UNION AND INTERSECTION OF FUZZY SETS

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Abstract

The notion of union and intersection of fuzzy sets as laid down by Zadeh is generalized in this paper. The justification behind such attempt is made with examples on real-life problems.

Keywords: Fuzzy union, fuzzy intersection.

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The notion of union and intersection in fuzzy sets as given by Prof. Zadeh [6] has been generalized here. The concept of the union of two fuzzy sets A and B in the universe X could be treated as a particular case of union of the following two fuzzy sets:

(I) fuzzy set A in the universe X, and

(II) fuzzy set B in the universe Y,

where X and Y are two different universes, in general.

The necessity of this kind of generalization of fuzzy union and fuzzy intersection is obvious because in real-life situation, many problems call for such type of operations. For example, suppose X is the set of all soldiers of Regiment-I and Y be the set of all leutinents of Regiment-II of an army. Now consider the collection of armymen who are either tall soldiers of Regiment-I or wise leutinents of regiment-II.
This collection is not a set. Besides, the present definition of union of two fuzzy sets [6] does not support this type of union.
Naturally question arises:- if A is the fuzzy set (tall soldiers) of X and B is the fuzzy set (wise leutinents) of Y, then what is the union of A and B ?

We propose here the following definitions on union and intersection of two fuzzy sets.

Definition 1

Let A be a fuzzy set of X with membership function μ_A and B be a fuzzy set of Y with membership function μ_B. Then the union of two fuzzy sets A and B denoted by $A\tilde{\cup}B$ is a fuzzy set of $X \cup Y$ with the membership function $\mu_{A\tilde{\cup}B}$ defined by

$$\begin{aligned}\mu_{A\tilde{\cup}B}(z) &= \mu_A(z), \quad if \quad z \in X - Y\\ &= \mu_B(z), \quad if \quad z \in Y - X\\ &= max\{\mu_A(z), \mu_B(z)\}, \quad if \quad z \in X \cap Y.\end{aligned}$$

Graphical representation of the union of two fuzzy sets is given in Fig.1

It may be observed that the fuzzy union defined by Zadeh [6] is a particular case of the above defined union when $X = Y$.

Graphical representation of the union of two fuzzy sets:-

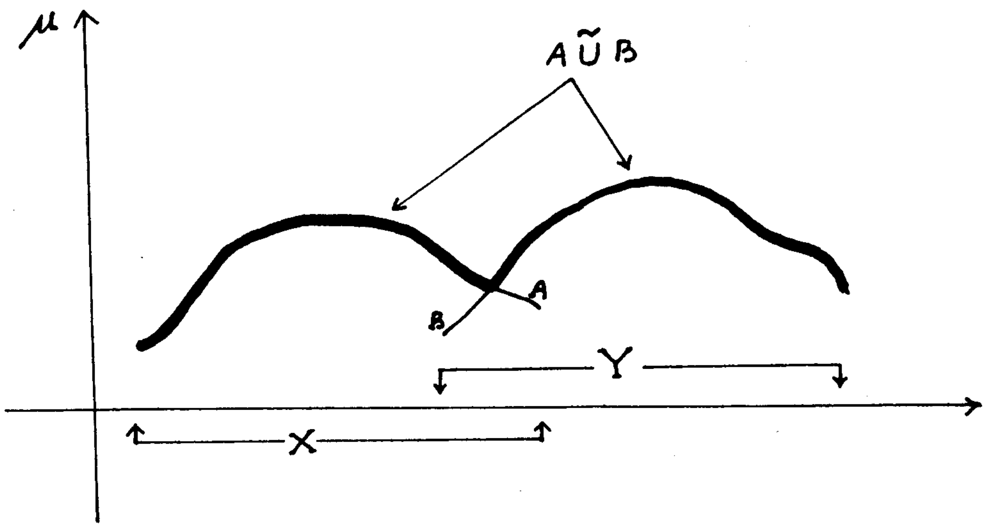

Fig.1

Graphical representation of the intersection of two fuzzy sets:-

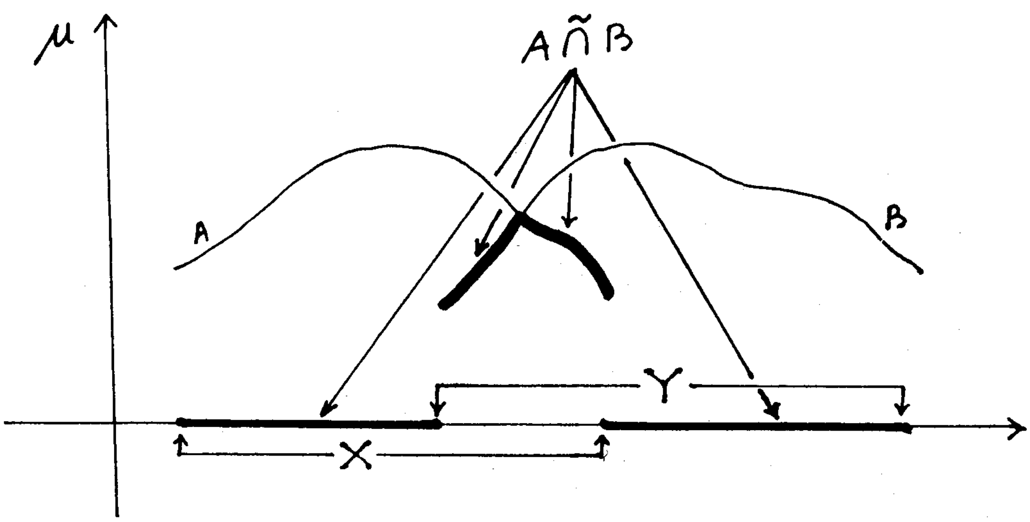

Fig.2

Definition 2

Let A be a fuzzy set of X with membership function μ_A and B be a fuzzy set of Y with membership function μ_B. Then the intersection of two fuzzy sets A and B denoted by $A\tilde{\cap}B$ is a fuzzy set of $X \cup Y$ with the membership function $\mu_{A\tilde{\cap}B}$ defined by

$$\mu_{A\tilde{\cap}B}(z) = min\{\mu_A(z), \mu_B(z)\}, \quad \forall z \in X \cup Y.$$

Graphical representation of the intersection of two fuzzy sets is given in Fig.2

It may be observed that the fuzzy intersection defined by Zadeh [6] is a particular case of the above defined intersection when $X = Y$.

It may also be observed that $\forall z \in (X \triangle Y)$, $\mu_{A\cap B}(z) = 0$, and in this sense $A\tilde{\cap}B$ is a fuzzy set in $X \cap Y$.

Example 1

Let $X = \{a, b, c, d\}$ be a set and $A = \{a/0.3, b/0.7, c/0.9, d/0.4\}$ be a fuzzy set of X. Also let $Y = \{p, b, q, c, r\}$ be a set and $B = \{a/0.2, b/0.8, q/0.4, c/0.6, r/0.1\}$ be a fuzzy set of Y.

Then, $A\tilde{\cup}B = \{a/0.3, b/0.8, c/0.9, d/0.4, p/0.2, q/0.4, r/0.1\}$

and $A\tilde{\cap}B = \{a/0, b/0.7, c/0.6, d/0, p/0, q/0, r/0\}$, which can be viewed as the fuzzy set $\{b/0.7, c/0.6\}$ of $X \cap Y$.

Proposition 1

Let A be a fuzzy set of X with membership function μ_A and B be a fuzzy set of Y with membership function μ_B. Then the following holds:-

(a) $(A\tilde{\cup}B)^c = A^c\tilde{\cap}B^c$

(b) $(A\tilde{\cap}B)^c = A^c\tilde{\cup}B^c$.

where $(A\tilde{\cup}B)^c$ and $(A\tilde{\cap}B)^c$ are the complements of $(A\tilde{\cup}B)$ and $(A\tilde{\cap}B)$ in $X \cup Y$ respectively, A^c is the complement of A in X and B^c is the complement of B in Y.

Proof (a):

Case-I

Consider $x \in X - Y$, where $\mu_A(x) = p$. Therefore, $\mu_B(x) = 0$.

Hence, $\mu_{A^c \tilde{\cap} B^c}(x) = min\{\mu_{A^c}(x), \mu_{B^c}(x)\} = min\{1 - p, 1\}$

$= 1 - p = 1 - \mu_{A \tilde{\cup} B}(x) = \mu_{(A \tilde{\cup} B)^c}(x)$.

Case-II

If $x \in Y - X$, then the proof is similar to that of Case-I.

Case-III

Suppose $x \in X \cap Y$.

Therefore, $\mu_{A^c \tilde{\cap} B^c}(x) = min\{\mu_{A^c}(x), \mu_{B^c}(x)\} = 1 - max\{\mu_A(x), \mu_B(x)\} = \mu_{(A \tilde{\cup} B)^c}(x)$.

Hence proved.

Proof (b): Proof is similar to that of (a).

The following results are straightforward.

Proposition 2

For any two fuzzy sets A and B of the sets X and Y respectively, the following holds:

(a) If $A = \phi$, $B = \phi$, then $A \tilde{\cap} B = \phi$ but not conversely,

(b) $\phi \tilde{\cup} \phi = \phi$, $A \tilde{\cup} \phi = A$, $A \tilde{\cap} \phi = \phi$, where ϕ is the null fuzzy set.

(c) $A \tilde{\cup} B = B \tilde{\cup} A$, $A \tilde{\cap} B = B \tilde{\cap} A$.

CONCLUSION: The occurence of union/intersection of two fuzzy sets in two different universes is very common in many real life problems. We therefore generalize Zadeh's notion of union and intersection in this work.

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