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$(\in', \in' \vee q')$-fuzzy subgroups

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Abstract: In this paper, we give two kinds of neighborhoods of a fuzzy point and a fuzzy set and give definition of $(\in', \in' \vee q')$-fuzzy subgroups of a group based on the new neighborhoods which is different from (α,β)-fuzzy subgroups of S. K. Bhakat and P. Das.

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Concept of fuzzy subgroups is introducted by A. Rosenfeld in 1971[4]. In 1979, J. M. Anthong and H. Sherwood redefined fuzzy subgroups by the use of t-norws[1]. In [2], S. K. Bhakt and P. Das gave definition of (α,β)-fuzzy subgroup and made some discussions for $(\in, \in \vee q)$-fuzzy subgroups, and obtained conclusion that A is a $(\in, \in \vee q)$-fuzzy subgroup of group if and only if $A_\lambda = \{x \mid x \in G$ and $A(x) \geqslant \lambda\}$ is a subgroup of group G for any $\lambda \in [0, 0.5]$[3].

In this paper, we give new neighborhoods of fuzzy point x_λ and fuzzy set A as following:

(i) $x_\lambda \in' A \Leftrightarrow A(x) \leqslant \lambda$ (ii) $x_\lambda q' A \Leftrightarrow \lambda + A(x) < 1$

(iii) $x_\lambda (\in' \vee q') A \Leftrightarrow x_\lambda \in' A$ or $x_\lambda q' A$.

Based on the neighborhoods $\in'$ and q', we are able to give new definiton of fuzzy subgroup of a group G as following:

Definition 1. Let G be a group and A be a fuzzy subset of G. If

(i) $x_t \in' A, y_r \in' A \Rightarrow (xy)_{\max\{t,r\}} (\in' \vee q') A$, for any $x, y \in G, t, r \in [0,1]$.

(ii) $x_t \in' A \Rightarrow (x^{-1})_t (\in' \vee q') A$, for any $x \in G, t \in [0,1]$.

then A is called as a $(\in', \in' \vee q')$-fuzzy subgroup of group G.

Theorem 1. A is a $(\in', \in' \vee q')$-fuzzy subgroup of group G if and only if (1) $A(xy) \leqslant \max\{A(x), A(y), 0.5\}$ (2) $A(x^{-1}) \leqslant \max\{A(x), 0.5\}$.

Proof: Let A is a $(\in', \in' \vee q')$-fuzzy subgroup of group G.

(1) If there are $x, y \in G$ such that $A(xy) > \max\{A(x), A(y), 0.5\}$ then there is $t \in [0,1]$ such that $A(xy) > t > \max\{A(x), A(y), 0.5\}$ it follows that $x_t \in' A, y_t \in' A$ and $t > 0.5$. Since A is a $(\in', \in' \vee q')$-fuzzy subgroup of group G, so $(xy)_t (\in' \vee q') A$, but $A(xy) > t$ and $t + A(xy) > 2t > 1$ and consequently this is a contradiction. Hence $A(xy) \leqslant \max\{A(x), A(y), 0.5\}$ for any $x, y \in G$.

(2) is clear.

On the other hand, let (1) and (2) of theorem 1 holds, let $x, y \in G$ and $t, r \in [0,1]$ such that $x_t \in' A, y_r \in' A$, then

$$A(xy) \leqslant \max\{A(x), A(y), 0.5\} \leqslant \max\{t, r, 0.5\}.$$

If $\max\{t,r\}<0.5$, then $A(xy)\leqslant 0.5$ and consequently $A(xy)+\max\{t,r\}<1$, i. e. , $(xy)_{\max\{t,r\}}$ $q'A$.

If $\max\{t,r\}\geqslant 0.5$, then $A(xy)\leqslant\max\{t,r\}$, i. e. , $(xy)_{\max\{t,r\}}\in' A$.

Hence $(xy)_{\max\{t,r\}}(\in'\vee q')A$, i. e, (i) of definition 1 holds.

(ii)is clear.

Corollary. A is a $(\in',\in'\vee q')$-fuzzy subgroup of group G if and only if $A(xy^{-1})\leqslant\max\{A(x),A(y),0.5\}$ for any $x,y\in G$.

Theorem 2. A is a $(\in',\in'\vee q')$-fuzzy subgroup of group G if and only if $A^t=\{x\mid x\in G$ and $A(x)\leqslant t\}$ is a subgroup of G for any $t\in[0.5,1]$.

Proof: Let A be a $(\in',\in'\vee q')$-fuzzy subgroup of group G, For $t\in[0.5,1]$and $x,y\in A^t$, We have

$$A(xy^{-1})\leqslant\max\{A(x),A(y),0.5\}\leqslant\max\{t,0.5\}=t$$

and consequently $xy^{-1}\in A^t$, i. e. , A^t is a subgroup of group G.

On the other hand, let A^t is a subgroup of group G for any $t\in[0.5,1]$. Assume that there are $x_o,y_o\in G$ such that

$$A(x_0y_0^{-1})>\max\{A(x_o),A(y_o),0.5\},$$

then there is a $t\in[0,1]$suh that

$$A(x_0y_0^{-1})>t>\max\{A(x_o),A(y_o),0.5\},$$

it follows that $t>A(x_o)$, $t>A(y_o)$ and $t>0.5$, and consequently $x_o\in A^t$, $y_o\in A^t$, they $x_oy_o^{-1}\in A^t$ and $A(x_oy_o^{-1})\leqslant t$. This is a contradiction. Hence $A(xy^{-1})\leqslant\max\{A(x),A(y),0.5\}$ for any $x,y\in G$, i. e. , A is a $(\in',\in'\vee q')$-fuzzy subgroups of group G.

Definition 2. Let A be a $(\in',\in'\vee q')$-fuzzy subgroup of group G. If A satisfies:

$x_t\in' A\Rightarrow(y^{-1}xy)_t(\in'\vee q')A$ for any $x,y\in G$ and $t\in[0,1]$, then A is called as normal $(\in',\in'\vee q')$-fuzzy subgroup of group G.

Theorem 3. Let A be a $(\in',\in'\vee q')$-fuzzy subgroup of group G, then A is a normal $(\in',\in'\vee q')$-fuzzy subgroup of G if and only if $A(y^{-1}xy)\leqslant\max\{A(x),0.5\}$.

Theorem 4. A is a normal $(\in',\in'\vee q')$-fuzzy subgroup of G if and only if $A^t=\{x\mid x\in G, A(x)\leqslant t\}$ is a normal subgroup of G for any $t\in[0.5,1]$.

References

1. J. M. Anthony and M. Sherwood. Fuzzy groups redefined. J. Math. and Appl. , 69(1979) 124～130.
2. S. K. Bhakat and P. Das. On the definition of a fuzzy subgroup. Fuzzy sets and systems, 51(1992) 235～241.
3. S. K. Bhakat and P. Das. $(\in,\in\vee q)$-fuzzy subgroup. Fuzzy sets and systems, 80(1996) 359～368.
4. A. Rosenfeld. Fuzzy groups, J. Math. Anal. Appl. , 35(1971) 512～517.