INTUITIONISTIC FUZZY RELATIONS

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Abstract :

We define intuitionistic fuzzy relations (IFRs), intuitionistic fuzzy tolerance relations (IFTRs), intuitionistic fuzzy equivalence relations (IFERs) and study some propositions.

Keywords :

Intuitionistic fuzzy set, fuzzy set, intuitionistic fuzzy relation, intuitionistic fuzzy tolerance relation, intuitionistic fuzzy equivalence relation.

1. INTRODUCTION

The notion of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov [1] as a generalizationof the notion of Zadeh's fuzzy sets [6]. Where fuzzy sets can be viewed as IFSs, but not conversely. Atnassov in [1] said it with an example. He also defined in [1,5] various operations on IFSs.

2. PRELIMINARIES

We given below some basic preliminaries.

Definition 2.1

If E is any set, a mapping

$$\mu_A : E \longrightarrow [0,1]$$

is called a fuzzy subset of E.

Definition 2.2

Let A be a fuzzy subset of a set E. Then complement of A is A^c with membership function μ_A^c defined by

$$\mu_A^c(x) = 1 - \mu_A(x), \ \forall x \in E .$$

Definition 2.3

Let a set E be fixed. An IFS A in E is an object having the form

$$A^* = \{ < x, \mu_A(x), \gamma_A(x) > \mid x \in E \}$$

where the functions $\mu_A : E \longrightarrow [0,1]$ and $\gamma_A : E \longrightarrow [0,1]$ define the degree of membership and the degree of non-membership respectvely of the element $x \in E$ to the set A, which is a subset of E, and for every $x \in E$:

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1.$$

Definition 2.4

If A and B are two IFSs of the set E, then

$A \subset B$ iff $(\forall x \in E)\ (\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x))$

$A \supset B$ iff $B \subset A$.

$A = B$ iff $(\forall x \in E)\ (\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x))$

$\overline{A} = \{ < x, \gamma_A(x), \mu_A(x) > \mid x \in E \}$

$A \cap B = \{ < x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) > \mid x \in E \}$

$A \cup B = \{ < x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x)) > \mid x \in E \}$

$A + B = \{ <x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \gamma_A(x).\gamma_B(x)> \mid x \in E \}$

$A.B = \{ < x, \mu_A(x).\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x).\gamma_B(x) > \mid x \in E \}$

$\square A = \{ < x, \mu_A(x), 1 - \mu_A(x) > \mid x \in E \}$

$\Diamond A = \{ < x, 1 - \gamma_A(x), \gamma_A(x) > \mid x \in E \}$.

Obviously every fuzzy set has the form

$$\{ <x, \mu_A(x), \mu_Ac(x) > : x \in E \}.$$

In [1], Atanassov gave an example of an IFS which is not a fuzzy set. From now onwards in this paper, by an IFS A we shall mean the IFS (A, μ_A, ν_A), where the meaning is obvious.

3. INTUITIONISTIC FUZZY RELATIONS

Definition 3.1

Let X and Y be two sets. An intuitionistic fuzzy relation (IFR) R from X to Y is an IFS of $X \times Y$

characterized by the membership function μ_R and non-membership function ν_R. An IFR R from X to Y will be denoted by R (X ⟶ Y).

Definition 3.2

If A is an IFS of X, the sup-inf composition of the IFR R (X ⟶ Y) with A is an IFS B of Y denoted by B = R o A, and is defined by the membership function

$$\mu_{RoA}(y) = \bigvee_{x} \{ \mu_A(x) \wedge \mu_R(x,y) \}$$

and the non-membership function

$$\nu_{RoA}(y) = \bigwedge_{x} \{ \nu_A(x) \vee \nu_R(x,y) \}, \qquad \forall\, y \in Y .$$

(where $\vee$ = sup, $\wedge$ = inf).

Definition 3.3

Let Q (X ⟶ Y) and R (Y ⟶ Z) be two IFRs. The sup-inf composition R o Q is an intuitionistic fuzzy relation from X to Z,

defined by the membership function

$$\mu_{RoQ}(x,z) = \bigvee_{y} \{ \nu_{Q}(x,y) \wedge \nu_R(y,z) \}$$

and the non-membership function

$$\mu_{RoQ}(x,z) = \bigwedge_{y} \{ \nu_Q(x,y) \vee \nu_R(y,z) \}$$

$\forall (x,z) \in X \times Z$ and $\forall\, y \in Y$.

Definition 3.4

An IFR R (X ⟶ X) is said to be

(i) reflexive : iff $\forall\, x \in X$, $\mu_R(x,x) = 1$, and $\nu_R(x,x) = 0$.

(ii) symmetric : iff $\forall\, x_1,\ x_2 \in X$,

$$\mu_R(x_1, x_2) = \mu_R(x_2, x_1) \text{ and}$$
$$\nu_R(x_1, x_2) = \nu_R(x_2, x_1).$$

Definition 3.5

If R is an IFR on X x Y, its inverse R^{-1} is an IFR on Y x X such that $\forall\, (y,x) \in Y \times X$

$$\mu_R^{-1}(y,x) = \mu_R(x,y) \text{ and}$$
$$\nu_R^{-1}(y,x) = \nu_R(x,y).$$

Proposition 3.1

If R and S are two IFRs on X x Y and Y x Z respectively, then

(i) $(R^{-1})^{-1} = R$

(ii) $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Proof : We prove only (ii)

Clearly, $S \circ R : X \longrightarrow Z$ and $R^{-1} \circ S^{-1} : Z \longrightarrow X$.

Now,
$$\begin{aligned}
\mu_{(SoR)^{-1}}(z,x) &= \mu_{SoR}(x,z) \\
&= \bigvee_y \{ \mu_R(x,y) \wedge \mu_S(y,z) \} \\
&= \bigvee_y \{ \mu_{R^{-1}}(y,x) \wedge \mu_{S^{-1}}(z,y) \} \\
&= \bigvee_y \{ \mu_{S^{-1}}(z,y) \wedge \mu_{R^{-1}}(y,x) \} \\
&= \mu_{R^{-1} \circ S^{-1}}(z,x)
\end{aligned}$$

Similarly we can show that

$$\nu_{(S \circ R)^{-1}}(z,x) = \nu_{R^{-1} \circ S^{-1}}(z,x)$$

Hence proved.

Definition 3.6

An IFR R on M x M is said to be transitive if $R^2 \leq R$ where $R^2 = R \circ R$ and the notion of " $\leq$ " is as defined by Definition 2.4.

Definition 3.7

The transitive closure of an IFR R on M x M is $\hat{R}$ defined by

$$\hat{R} = R \cup R^2 \cup R^3 \cup \ldots$$

where the operation of union is as defined in Definition 2.4.

Definition 3.8

An IFR R on M x M is called an intuitionstic fuzzy transitive relation (IFTR) if R is reflexive and symmetric.

Definition 3.9

An IFR R on M x M is called an intuitionistic fuzzy equivalence relation (IFER) if R is reflexive, symmetric and transitive.

Proposition 3.2

If R is an IFTR on M x M and R_1 is an IFER on M x M such that $R \leq R_1$, then $\hat{R} \leq R_1$ where $\hat{R}$ is the transitive closure of R.

Proof. $\hat{R} = \bigcup_{n=1}^{\alpha} R^n \leq \bigcup_{n=1}^{\alpha} R_1^n = \hat{R}_1 = R_1.$

Proposition 3.3

If R_1 and R_2 are two IFTRs on M x M, then $R_1 \cup R_2$, $R_1 \cap R_2$, R_1^{-1} and $\hat{R}_1$ are also IFTRs.

Proof : We prove only for $R_1 \cup R_2$. We have $\forall\, x \in M$

$$\mu_{R_1 \cup R_2}(x,x) = \mu_{R_1}(x,x) \vee \mu_{R_2}(x,x) = 1$$

$$\nu_{R_1 \cup R_2}(x,x) = \nu_{R_1}(x,x) \wedge \nu_{R_2}(x,x) = 0$$

=> $R_1 \cup R_2$ is reflexive.

Again, $\forall\, x_1 , x_2 \in M$

$$\begin{aligned}\mu_{R_1 \cup R_2}(x_1,x_2) &= \mu_{R_1}(x_1,x_2) \vee \mu_{R_2}(x_1,x_2)\\ &= \mu_{R_1}(x_2,x_1) \vee \mu_{R_2}(x_2,x_1)\\ &= \mu_{R_1 \cup R_2}(x_2,x_1)\end{aligned}$$

Similarly, $\nu_{R_1 \cup R_2}(x_1,x_2) = \nu_{R_1 \cup R_2}(x_2,x_1)$

=> $R_1 \cup R_2$ is symmetric. Hence proved.

Proposition 3.4

If R is an IFTR on M x M, then $\square R$ and $\Diamond R$ are also so.

Proof : Straight forward.

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