Theory of Interval-valued Fuzzy Comprehensive Evaluation and Its Application to Teaching Competition

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Abstract: A method of ranking interval numbers is proposed by introducing an index of optimism to reflect the decision maker's optimistic attitude. Based on this, a theory of interval-valued fuzzy comprehensive evaluation is built, it is used to solve some fuzzy comprehensive evaluation problems whose evaluated objects have some uncertainty. As application, some teachers, which take part in classroom teaching competition held by Liaocheng Teacher's College, are comprehensively evaluated.

Keywords: Fuzzy comprehensive evaluation; ranking interval numbers; index of optimism; uncertainty; classroom teaching; competition.

1. Introduction

In fuzzy comprehensive evaluation problem [1], very often, evaluated objects have some uncertainty for some evaluating indexes, this makes it are not definite numbers that degree of these evaluated objects belong to some comments. For example, we comprehensively evaluat a class taught by Teacher Zhao, the writing on the blackboard (WB, for short)is one of evaluating indexes. Five judges give Zhao's mark for WB are respectively 90, 92, 89, 88, 91. We generally take their mean value 0.2×(88+89+90+91+92)=90 as his mark for the index, but this lose some information. In order to make full use of these information, we take interval number [88, 92] as zhao's mark for the index. So, the degree of zhao's WB belonges to some comment, say "excellent", will is not a definite number, but a interval number . This is just so-called interval-valued fuzzy comprehensive evaluation problem. The key solving the problem is to rank interval numbers. However, the order of interval numbers varies from person to person. For example, [2, 4] and [1, 5], who is greater? opinions vary . In fact , this relate to decision maker's attitude. It is in view of the viewpoint that we introduce an index of optimism to reflect the decision maker's optimistic attitude, and by it we present a method of ranking interval numbers . The order will change along with change of the decision maker's attitude. Based on this , we build a theory of interval-valued fuzzy comprehensive evaluation.

2. Ranking interval numbers

We begin by recalling the operation of interval numbers.

Let $[R]=\{[a,b]: a\leqslant b,\ a,b\in R\}$, specifically, for $r\in R$, $r=[r,r]$. Given $[a^-,a^+],[b^-,b^+]\in[R]$ and $a^->0, b^->0$, we have

(1) $[a^-,a^+]+[b^-,b^+]=[a^-+b^-,a^++b^+]$;

(2) $[a^-,a^+][b^-,b^+]=[a^-b^-,a^+b^+]$,

specifically, for $r\in R$ and $r>0$, we have

$r[a^-,a^+]=[ra^-,ra^+]$;

(3) $[a^-,a^+]\div[b^-,b^+]=[a^-\div b^+,a^+\div b^-]$.

Definition 2.1. For $[a^-,a^+]\in[R]$, let $m_a=0.5\times(a^-+a^+)$, $r_a=0.5\times(a^+-a^-)$. For $\alpha\in[0,1]$, define

$$I_a(\alpha)=m_a+(2\alpha-1)r_a,$$

α is called the index of optimism, which reflect the decision maker's optimistic attitude.

Clearly, when $\alpha=1$, $I_a(1)=a^+$, this is an optimistic decision maker; When $\alpha=0$, $I_a(0)=a^-$, this is a pessimistic decision maker; When $\alpha=0.5$, $I_a(0.5)=m_a=0.5\times(a^-+a^+)$, this is a moderate decision maker.

Definition 2.2. For $[a^-,a^+],[b^-,b^+]\in[R]$, $\alpha\in[0,1]$.

(1) If $I_a(\alpha)<I_b(\alpha)$, then $[a^-,a^+]<[b^-,b^+]$;

(2) If $I_a(\alpha)=I_b(\alpha)$, then $[a^-,a^+]=[b^-,b^+]$;

(3) If $I_a(\alpha)$ and $I_b(\alpha)$ are incomparable for general α , then according to decision maker's attitude, take a fixed α-value, say α_o , then

1° if $I_a(\alpha_o)<I_b(\alpha_o)$, then $[a^-,a^+]<[b^-,b^+]$;

2° if $I_a(\alpha_o)>I_b(\alpha_o)$, then $[a^-,a^+]>[b^-,b^+]$;

3° if $I_a(\alpha_o)=I_b(\alpha_o)$, then $[a^-,a^+]=[b^-,b^+]$.

Example 2.3. (1) Let $[a^-,a^+]=[1,2]$, $[b^-,b^+]=[1,3]$.

Then $I_a(\alpha)=1.5+(2\alpha-1)\times 0.5=1+\alpha$, $I_b(\alpha)=2+(2\alpha-1)=1+2\alpha$.
Hence $[1,3]\succ[1,2]$.

(2) Let $[a^-,a^+]=[2,4]$, $[b^-,b^+]=[1,5]$. Then
$I_a(\alpha)=2+2\alpha$, $I_b(\alpha)=1+4\alpha$.

1° Take $\alpha=0.5$, then $[2,4]=[1,5]$;

2° Take $\alpha<0.5$, then $[1,5]\prec[2,4]$;

3° Take $\alpha>0.5$, then $[1,5]\succ[2,4]$.

3. Interval-valued fuzzy comprehensive evaluation

Suppose $X=\{x_1, x_2, \cdots, x_n\}$ is index set, $Y=\{y_1, y_2, \cdots, y_m\}$ is comment set , A is evaluated object . For $x_i \in X$, assume degree of A belongs to y_j is $[r_{ij}^-, r_{ij}^+]\subset[0,1]$, then we get a mapping $f: X\to \mathbb{IF}(Y)$,

$f(x_i)=([r_{i1}^-, r_{i1}^+], [r_{i2}^-, r_{i2}^+], \cdots, [r_{im}^-, r_{im}^+])$, $i=1,2,\cdots,n$,

where, $\mathbb{IF}(Y)$ is set formed by all interval-valued fuzzy sets[2] on Y. So , we get a comprehensive evaluation matrix of A:

$$R_A=\begin{bmatrix} [r_{11}^-, r_{11}^+] & [r_{12}^-, r_{12}^+] & \cdots & [r_{1m}^-, r_{1m}^+] \\ [r_{21}^-, r_{21}^+] & [r_{22}^-, r_{22}^+] & \cdots & [r_{2m}^-, r_{2m}^+] \\ \cdots & \cdots & \cdots & \cdots \\ [r_{n1}^-, r_{n1}^+] & [r_{n2}^-, r_{n2}^+] & \cdots & [r_{nm}^-, r_{nm}^+] \end{bmatrix}$$

Let c_i be weight of x_i, $i=1,2,\cdots,n$, $c_1+c_2+\cdots+c_n=1$, put $Z=(c_1, c_2, \cdots, c_n)$. Then

$$ZR=([d_1^-, d_1^+], [d_2^-, d_2^+], \cdots, [d_m^-, d_m^+]), \tag{3.1}$$

where $[d_j^-, d_j^+] = \sum_{i=1}^{n} [c_i r_{ij}^-, c_i r_{ij}^+]$, $j=1, 2, \cdots, m$. (3.2)

$$\Delta = [\Delta^-, \Delta^+] = \sum_{j=1}^{m} [d_j^-, d_j^+], \quad (3.3)$$

$$[h_j^-, h_j^+] = [d_j^- \div \Delta^+, d_j^+ \div \Delta^-], \quad j=1, 2, \cdots, m, \quad (3.4)$$

$$H = ([h_1^-, h_1^+], [h_2^-, h_2^+], \cdots, [h_m^-, h_m^+] \quad (3.5)$$

If $[h_k^-, h_k^+] = \max\{[h_j^-, h_j^+]: j=1, 2, \cdots, m\}$, then we say evaluated object A belongs to comment $y_k \in Y$. If we don't find the maximum from all $[h_j^-, h_j^+]$, $j=1, 2, \cdots, m$, for general α-value, then according to decision maker's attitude, take a fixed α_o-value, then it follows from Definition 2.2 (3) that A belongs to $y_k \in Y$.

4. Comprehensive evaluation for teaching competition

The indexes of the 4th classroom teaching competition held by Liaocheng Teacher's College are as follows:

x_1 — full prepare, and skilfully teach ;

x_2 — form a connecting link between the preceding and the following, and logic is well-knit ;

x_3 — explain concepts in precise terms , and thoroughly analyse problems ;

x_4 — make the key points stand out , and difficult points are resolved ;

x_5 — watch what is going on , and arouse thinking ;

x_6 — succinct language, and lively ;

x_7 —writing on the blackboard is neat and orderliness .

The weights of $x_1 \sim x_7$ are as follows :

0.14 , 0.12 , 0.15 , 0.15 , 0.16 , 0.16 , 0.12 .

Hence, we have index set $X=\{x_1, x_2, \cdots, x_7\}$ and weight vector

$Z=\{0.14, 0.12, 0.15, 0.15, 0.16, 0.16, 0.12\}$.

Comment set is $Y=\{y_1, y_2, y_3, y_4, y_5\}$, where y_1— poor ; y_2 —mediocre ; y_3— medium ; y_4— good; y_5—excellent. They are fuzzy sets on [0, 100], and the definition are as follows :

$$y_1=\begin{cases} 1 & , \quad 0\leqslant t\leqslant 40 \\ \dfrac{60-t}{20} & , \quad 40\leqslant t\leqslant 60 \\ 0 & , \quad 60\leqslant t\leqslant 100 \end{cases} ,$$

$$y_2=\begin{cases} 0 & , \quad 0\leqslant t\leqslant 50 \quad \text{or} \quad 85\leqslant t\leqslant 100 \\ \dfrac{t-50}{10} & , \quad 50\leqslant t\leqslant 60 \\ 1 & , \quad 60\leqslant t\leqslant 75 \\ \dfrac{85-t}{10} & , \quad 75\leqslant t\leqslant 85 \end{cases} ,$$

$$y_3=\begin{cases} 0, & 0\leqslant t\leqslant 60 \\ \dfrac{t-60}{15}, & 60\leqslant t\leqslant 75 \\ 1, & 75\leqslant t\leqslant 85 \\ \dfrac{100-t}{15}, & 85\leqslant t\leqslant 100 \end{cases},$$

$$y_4=\begin{cases} 0, & 0\leqslant t\leqslant 78 \\ \dfrac{t-78}{7}, & 78\leqslant t\leqslant 85 \\ 1, & 85\leqslant t\leqslant 93 \\ \dfrac{100-t}{7}, & 93\leqslant t\leqslant 100 \end{cases},$$

$$y_5=\begin{cases} 0, & 0\leqslant t\leqslant 85 \\ \dfrac{t-85}{8}, & 85\leqslant t\leqslant 93 \\ 1, & 93\leqslant t\leqslant 100 \end{cases},$$

The teachers participating in the competition are Zhao , Qian , Sun and Li . The judges give their mark as follows :

	Zhao	Qian	Sun	Li
x_1	[90, 95]	[85, 89]	[93, 95]	[92, 94]
x_2	[80, 83]	[84, 86]	[89, 91]	[93, 94]
x_3	[84, 88]	[89, 90]	[92, 94]	[93, 94]
x_4	[75, 78]	[88, 89]	[91, 93]	[90, 92]
x_5	[71, 74]	[90, 91]	[93, 95]	[95, 96]
x_6	[75, 78]	[88, 90]	[90, 91]	[94, 96]
x_7	[92, 96]	[85, 87]	[80, 83]	[92, 93]

For Zhao's 7 marks [90, 95], [80, 83], ⋯ , [92, 96] , from membership functions of $y_1 \sim y_5$ we get comprehensive evaluation matrix of Zhao :

$$R_{Zhao}=\begin{bmatrix} 0 & 0 & [0.33, 067] & [0.71, 1] & [0.62, 1] \\ 0 & [0.2, 0.5] & 1 & [0.28, 0.71] & 0 \\ 0 & [0, 0.1] & [0.8, 1] & [0.86, 1] & [0, 0.37] \\ 0 & [0.7, 1] & 1 & 0 & 0 \\ 0 & 1 & [0.73, 0.93] & 0 & 0 \\ 0 & [0.7, 1] & 1 & 0 & 0 \\ 0 & 0 & [0.27, 0.53] & [0.57, 1] & [0.87, 1] \end{bmatrix}$$

From (3.1) and (3.2) we have

$$ZR_{Zhao}=(0, [0.617, 1.085], [0.746, 0.887], [0.332, 0.496], [0.193, 0.316]).$$

It follows from (3.3) $\sim$ (3.5) that

H_{Zhao}=(0, [0. 222, 0. 575], [0. 268, 0. 470], [0. 119, 0. 263], [0. 069, 0. 168]) .

Clearly, the maximum of 5 interval numbers in H_{Zhao} is [0. 268, 0. 470], hence Zhao is"medium" .

Similarly , we have

H_{Qian}=(0, [0, 0. 006], [0. 310, 0. 421], [0. 416, 0. 499], [0. 123, 0. 251]),

H_{Sun}=(0, [0. 010, 0. 029], [0. 211, 0. 304], [0. 329, 0. 470], [0. 286, 0. 394]),

H_{Li}=(0, 0, [0. 161, 0. 229], [0. 336, 0. 443], [0. 381, 0. 467]).

Clearly, Qian is "good" , Sun is "good" too , Li is "excellent". But [0. 416, 0. 499] > [0. 329, 0. 470] , Hence Qian excels Sun . So , four teachers are arranged the names of contestants in the order of their results as follows : Li , Qian , Sun , Zhao .

References

[1] Didier Dubois and Henri Prade , Fuzzy Sets and Systems , Academic Press , 1980.

[2] Meng Guangwu , Basic theory for interval-valued fuzzy sets , Mathematica Applicata, 2(1993) , 129～ 135 .