SQUARE ZERO (□) : REDUCING VAGUENESS IN ZERO (O)

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Abstract : Mathematically, answers to some questions are zero. In real world, some of these zero are in connection with the possible "absence" of objects where some are in connection with the "absence" of impossible or indefinable or absurd objects. Besides, the latter type of "absence" could be sometimes due to the impossibility of the presence of some types of objects. In the present paper, the author defines a new zero called by square zero (□) which may replace zero (0) very suitably in some situations. Some possible applications of □ are also discussed.

Keywords : Square zero (□).

1. INTRODUCTION

The number zero (0) possesses many interesting properties, many beautiful and far reaching applications. It is not just "nothing" or "void"; it really means something. In another terminology it is for the measurement of "something" but not for the measurement of "nothing". As an example, imagine the case of a student who is unable to answer to any question in Mathematics - Test of his annual examination and submit blank answerscript. In parallel, imagine the case of another student who could not appear at the same test (Mathematics-Test) of the same examination. Naturally, in the first case the mark to be awarded by the examiner is zero (0) whereas in the second case the question of awarding a mark does not arise. Now, problem arises to calculate the total marks (which is the sum of the marks for all subjects in this annual examination obtained by the student) in the second case; although the conventional method is to put "A", say (where "A" stands for "ABSENT") in the Grand Mark-Sheet against the subject "Mathematics" and to assume it to be zero (0) while calculating the total mark. Consider another case. Draw a point on a white paper. Imagine the situations "before" and "after" this drawing. What is the difference between these two different situations ? Clearly in the "before" case there was nothing on this white paper and in the "after" case there is something because of some sort of existence. But, what type of existence is it ? Yes, it is the existence of something whose every dimension is zero (0), where in the "before" case there is no existence at all due to the absence of something, whatever it be.

Consider one case more. The statement "I have no money in my pocket" means that I have zero dollar in my pocket. But what does the statement "I have no electric train in my pocket" mean ? Does it mean that I have zero number of train in my pocket ? In fact it means so mathematically. Now question arises whether the mathematical answers to the above two questions, being zero (0) for both, are the same zero ? The first case i.e. the possibility of no money in the pocket is quite acceptable, whereas the second case i.e. the possibility of no train in the pocket is not acceptable; however the possibility of some other number of trains to be in the pocket is also not acceptable at all. Thus in the first case the answer "zero" is for a "possible type of absence" while in the second case the answer "zero" is for an absence caused by an "impossible type of presence". As a consequence, the author feels the need to separate these two zeros, and calls the second zero by Square Zero (□). Whenever an information carries data □ , an user of this information may become careful during the processing of his job. He may investigate for the existence of any vagueness or fuzziness in this information. The author also gives some applications of □ and redefine fuzzy sets of Zadeh in order to make a little step ahead in exploring vagueness, if exists, in some cases.

2. SQUARE ZERO (□)

A square zero is a real number with the following properties :

(P.1) it is denoted by the symbol □

(P.2) $\square \neq 0$

(P.3) it's position on the real number line is coincident with that of zero (0).

(P.4) □ equals the number of impossible or invalid or indefinable or absurd objects which are absent at the time of counting. It also equals the number of absent objects whose presence is impossible.

(P.5) if x is any real number, then

(i) $x + \square = \square + x = x$

(ii) $x - \square = x$

(iii) $\square - x = - x$

(iv) $x . \square = \square . x = \square$

(v) $\square + \square = \square$

(vi) $\square - \square = \square$

(vii) $\square . \square = \square$

(viii) $x / \square$, $\square / \square$ are meaningless.

3. REDEFINING FUZZY SETS

The properties of $\square$ encourage to redefine Zadeh's fuzzy sets. Consider a set $X = \{ x_1 , x_2 , x_3 \}$. Let F be a fuzzy set of X with membership function m_F given by

$$m_F : X \longrightarrow [0,1]$$

There is no specific rule or formula or algorithm for the construction of membership functions. Now, question arises whether it is always possible to find or to define or to choose value of the membership function for each element of X ? If not, then what to do ? Should we take $m_F(x) = 0$ for such an element x ? Our answer is "NO". Instead, we write $m_F = \square$, which will certainly mean, by definition of $\square$, that the membership value of x is either indefinable or to be postponed for the time being assuming it as a "don't care" case.

We may redefine Zadeh's fuzzy sets as below :

Definition 3.1 :

Let X be a set. A fuzzy set F of X is defined by the mapping

$$m_F : X \longrightarrow V$$

where $V = \{ \square \} \cup [0,1]$.

Example 3.1 :

Consider the set X of all students of Class-V in Ram Krishna Mission School, Calcutta. Construct a fuzzy set F of X which is :

"Set of all brilliant students in Class-V"

with the membership function m_F given by :

$$m_F = \square, \quad \text{if } x \text{ is a newly admitted or failed student in Class-V}$$

$$= \frac{n - r_x}{n}, \quad \text{otherwise.}$$

where, r_x = rank of x in his annual examination of Class-IV, and

n = total number of students in Class-V.

4. MORE APPLICATIONS OF □ :

We illustrate below, with the help of examples, three more applications of square zero □ , where zero (0), if be used, will stand with much more vagueness.

Example 4.1 :

Consider the marksheet of a student named by RATAN BOSE in connection with his Class-IV Annual Examination which reveals the following information :

Subjects	Marks Obtained
English Language	53
Bengali Language	60
Social Studies	72
Work-education	absent
Science	80
Mathematics	94
TOTAL MARK	359
REMARK	Promoted

RATAN BOSE was absent in his examination on the subject "Work-education". So he is not awarded any mark, even not zero (0), against this subject in his mark-sheet. Let us denote this case by CASE-I. Imagine now another case CASE-II as furnished below :

CASE - II :

RATAN BOSE appeared in his Work-education examination but scored zero (0) as he could not write any answer correctly. His marksheet looks like below :

Subjects	Marks Obtained
English Language	53
Bengali Language	60
Social Studies	72
Work-education	00
Science	80
Mathematics	94
TOTAL MARK	359
REMARK	Promoted

Usually the TOTAL MARK is calculated by adding all the marks. But how to do so in case of CASE-I where all the data are not numerical ? Well, our conventional method is to imagine (without mentioning in the marksheet) zero(0) mark in place of "absent" and then to add. We see that this is neither a fair job nor an unfair. We suggest to award ◻ mark instead of putting "absent" and then to do the addition by using the properties of ◻ as laid down in Definition 2.1. Undoubtedly, doing this is not unfair at least, although not very fair.

Example 4.2 :

Suppose Mr. Ram has 50 dollars in his pocket. Going to market, he spends the whole amount of money. Now, consider the following two questions :

Q.1 How many dollars does Mr. Ram have now in his pocket ?

Q.2 How many BOEING-737 aircrafts does Mr. Ram have now in his pocket ?

The answer to Q.1 is zero (0) and to Q.2 is ◻ .

Example 4.3 :

Here we suggest an application of square zero (◻) in Genetic Algorithm (GA). We give, first of all, some basic preliminaries in brief. For details one can see [2].

The theory of Genetic Algorithm is based on Schema analysis. Theory of evaluation deals with the manipulation of **population** of possible **individuals** over a number of **generations** in an attempt to find the optimal design. Here **population** means a number of **individuals**, an **individual** means a number of **genes**, where each **gene** is one **allele** or value.

Each parameter of an **individual** consists of one or more genes and their associated alleles.

A **schema** may be viewed as a set of chromosomes which share some specified subset of their genes.
A **forma** may be viewed as a set of chromosomes which are related by some specific characteristic.
A **string** may be of the form $a_1a_2a_3 \ldots a_n$, where $a_i \in \{0,1\}$.

Let us discuss an application of square zero (□) in Genetic Algorithm. Holland [2] introduced the symbol □ (this is not our square zero), which stands for "don't care" character and the symbol ■ , which stands for "care" character where □ and ■ belong to { 0,1 }.
And, thus a schemat over a string of length n can be viewed as equivalent classes of equivalence relation described by the members of $\psi = \{ \square , \blacksquare \}^n$.

We see that this don't care symbol □ is, in another terminology, our square zero □ . And, for the sake of instance, we have :

schema □ 10 □ = set of all chromosomes which have a one at their 2nd locus and zero at their third locus and square zero at first and fourth.

and **Similarity Set** of two chromosomes $a_1a_2 \dots a_n$ and $b_1b_2 \dots b_n$ (where $a_i, b_i \in \{0,1\}$) with respect to schemata can be defined as below :

$$\text{Similarity} (a_1a_2 \dots a_n , b_1b_2 \dots b_n) = c_1c_2 \dots c_n ,$$

where $c_i = a_i$, if $a_i = b_i$

$= \square$ (square zero), if $a_i \neq b_i$.

Thus, similarity (110, 011) = $\square$ 1 $\square$

We may view a string of length n as $a_1a_2a_3 \dots a_n$, where $a_i \in \{ 0, 1, \square \}$, $\square$ being the square zero.

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