATING THE EPICENTRE OF A GROUP OF TREMORS IN A MINE WITH THE APPLICATION OF A THEORY OF FUZZY SETS

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Abstract

Tremors in a mine are monitored by means of seismometers. Different calculation methods give different results. Seismometer measurements by means of fuzzy sets have been presented in the paper. The method of locating the epicentre of group tremors in a mine, using the described fuzzy sets, has been presented. This method consists in determining the argument, for which the membership function has its maximum. The example of locating the epicentre of group tremors on the basis of the data from coal mine "Szombierki" has been described (for using different measurement methods).

1. Introduction

Tremors which occur in a mine are recorded by a set of seismometers. Different mathematical methods [5] are used to locate the epicentre of a group of tremors. These methods can give different results. The aim of this paper is to locate the epicentre of a group of tremors on the basis of different results given by different methods. Fuzzy sets theory [1], [2], [3], [4] and [6] will be used to achieve this purpose. Each method calculates coordinates x, y and z in three dimensional space and gives positional tolerance (measurement error) for each coordinate [4], [5]. Coordinate x measurement will be considered as fuzzy set X_{fuzzy} with membership function $\mu_{Xfuzzy}(x)$. This function is shown in figure 1, where:

x_p - is coordinate x measurement,

t - is tolerance for coordinate x.

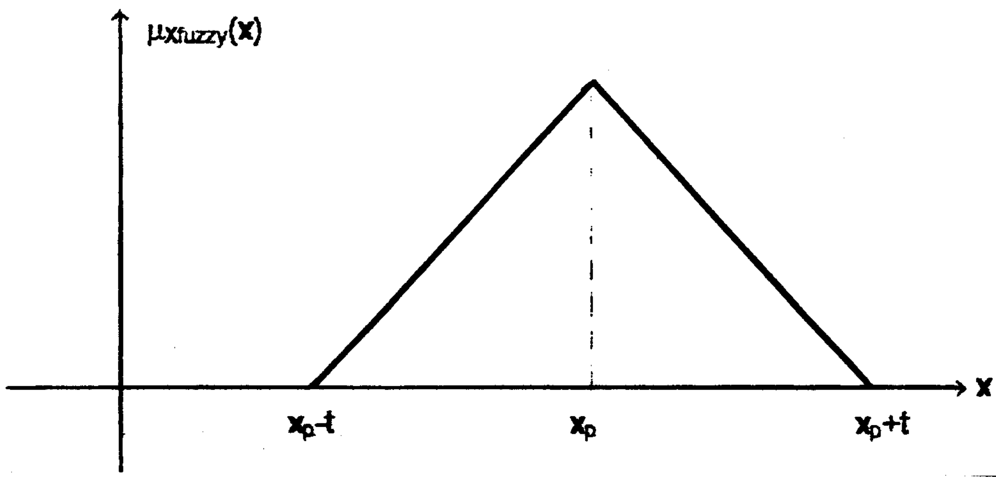

Figure 1: The membership function for one measurement of coordinate x.

The data from the paper [5] collected during tremors in coal mine "Szombierki" in the region 2A/509 have been used for calculation.

2. The example of the epicentre of a group of tremors calculation.

For example, we will consider one group of tremors which in paper [5] have the numbers 124, 126, ...,131. The tremor have been located on the basis of the data from seismometers by five different mathematical methods:

a) by geophysical mine service using A. Kijki algorithm,

b) using P method for weighed function of the location error,

c) using P method for not weighed function of the location error,

d) using PA method for weighed function of the location error,

e) using PA method for not weighed function of the location error.

The epicentre of tremors coordinates are designated by x, y, z. They are measured in meters from a given reference point in the mine. Coordinate z is the depth under the surface.

The following results of the epicentre location were calculated [5]:

Table 1

Locating a group of tremors

Number of tremor		Method a	Method b	Method c	Method d	Method e
	x	6800	6794.32	6794	7009	6726
124	y	-1650	-1631.06	-1631	-1801	-1694
	z	- 600	-922.68	-919	-656	-664
	x	6770	6732.86	6732	6928	6721
126	y	-1680	-1651.38	-1651	-1808	-1717
	z	-620	-397.85	-397	-560	-566
	x	6810	6780.51	6903	7034	6814
127	y	-1600	-1596.20	-1700	-1773	-1670
	z	-570	-499.86	-684	-574	-574
	x	6830	6819.17	6819	7206	6932
128	y	-1630	-1638.27	-1638	-1561	-1730
	z	-540	-575.49	-574	-576	-574
	x	6770	6767.52	6767	6990	6749
129	y	-1640	-1631.23	-1631	-1800	-1699
	z	-520	-603.34	-602	-596	-599
	x	6880	6945.36	6944	7699	7281
130	y	-1640	-1712.61	-1712	-2060	-1864
	z	-540	-852.41	-850	-596	-586
	x	6820	6765.70	6765	7048	6831
131	y	-1600	-1634.36	-1634	-1819	-1714
	z	-570	-369.39	-369	-547	-549

Literature [5]

The table contains 35 measurement values x, 35 measurement values y and 35 measurement values z. Each number of this table contains the method's error. The following values have been assumed in calculations [5] for tolerances:

58.33 [m] - for method a),

52.01 [m] - for method b), d),

99.27 [m] - for method c), e).

Arithmetic averages obtained from the results in methods a), b), c), d), e) have been assumed as the epicentre of group tremors coordinates by the author of the paper [5]. He has obtained: x = 6881.56, y= -1689.66, z = -586.23. In this way all the measurements have been dealt with in the same manner. We will pay greater attention to more focused measurements than to measurements that differ considerably from the other ones.

For each measurement on the basis of the above data we create a fuzzy set, which is shown in figure 1. These sets have the following membership functions.

$$\mu_{X_i}(x)=\begin{cases}\dfrac{x-x_p+t}{t} & \text{dla} \quad x\in(x_p-t,\quad x_p],\\ \dfrac{x-x_p-t}{-t} & \text{dla} \quad x\in(x_p,\quad x_p+t],\\ 0 & \text{dla} \quad x\notin(x_p-t,\quad x_p+t].\end{cases} \qquad (1)$$

$$\mu_{Y_i}(y)=\begin{cases}\dfrac{y-y_p+t}{t} & \text{dla} \quad y\in(y_p-t,\quad y_p],\\ \dfrac{y-y_p-t}{-t} & \text{dla} \quad y\in(y_p,\quad y_p+t],\\ 0 & \text{dla} \quad y\notin(y_p-t,\quad y_p+t].\end{cases} \qquad (2)$$

$$\mu_{Z_i}(z)=\begin{cases}\dfrac{z-z_p+t}{t} & \text{dla} \quad z\in(z_p-t,\quad z_p],\\ \dfrac{z-z_p-t}{-t} & \text{dla} \quad z\in(z_p,\quad z_p+t],\\ 0 & \text{dla} \quad z\notin(z_p-t,\quad z_p+t].\end{cases} \qquad (3)$$

Values x_p-t, x_p, x_p+t, y_p-t, y_p, y_p+t, z_p-t, z_p, z_p+t are shown in tables 2,3,4.

Table 2

Fuzzy sets X_i characterizing coordinate x

Number of tremor		Method a	Method b	Method c	Method d	Methoa e
	x_p-t	6741.67	6742.31	6694.73	6956.99	6626.73
124	x_p	6800.00	6794.32	6794.00	7009.00	6726.00
	x_p+t	6858.33	6846.33	6893.27	7061.01	6825.27
	x_p-t	6711.67	6680.85	6632.73	6875.99	6621.73
126	x_p	6770.00	6732.86	6732.00	6928.00	6721.00
	x_p+t	6828.33	6784.87	6831.27	6980.01	6820.27
	x_p-t	6751.67	6728.50	6803.73	6981.99	6714.73
127	x_p	6810.00	6780.51	6903.00	7034.00	6814.00
	x_p+t	6868.33	6832.52	7002.27	7086.01	6913.27
	x_p-t	6771.67	6767.16	6719.73	7153.99	6832.73
128	x_p	6830.00	6819.17	6819.00	7206.00	6932.00
	x_p+t	6888.33	6871.18	6918.27	7258.01	7031.27
	x_p-t	6711.67	6715.51	6667.73	6937.99	6649.73
129	x_p	6770.00	6767.52	6767.00	6990.00	6749.00
	x_p+t	6828.33	6819.53	6866.27	7042.01	6848.27
	x_p-t	6821.67	6893.35	6844.73	7646.99	7181.73
130	x_p	6880.00	6945.36	6944.00	7699.00	7281.00
	x_p+t	6938.33	6997.37	7043.27	7751.01	7380.27
	x_p-t	6761.67	6713.69	6665.73	6995.99	6731.73
131	x_p	6820.00	6765.70	6765.00	7048.00	6831.00
	x_p+t	6878.33	6817.71	6864.27	7100.01	6930.27

Table 3

Fuzzy sets Y_i characterizing coordinate y

Number of tremors		Method a	Method b	Method c	Method d	Method e
	y_p-t	-1708.33	-1683.07	-1730.27	-1853.01	-1793.27
124	y_p	-1650.00	-1631.06	-1631.00	-1801.00	-1694.00
	y_p+t	-1591.67	-1579.05	-1531.73	-1748.99	-1594.73
	y_p-t	-1738.33	-1703.39	-1750.27	-1860.01	-1816.27
126	y_p	-1680.00	-1651.38	-1651.00	-1808.00	-1717.00
	y_p+t	-1621.67	-1599.37	-1551.73	-1755.99	-1617.73
	y_p-t	-1658.33	-1648.21	-1799.27	-1825.01	-1769.27
127	y_p	-1600.00	-1596.20	-1700.00	-1773.00	-1670.00
	y_p+t	-1541.67	-1544.19	-1600.73	-1720.99	-1570.73
	y_p-t	-1688.33	-1690.28	-1737.27	-1613.01	-1829.27
128	y_p	-1630.00	-1638.27	-1638.00	-1561.00	-1730.00
	y_p+t	-1571.67	-1586.26	-1538.73	-1508.99	-1630.73
	y_p-t	-1698.33	-1683.24	-1730.27	-1852.01	-1798.27
129	y_p	-1640.00	-1631.23	-1631.00	-1800.00	-1699.00
	y_p+t	-1581.67	-1579.22	-1531.73	-1747.99	-1599.73
	y_p-t	-1698.33	-1764.62	-1811.27	-2112.01	-1963.27
130	y_p	-1640.00	-1712.61	-1712.00	-2060.00	-1864.00
	y_p+t	-1581.67	-1660.60	-1612.73	-2007.99	-1764.73
	y_p-t	-1658.33	-1686.37	-1733.27	-1871.01	-1813.27
131	y_p	-1600.00	-1634.36	-1634.00	-1819.00	-1714.00
	y_p+t	-1541.67	-1582.35	-1534.73	-1766.99	-1614.73

Tablie 4

Fuzzy sets Z_i characterizing coordinate z

Number of trmors		Method a	Method b	Method c	Method d	Method e
	z_p-t	-658.33	-974.69	-1018.27	-708.01	-763.27
124	z_p	-600.00	-922.68	-919.00	-656.00	-664.00
	z_p+t	-541.67	-870.67	-819.73	-603.99	-564.73
	z_p-t	-678.33	-449.86	-496.27	-612.01	-665.27
126	z_p	-620.00	-397.85	-397.00	-560.00	-566.00
	z_p+t	-561.67	-345.84	-297.73	-507.99	-466.73
	z_p-t	-628.33	-551.87	-783.27	-626.01	-673.27
127	z_p	-570.00	-499.86	-684.00	-574.00	-574.00
	z_p+t	-511.67	-447.85	-584.73	-521.99	-474.73
	z_p-t	-598.33	-627.50	-673.27	-628.01	-673.27
128	z_p	-540.00	-575.49	-574.00	-576.00	-574.00
	z_p+t	-481.67	-523.48	-474.73	-523.99	-474.73
	z_p-t	-578.33	-655.35	-701.27	-648.01	-698.27
129	z_p	-520.00	-603.34	-602.00	-596.00	-599.00
	z_p+t	-461.67	-551.33	-502.73	-543.99	-499.73
	z_p-t	-598.33	-904.42	-949.27	-648.01	-685.27
130	z_p	-540.00	-852.41	-850.00	-596.00	-586.00
	z_p+t	-481.67	-800.40	-750.73	-543.99	-486.73

	z_p-t	-628.33	-421.40	-468.27	-599.01	-648.27
131	z_p	-570.00	-369.39	-369.00	-547.00	-549.00
	z_p+t	-511.67	-317.38	-269.73	-494.99	-449.73

Next each fuzzy set from tables 2, 3, 4 has been multiplied by scale coefficient $\alpha=1/35$, because for coordinates x, y, z there are 35 measurements. The multiplication of a fuzzy set by coefficient α is executed according to the following definition.

Definition 1.

The product of fuzzy set A by real number $\alpha\in[0, 1]$ is set αA defined by the membership function:

$$\forall_{x\in X}\ \mu_{\alpha A}(x) = \alpha\mu_A(x). \tag{4}$$

Fuzzy sets from table 2, 3, 4 multiplied by scale coefficient $\alpha=1/35$ are added using the definition of limited sum.

Definition 2

Limited sum of fuzzy sets A and B is set $A\oplus B$ defined by the membership function:

$$\forall_{x\in X}\ \mu_{A\oplus B}(x) = \min(\mu_A(x) + \mu_B(x), 1). \tag{5}$$

After calculating the sum of fuzzy sets from table 2 on the ground of definition 2 we obtain fuzzy set

$$S_X = \alpha X_1 \oplus \alpha X_2 \oplus \ldots \oplus \alpha X_{35} = \alpha(X_1 \oplus \ldots \oplus X_{35}) =(1/35) (X_1 \oplus \ldots \oplus X_{35}) \tag{6}$$

After calculating the sum of fuzzy sets from table 3 on the ground of definition 2 we obtain fuzzy set

$$S_Y = \alpha Y_1 \oplus \alpha Y_2 \oplus \ldots \oplus \alpha Y_{35} = \alpha(Y_1 \oplus \ldots \oplus Y_{35}) =(1/35) (Y_1 \oplus \ldots \oplus Y_{35}) \tag{7}$$

After calculating the sum of fuzzy sets from table 4 on the ground of definition 2 we obtain fuzzy set

$$S_Z = \alpha Z_1 \oplus \alpha Z_2 \oplus \ldots \oplus \alpha Z_{35} = \alpha(Z_1 \oplus \ldots \oplus Z_{35}) =(1/35) (Z_1 \oplus \ldots \oplus Z_{35}) \tag{8}$$

For fuzzy set S_X we calculate point x_0, in which membership function $\mu_{S_X}(x)$ reaches maximum

$$\max_x \mu_{S_X}(x) = \mu_{S_X}(x_0). \tag{9}$$

On the ground of table 2 we obtain

$$\max_x \mu_{S_X}(x) = \mu_{S_X}(6780.51) = 0.3771. \tag{10}$$

For fuzzy set S_Y we calculate point y_0, in which membership function $\mu_{S_Y}(y)$ reaches maximum

$$\max_{y} \mu_{S_Y}(y) = \mu_{S_Y}(y_0). \tag{11}$$

On the ground of table 3 we obtain

$$\max_{y} \mu_{S_Y}(y) = \mu_{S_Y}(-1638.27) = 0.4715. \tag{12}$$

For fuzzy set S_Z we calculate point z_0, in which membership function $\mu_{S_Z}(z)$ reaches maximum

$$\max_{z} \mu_{S_Z}(z) = \mu_{S_Z}(z_0). \tag{13}$$

On the ground of table 4 we obtain

$$\max_{z} \mu_{S_Z}(z) = \mu_{S_Z}(-574.00) = 0.4679. \tag{14}$$

Therefore the epicentre of tremors in the mine has the following coordinates:

$$x_0 = 6780.51 \text{ [m]}, \tag{15}$$

$$y_0 = -1638.27 \text{ [m]}, \tag{16}$$

$$z_0 = -574.00 \text{ [m]}. \tag{17}$$

Comparing coordinates x_0, y_0, z_0 with the arithmetic averages of the results calculated in methods a), b), c), d), e) [5] shows that there are differences in the location of the epicentre of tremors. For x coordinate there is difference of about 101 [m], for y 51[m] and for z 12 [m]. Such small differences were obtained, because in the method presented in the paper, the measurements which were to far away from the focused group were rejected and had a smaller influence on the calculated results.

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