ON FUZZY LINEAR SPACES AND FUZZY NORM FUNCTIONS

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Abstract :

Some characterizations of fuzzy linear spaces are made. A partition technique of finite linenar spaces is introcued with the help of fuzzy linear spaces and proposition s are proved. Fuzzy norms are redefined.

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1. Introduction

Fuzzy fields and fuzzy linear spaces were defined by Nanda [3] and redefined by Biswas in [1]. In this paper some characterizations are made on fuzzy linear spaces (for definition one can see [1]). A partition of a finite linear space is introduced with the help of fuzzy linear spaces. The definition of fuzzy norms in [2] contains a superfluous condition which is removed out.

2. Our Results

Let X be a field of real numbers, F a fuzzy field in X, Y be a linear space over F ad V be a fuzzy linear space of Y. The following proposition is obvious.

Proposition 2.1

(i) $\mu_V(-x) = \mu_V(x) \quad \forall\, x \in Y$

(ii) $\mu_V(0) \geq \mu_V(x) \quad \forall\, x \in Y$

Proposition 2.2

Let μ be a fuzzy linear space of a linear space Y defined over the field F. If for $x, y \in Y$, $\mu(x) \neq \mu(y)$, then

$$\mu(x+y) = \min\{\mu(x), \mu(y)\}$$

Proof :

See that if $\mu(x+y) > \min \{ \mu(x), \mu(y) \}$ then $\mu(x) = \mu(y)$.

$\Rightarrow$ If $\mu(x) \neq \mu(y)$, $\mu(x+y) = \min \{ \mu(x), \mu(y) \}$.

Hence proved.

Corollary : The case $\mu(x) < \mu(x+y) < \mu(y)$ is never possible $\forall$ x, y $\in$ Y.

Propositionn 2.3

Consider the set $S = \{ x \in Y : x \neq 0, \ \mu(x) = \min_{y \in Y} \mu(y) \}$,

where μ is a fuzzy linear space of the linear space Y.

(a) If # (S) = 1, # (X) = 2 and conversely.

(b) $\forall$ y $\in$ Y, at least one of y and x+y belongs to S where x $\in$ S.

[Here # stands for cordinality of a set].

Proof :

(a) If $x \in S$, $x = -x$. If $y \in Y$ where $y \neq x$, $y \neq 0$, then $x + y \notin \{0,x,y\}$.

Now if $\mu(y) = \mu(x)$, # (S) $\neq$ 1 . If $\mu(y) \neq \mu(x)$, then $\mu(x+y) = \mu(x)$ and again # (S) $\neq$ 1.

Hence proved.

Converse is straightforward.

(b) If $y \notin S$, then $\mu(y) > \mu(x)$

$\Rightarrow$ $\mu(x+y) = \mu(x)$. Hence proved.

Definition 2.1

Consider a fuzzy linear space μ of a finite linear space Y. Clearly the membership values are finite. Arrange the distinct membership values only, in ascending order of magnitude in the following way :

$$m_1 < m_2 < \ldots < m_{n+1}.$$

Consider the set

$$C_r = \{ x \in Y : \mu(x) = m_r \}, \quad r = 1,2,3,\ldots,n+1.$$

Clearly, these sets make a partition of the linear space Y. Call it μ-partition of linear space Y. For a given fuzzy linear space μ, this partition is unique.

Proposition 2.4

If C_r, $r = 1,2,\ldots,n+1$ be the μ-partition of a finite linear space Y with respect to the fuzzy linear space μ, then

(i) $\#(C_r) > \#(C_{r+1})$, $r = 1,2,\ldots,n-1$.

(ii) $\#(C_n) \geq \#(C_{n+1})$.

Proof :

(i) Suppose $x \in C_r$. $\forall\, y \in C_{r+1}$ there exists an element $x+y$ (other than x) in C_r. Hence proved.

(ii) Straightforward.

3. A Correction

In [2], Biswas defined fuzzy inner product spaces and fuzzy norm functions as below :

Let S be a linear space over the field F of which V is a fuzzy field. Let μ be a fuzzy inner product space in S. Then the fuzzy set $\|\cdot\|$ of S is called the fuzzy norm function in S if

(i) $\|x\| \leq \|o\|$

(ii) $\|\lambda x\| \geq \min\{V(\lambda), V(\bar{\lambda}), \|x\|\}$

(iii) $\|x+y\| \geq \min\{\|x\|, \|y\|, x.y, y.x\}$

where $x, y \in S$, $\lambda \in F$.

We find that condition (i) is automatically followed from condition (ii) for $\lambda = 0$, $V(o) = 1$ is obvious because μ is an inner product space. Therefore, we redefine fuzzy norm functions as below :

Definition 3.1

Let S be a linear space over the field F of which V is a fuzzy subfield with $V(o) = V(1) = 1$. Let μ be a fuzzy inner product space in S. Then the fuzzy set $\|\cdot\|$ of S is called the fuzzy norm in S if

(i) $\|x+y\| \geq \min\{\|x\|, \|y\|, x.y, y.x\}$

(ii) $\|\lambda x\| \geq \min\{V(\lambda), V(\bar{\lambda}), \|x\|\}$

where $x, y \in S$, $\lambda \in F$.

4. Conclusion

A finite linear space Y is partitioned with the help of a fuzzy linear space μ. If $\#(C_{n+1}) = r$, $\#(Y) \geq (n+1)r + \frac{n(n-1)}{2}$. If UC_i be the μ-partition of the linear space Y, then $\forall\, x \in C_i$ and $\forall\, y \in C_j$, $\mu(x) > \mu(y)$ provided $i > j$. This type of partitioning of a linear space is unique for a given fuzzy linear space of it.

REFERENCES

[1] Biswas, R., Fuzzy fields and fuzzy linear spaces redefined, Fuzzy Sets and Systems 33 (1989) 257-259.

[2] Biswas, R., Fuzzy inner product spaces and fuzzy norm functions, Inform. Sc. 53 (1991) 185-190.

[3] L.A. Zadeh., Fuzzy sets, Inform. and Control 8 (1965) 338-353.

[4] Nanda, S., Fuzzy fields and fuzzy linear spaces, Fuzzy Sets and Systems 19 (1986) 89-94.