FUZZY INTERPOLATION

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Abstract :

A method of fuzzy number interpolation (in short, which is called by "fuzzy interpolation") is introduced which will be useful for linguistic interpolation.

Keywords : Fuzzy interpolation, linguistic interpolation, fuzzy advance operator, fuzzy forward difference operator, fuzzy backward difference operator, fuzzy forward interpolation, fuzzy backward interpolation.

INTRODUCTION :

Numerical interpolation or extrapolation is a very important subject in Mathematics, Engineering Sciences and Technology because of its tremendous applications. But all types of data or information available are not always numerical. In many cases they are subjective and expressed using linguistic hedges like good, very good, bad etc. In literature, there is no modelling for interpolation with such linguistic values. For example, suppose the weather condition to the cricket players of Calcutta on 1st March 1960 was good, on 1st March 1970 was extremely good, on 1st March 1980 was not bad, on 1st March 1990 was very good. What quality of weather Calcutta could expect on 1st March 2000 ? This type of predictions are very much essential in many real life situations.

In the present paper, we introduce a method of linguistic interpolation. It is in fact to be called "fuzzy number interpolation" or "fuzzy interpolation", because our method first of all converts all linguistic values into suitable fuzzy numbers, then performs interpolation of fuzzy numbers and finally converts the result back into a linguistic value. The interpolation involves additions $\oplus$, subtractions

$\ominus$, and multiplication $\otimes$ of fuzzy numbers. For details on fuzzy numbers and operations on fuzzy numbers, one can see [4-7], [9-11].

On a method and application of "conversion of linguistic values into fuzzy numbers and then back to linguistic values" the work in [8] may be seen as an example. So we start now with some useful definitions. Throughout this paper, we consider tabular form of fuzzy number valued (linguistic valued) function f(x). An example (hypothetical) of such function in tabular form is :

x	f(x)
2	approximately 12
5	little less than 16
8	approximately 17
11	almost 20

Definition 1

Suppose that there is a table of fuzzy numbers $f_j = f(x_j)$, $j = 0, 1, 2, \ldots, n$ of an ill-defined function f at equally spaced crisp points $x_o, x_1, x_2, \ldots, x_n$, where $x_r = x_o + rh$, $r = 0,1,2,\ldots,n$, h being the fixed space length.

The fuzzy operators $\tilde{E}$, $\tilde{\Delta}$, $\tilde{\nabla}$ (or, simply expressing with the notations E, Δ and ∇ respectively) are defined as below :

(i) $E\, f_r = f_{r+1}$, fuzzy advance operator.

(ii) $\Delta f_r = f_{r+1} \ominus f_r$, fuzzy forward difference operator.

(iii) $\nabla f_r = f_r \ominus f_{r-1}$, fuzzy backward difference operator.

Thus fuzzy difference tables can be constructed using these fuzzy operators. The following results in Proposition 1 are straightforward.

Proposition 1

(i) $E^m f_r = f_{m+r}$

(ii) $E^r \Delta^s \equiv \Delta^s E^r$

(iii) $E \equiv 1 \oplus \Delta$,

where $(1 \oplus \Delta) f_r = f_r \oplus \Delta f_r \quad \forall\, r$

(iv) $\nabla \equiv 1 \ominus E^{-1}$

It can be easily seen that Newton's forward interpolation formula and Newton's backward interpolation formula are true in case of fuzzy difference tables. Those are stated below without proof.

Proposition 2 (Fuzzy forward interpolation formula)

If $x = x_o + rh$ where $0 \le r \le n$,

$$f(x) \approx f_o \oplus r \bullet (\Delta f_o) \oplus \frac{r(r-1)}{2\,!} \bullet (\Delta^2 f_o) \oplus \ldots.$$

$$\oplus \frac{r(r-1)\,(r-2)\,\ldots\,(r-n+1)}{n\,!} \bullet (\Delta^n f_o) ,$$

where the symbol $\bullet$ stands for scalar multiplication (P-49 in [6]), and $\oplus$ stands for addition of fuzzy numbers.

Proposition 3 (Fuzzy backward interpolation formula)

If $x = x_n + rh$ where $0 \le r \le n$,

$$f(x) \approx f_n \oplus r \bullet (\nabla f_n) \oplus \frac{r(r+1)}{2\,!} \bullet (\nabla^2 f_n) \oplus \ldots.$$

$$\oplus \frac{r(r+1)\,(r+2)\,\ldots\,(r+n-1)}{n\,!} \bullet (\nabla^n f_n) .$$

CONCLUSION :

Linguistic interpolation can be made with the help of fuzzy interpolation. For this, linguistic values are converted into fuzzy numbers and after interpolation, the resultant fuzzy number is converted back into linguistic value by the method

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