Fuzzy weakly semi-irresolute mappings

Ma Baoguo
departement of Mathmatics,Yanan University, Yanan China 71600

Abstract:The aim of this paper is to introduce and discuss the fuzzy weakly semi-irresolute mapping.
Keywords: Fuzzy semi-irresolute mapping, Fuzzy weakly semi -irresolute mapping, Fuzzy semi-q-neighborhood, Fuzzy semi-θ -cluster, Fuzzy s-closed spaces.

1. Introduction

Throughout the paper,by (X,T) and (Y,T_1) (or simply X and Y) we shall mean fuzzy toplogical spaces (fts, for short) in Chang's [2] sense. A fuzzy singleton [4] with the singleton support $x\in X$ and the value α $(0<\alpha\leqslant 1)$ at x will be denoted by x_α .For two fuzzy sct A and B , wc shall write AqB to mean that A is quasi- coincident(q -coincident,for short) [4] with B. B is a quasi -neighborhood(q-nbd, for short) of A iff there exists a fuzzy open set $U\leqslant B$ such that AqU .A fuzzy set A in an fts X is called fuzzy semi-open (semi-closed) iff there exists a fuzzy open (resp, fuzzy closed) set B such that $B\leqslant A\leqslant B^-$ (resp. $B^o\leqslant A\leqslant B$)[6]. For a fuzzy set A in X, A^- $A^o$and A', will respectively denote the closure and interior and complement in X of A .

Definition 1.1 A mapping f: X→Y is said to be fuzzy semi- irresolute (fuzzy irresolute, fuzzy strongly irresolute, fuzzy strongly semi-irresolute) iff for any fuzzy singleton x_α in X and for any fuzzy semi-open set V in Y containing $f(x_\alpha)$, there exists a fuzzy semi- open set U in X containing x_α such that $f(U) \leqslant V_{-}$[1]. ($f(U) \leqslant V$ $f(U_{-}) \leqslant V$[5], $f(U_{-}) \leqslant V_{-}$ [9]).

Definition 1.2 A fuzzy set A in X is said to be a semi-q-nbd of x_α iff there exists a fuzzy semi-open set $V \leqslant A$ in X such that $x_\alpha qV$.

Theorem 1.3 A mapping $f: X \rightarrow Y$ is fuzzy semi- irresolute (irresolute, strongly irresolute, strongly semi-irresolute) iff for each fuzzy singleton x_α in X and each semi-opcn scmi-q -nbd V of $f(x_\alpha)$ thcrc cxists a fuzzy scmi-open semi-q-nbd U of x_α in X such that $f(U) \leqslant V_{-}$ ($f(U) \leqslant V$ $f(U_{-}) \leqslant V$[5], $f(U_{-}) \leqslant V_{-}$).

2. Fuzzy weakly semi-irresolute mappings

Let us now define as follows.

Definition 2.1 A mapping $f: X \rightarrow Y$ is said to be fuzzy weakly semi-irresolute iff for any fuzzy singleton x_α in X and for any semi-opsn set V in Y containing $f(x_\alpha)$, there exists a fuzzy semi-open set U in X containing x_α such that $f(U) \leqslant V^{-}$.

We first show that in the above definition, containment by fuzzy sem i-open set can be replaced by semi-q-nbd.

Theorem 2.2 A mapping $f: X \rightarrow Y$ is fuzzy weakly semi-irresolute iff for each fuzzy singleton x_α in X an each semi -open semi-q-nbd V of $f(x_\alpha)$, there exists a fuzzy semi-open semi-q-nbd U of x_α in X such that $f(U) \leqslant V^{-}$.

proof. The proof being similar to that of Theorem 1. 2 omitted.

Remark 2.3 It is obvious that fuzzy semi- irresolute → fuzzy weakly semi-irresolute.

We have the following diagram:

fuzzy strongly irresolute → strongly semi-irresolute
↓
fuzzy irresolute
↓ ↓
fuzzy semi-irresolute →fuzzy weakly semi-irresolute

Theorem 2.4 For a mapping $f: X \to Y$ following are cquivalcnt:

(a) f is fuzzy weakly semi-irresolute;

(b) $f^{-1}(V) \leqslant (f^{-1}(V^{-}))_{o}$,for sech fuzzy semi-open set V in X;

(c) $(f^{-1}(V))^{-} \leqslant x_{\alpha}\ qf^{-1}(U) \leqslant f^{-1}(V^{-})$,for each fuzzy semi -open set V in X.

Proof.It is immediate.

Theorem 2.5 A mapping $f: X \to Y$ is fuzzy weakly semi -irresolute iff for every fuzzy semi-open set U in Y , then $x_{\alpha}\ qf^{-1}(U)$ such that $x_{\alpha}\ q(f^{-1}(U^{-}))_{o}$.

Proof. Let f be fuzzy weakly semi-irresolute an U be any fuzzy semi-open set in Y, such that $x_{\alpha}\ qf^{-1}(U)$, then $f(x_{\alpha})qU$,By hypothesis ,there exists a fuzzy semi -open set V in X ,such that $x_{\alpha}\ qV$ and $f(V) \leqslant U^{-}$, this means $V \leqslant f^{-1}(U^{-})$, and since V is semi-open set in X , we have $V = V_{o} \leqslant (f^{-1}(U^{-}))_{o}$,hence $x_{\alpha}\ q(f^{-1}(U^{-}))_{o}$.

Conversely,let x_{α} be any fuzzy singleton in X, and U any fuzzy semi-open semi-q-nbd in Y containing $f(x_{\alpha})$.

Then x_α $q(f^{-1}(U)$,say $f(^{-1}(U^{-}))_{o}=V$,then by hypothesis, V be semi-open semi-q-nbd of x_α and $f(V)\leqslant f(f^{-1}(U^{-}))_{o}\leqslant U^{-}$. Hence f is fuzzy weakly semi-irresolute.

Definition 2.6 A fuzzy singleton x_α is said to be a fuzzy θ -clouster point of a fuzzy set A in X iff the fuzzy closure of every fuzzy open q-nbd of x_α is q -coincident with A. The union of all fuzzy θ -cluster points of A is called the fuzzy θ -clsure of A and is denoted by $[A]_\theta$. A fuzzy set A called fuzzy θ -closed iff $A=[A]_\theta$,and complement of a fuzzy θ -closed set is fuzzy θ -open.

Theorem 2.7 Let $f:X\to Y$ be a fuzzy weakly semi-irresolute ,then the following statements are ralid:

(a)$(f^{-1}(B))_{-}\leqslant f^{-1}([A]_\theta)$, for each fuzzy set B in Y;

(b) $f(A_{-})\leqslant[f(A)]_\theta$,for each fuzzy set A in X;

(c) $f((f^{-1}(B))_{-o-})\leqslant[B]_\theta$,for each fuzzy set B in Y;

(d) $f^{-1}(F)$ is fuzzy semi-closed for every fuzzy θ -closed set F in Y;

(e) $f^{-1}(U)$ is fuzzy semi-open for every fuzzy θ -open set U in Y.

Proof. It is immediate.

3. Application

Definition 3.1 An fts X is said to be fuzzy s-regular iff for each fuzzy singleton x_α in X and each fuzzy semi-open semi-q-nbd U of x_α ,there exists a fuzzy semi -open semi-q-nbd V of x_α such that $V^{-}\leqslant U$.

Let us now difine as follows.

Definition 3.2 An fts X is said to be fuzzy r-regular iff for each fuzzy singleton x_α in X and each fuzzy semi-open semi-q-nbd U of x_α, there exists a fuzzy semi-open semi-q-nbd V of x_α such that $V^- \leqslant U$.

Theorem 3.3 Let $f: X \to Y$ be a mapping, if f is a fuzzy weakly semi-irresolute and Y is fuzzy r-regular, then f is fuzzy irresolute.

Proof. Let $f: X \to Y$ be a fuzzy weakly semi-iresolute and Y is fuzzy r-regular. Let x_α be any fuzzy singleton in X and V any fuzzy semi-open semi-q-nbd of $f(x_\alpha)$, since Y is fuzzy r-regular, there exists a fuzzy semi-open semi-q-nbd U of $f(x_\alpha)$ such that $U^- \leqslant V$, by fuzzy weakly semi-irresolute property of f, there is a fuzzy semi-open semi-q-nbd W of x_α such that $f(W) \leqslant U^- \leqslant V$, then by Theorem 2.2 [1] f is fuzzy irresolute.

Theorem 3.4 Let $f: X \to Y$ be a mapping. If f is a fuzzy weakly semi-irresolute, X is s-regular and Y is fuzzy r-regular, then f is fuzzy strongly irresolute.

proof. The proof being similar to that of Theorem 3.3 is omitted.

In [1] Chang defined an fts X to be fuzzy compact iff erery fuzzy open cover of X has a finite subcover. In a similar manner, one can call an fts X fuzzy semi-compact iff erery fuzzy semi-open cover of X admits a finite subcover.

Definition 3.5 An fts X is said to be fuzzy s-closed iff for every fuzzy semi-open cover $\{U_\alpha : \alpha \in \Lambda\}$ of X. there exists a finite subset Λ of Λ_0 such that $\{U_\alpha^- : \alpha \in \Lambda_0\}$ is a fuzzy cover of X.

Theorem 3.6 Let f:X→Y be a fuzzy weakly semi-irresolute surjection, If X is fuzzy semi-compact, then Y is fuzzy s-closed.

References

[1] S.Malakar, On fuzzy semi-irresolute and strongly irresolute function ,Fuzzy set and systems 45(1992) 239-244.

[2] C.L.Chang, Fuzzy topological spaces , J.Math.Anal. Appl. 24(1968) 182-190.

[3] B.Ghosh, Semi-continuous and semi-closed mapping and semi-connectedness in fuzzy setting, Fuzzy set and systemes, 35(1990) 345-355.

[4] Pao-Ming Pu and Ying- Ming Lu, Fuzzy topology I :Neigbourhood structure of a fuzzy point and Moore-Smith convengence. J.Math. Anal.Appl. 76(1980) 571-599.

[5] M.N.Mukherjee and S.P.Sinha, Irresolote and almost open function between fuzzy topological spaces ,Fuzzy set and systems. 29(1989) 381-388.

[6] K. K. Azad, On fuzzy semi- continuity, fuzzy almost continuity and fuzzy weakly continuity , J.Math.Anal.Appl. 82(1981) 14-32.

[7] M.N.Mukherjee andS.P.Sinha, On some near fuzzy continuous function between fuzzy topological spaces, Fuzzy sets and systems 34(1990) 245-254.

[8] M.N.Mukherjee and B.Ghosh, On fuzzy s-closed spaces and FSC-set, Bull, Malysian Math.Soc. 12(1989) 1-14.

[9] Ma Bao-Guo, Some stronger forms of fuzzy semi-irresolute function on fuzzy topological spaces. Journal of Yanan Universty. 12(1993)