INTUITIONISTIC FUZZY LOGIC ON OPERATOR LATTICE

CHEN TUYUN ZOU LI

Department of Mathematics, Liaoning Normal University, DaLian, 116029, *P. R. China*

ABSTRACT

The basic principles and results of intuitionistic fuzzy set introduced by K. Atanassov are extended to intuitionistic fuzzy logic[1~2]. In 1989, Liu Xuhua introduced operator fuzzy logic and its λ − resolution method[3]. In this paper, intuitionistic operator fuzzy logic (IOFL in simple) on operator lattice $L=\{(\mu,\upsilon) \mid \mu,\upsilon\in[0,1]\}$ is defined. Then some properties of them and (μ,υ) − resolution methed are discussed.

Keywords: fuzzy logic, intuitionistic fuzzy logic, operator lattice.

Definition 1 The operation '$*$', '$\oplus$' and "'" on $L=\{(\mu,\upsilon) \mid \mu,\upsilon\in[0,1], \mu+\upsilon\leqslant 1\}$ are defined as follows, for $\forall\ (\mu_1,\upsilon_1),(\mu_2,\upsilon_2)\in L$,

$(\mu_1,\upsilon_1) * (\mu_2,\upsilon_2) \triangleq (\min(\mu_1,\mu_2), \max(\upsilon_1,\upsilon_2))$,

$(\mu_1,\upsilon_1) \oplus (\mu_2,\mu_2) \triangleq (\max(\mu_1,\mu_2), \min(\upsilon_1,\upsilon_2))$,

$(\mu_1,\upsilon_1)' \triangleq (\upsilon_1,\mu_1)$.

Note that, these three operations is closed in L.

Theorem 1 Algebra system $(L, *, \oplus)$ is a lattice.

Proof. It's obviously that binary operator " $*$ " and " $\oplus$" can be exchanged and associated. The absorption law is satisfied.

Theorem 2 $(L, *, \oplus, ')$ is a complete complementary distributive lattice.

Theorem 3 The operator operation " $\cdot$ " in $(L, *, \oplus, ')$ is defined as follows, $\forall\ (\mu_1,\upsilon_1),(\mu_2,\upsilon_2)\in L$, $(\mu_1,\upsilon_1)\circ(\mu_2,\upsilon_2)=((\mu_1+\mu_2)/2, (\upsilon_1+\upsilon_2)/2)$, so L is a operator lattice.

As above, L is satisfied with the condition of the operator lattice which is defined in article[3].

According to the result of Theorem 3, IOFL on operator lattice can be discussed.

Definition 2 Let P be a symbol of atom, $(\mu,\upsilon)\in L$, $(\mu,\upsilon)P$ is regarded as an intuitionistic fuzzy atom (IF atom).

Definition 3 The formulas in IOFL are symbol strings which are defined successively as follows: (1) An IF atom is a formula; (2) If G is a formula, $(\mu,\upsilon)G$, $(\sim G)$ are formulas; (3) If G,H are formulas, $(G\wedge H)$, $(G\vee H)$, $(G\rightarrow H)$, $(G\rightarrow H)$ are formulas; (4) If G is a formula, x is a free variable in G, $(\forall\ x)G(x)$, $(\exists\ x)G(x)$, $((\mu,\upsilon)\forall\ x)G(x)$, $((\mu,\upsilon)\exists\ x)G(x)$ are formulas; (5) All the formulas is the strings which use (1)~(4) finite times.

Definition 4 The true value $V_I(G)$ of formula G under interpret I is fixed as follows:

(1) If $(\mu,\upsilon)P$ is an IF atom, $V_I((\mu,\upsilon)P)=\begin{cases}(\mu,\upsilon), \text{when } P \text{ is appointed } T \text{ by I}\\(\upsilon,\mu), \text{when } P \text{ is appointed } F \text{ by } I\end{cases}$

(2) If G,H are formulas, let $V_I(G)=Q(a,b)$, $V_I(H)=(c,d)$, $\forall\ (x)\in D, V_I(G(x))=(a_x,b_x)$,

Ⅰ) $V_I((\mu,\upsilon)G)\triangleq(\mu,\upsilon)\cdot V_I(G)$;

Ⅱ) $V_I(\sim G)\triangleq(b,a)$;

Ⅲ) $V_I(G\vee H)\triangleq(\max(a,c),\min(b,d))$;

Ⅳ) $V_I(G\wedge H)\triangleq(\min(a,c),\max(b,d))$;

Ⅴ) $V_I(G\rightarrow H)\triangleq V_I(\sim G\vee H)$

Ⅵ) $V_I(G\rightarrow H)\triangleq V_I((G\rightarrow H)\wedge(H\rightarrow G))$;

Ⅶ) $V_I((\forall\ x)G(x))\triangleq(\inf_{x\in D}a_x,\sup_{x\in D}b_x)$;

Ⅷ) $V_I((\exists\ x)G(x))\triangleq(\sup_{x\in D}a_x,\inf_{x\in D}b_x)$;

Ⅸ) $V_I(((\mu,\upsilon)\forall\ x)G(x))\triangleq V_I((\mu,\upsilon)((\forall\ x)G(x)))$;

Ⅹ) $V_I(((\mu,\upsilon)\exists\ x)G(x))\triangleq V_I((\mu,\upsilon)((\exists\ x)G(x)))$.

Definition 5 If for the arbitrary interpret I there exist $V_I(G)=V_I(H)$, formula G is equal to formula H, which can be signed $G\equiv H$.

Definition 6 Let G be a formula, $(\mu,\upsilon)\in L$, let $V_I(G)=(\mu_G,\upsilon_G)$, If for the arbitrary interpret I, there is $\mu_G\geqslant\mu,\upsilon_G\leqslant\upsilon$, G is called (μ,υ)—identically true; If $\mu_G\leqslant\mu,\upsilon_G\geqslant\upsilon$, G is called (μ,υ)—identically false.

From the defintion 4, some properties of IOFL can be obtained, hence it is obviously that G is (μ,υ)—identically true iff $\sim$G is (μ,υ)—identically false.

From afore said, an arbitrary formula in IOFL is equal to a prefix normal form. It's easy to prove that formula G is (μ,υ)—identically true iff the skolem form is (μ,υ)—identically false. Hence, an arbitrary formula G corresponds a set S of clauses and G is (μ,υ)—identically false iff S is (μ,υ)—identically false.

Definition 7 $(\mu_1,\upsilon_1)P$ and $(\mu_2,\upsilon_2)P$ are called (μ,υ)—complementary literals if $\mu_1>\mu,\upsilon_1<\upsilon$ and $\mu_2<\upsilon,\upsilon_2>\mu$ or $\mu_1<\upsilon,\upsilon_1>\mu$ and $\mu_2>\mu,\upsilon_2<\upsilon$ when $\mu\geqslant0.5,\upsilon\leqslant0.5$ for given $(\mu,\upsilon)\in L$. (It's just contrary to this when $\mu_1<0.5,\upsilon_1>0.5$) .

Definition 8 $(\mu_1,\upsilon_1)P$ and $(\mu_2,\upsilon_2)P$ are called (μ,υ)—similar literals if $\mu_1>\mu,\upsilon_1<\upsilon$ and $\mu_2>\mu,\upsilon_2<\upsilon$ or $\mu_1<\upsilon,\upsilon_1>\mu$ and $\mu_2<\upsilon,\upsilon_2>\mu$ when $\mu\geqslant0.5,\upsilon\leqslant0.5$ for given $(\mu,\upsilon)\in L$. (It's just contrary to this when $\mu<0.5,\upsilon>0.5$).

Definition 9 Let C_1 and C_2 be sentences with no variables in common, let $(\mu_1,\upsilon_1)P_1$ and $(\mu_2,\upsilon_2)P_2$ be literals in C_1 and C_2 respectively, If P_1 and P_2 have a most general unifier σ (MGUσ[4]), and $(\mu_1,\upsilon_1)P_1^\sigma$ and $(\mu_2,\upsilon_2)P_2^\sigma$ are (μ,υ)—complementary literal each other, the clause $(C_1^\sigma-S_1)\cup(C_2^\sigma-S_2)$ is called two variables resolution formal of C_1 and C_2, it can be signed $R(\mu,\upsilon)(C_1,C_2)$, in which

$S_1=\{(\mu^*,\upsilon^*)P^\sigma|(\mu^*,\upsilon^*)P^\sigma\in C_1^\sigma,(\mu^*,\upsilon^*)P^\sigma$ and $(\mu_1,\upsilon_1)P_1^\sigma$ is (μ,υ)—similar$\}$,

$S_2=\{(\mu^*,\upsilon^*)P^\sigma|(\mu^*,\upsilon^*)P^\sigma\in C_2^\sigma,(\mu^*,\upsilon^*)P^\sigma$ and $(\mu_2,\upsilon_2)P_2$ is (μ,υ)—similar$\}$.

Definition 10 Let $(\mu_1,\upsilon_1)P_1,\cdots,(\mu_n,\upsilon_n)P_n$ be literals in clause C, if $P_1,P_2,\cdots,P_n$ have MGUσ and $(\mu_1,\upsilon_1)P_1^\sigma,\cdots,(\mu_n,\upsilon_n)P_n^\sigma$ are (μ,υ)—similar literals, C^σ is called a factor of C.

Definition 11 Two variables resolution formal of C_1 (or the factor of C_1) and C_2 (or the factor of C_2) is called (μ,υ)—resolution formal of C_1 and C_2.

Definition 12 Let S be a set of clause, $S_{PR}^{(\mu,\upsilon)}$ is called primary reduced set of S in which $(\mu,\upsilon)\in L$. $S_{PR}^{(\mu,\upsilon)}$ is obtained with the method as follows, for $\forall\ (\mu^*,\upsilon^*)P\in S$.

(1) When $\mu\geqslant 0.5,\upsilon\leqslant 0.5$ if $\upsilon\leqslant\mu^*\leqslant\mu$ or $\upsilon\leqslant\upsilon^*\leqslant\mu$, delete $(\mu^*,\upsilon^*)P$ *from* S;

(2) When $\mu<0.5,\upsilon>0.5$, if $\mu\leqslant\mu^*\leqslant\upsilon$ or $\mu\leqslant\upsilon^*\leqslant\upsilon$, delete (μ^*,P) from S.

Theorem 4 Let S be a clause set and $(\mu,\upsilon)\in L$, so S is (μ,υ)—identically false iff $S_{PR}^{(\mu,\upsilon)}$ is (μ,υ)—identically false.

Proof. Use the properties of IOFL, Definition 6 and Definition 12.

Definition 13 Let $S_{PR}^{(\mu,\upsilon)}$ be a (μ,υ)—primary reduced set, and $S_{PR}^{(\mu,\upsilon)}$ is called (μ,υ)—reduced set of S if $\forall\ (\mu^*,\upsilon^*)P\in S_{PR}^{(\mu,\upsilon)}$ can be done these replacement:

(1) When $\mu^*\geqslant 0.5,\upsilon^*\leqslant 0.5$, if $\mu^*>\mu,\upsilon^*<\upsilon$, $(\mu^*,\upsilon^*)P$ is replaced with P; if $\mu^*<\upsilon$, $\upsilon^*>\mu^*$, $(\mu^*,\upsilon^*)P$ is replaced with $\sim P$.

(2) When $\mu^*<0.5,\upsilon^*>0.5$, if $\mu^*>\upsilon,\upsilon^*<\mu$, $(\mu^*,\upsilon^*)P$ is replaced with P; if $\mu^*<\mu$, $\upsilon^*>\upsilon$, $(\mu^*,\upsilon^*)P$ is replaced with $\sim P$.

It is obviously that $S_R^{(\mu,\upsilon)}=S_R^{(\upsilon,\mu)}$.

Theorem 5 Let C_1 and C_2 be two clause sets, $C_{1R}^{(\mu,\upsilon)}$ and $C_{2R}^{(\mu,\upsilon)}$ are (μ,υ)—reduced set of C_1 and C_2 respectively. If $C=R_{(\mu,\upsilon)}(C_1,C_2)$, there exist $C'=R(C_{1R}^{(\mu,\upsilon)},C_{2R}^{(\mu,\upsilon)})$ which makes $C'=C_R^{(\mu,\upsilon)}$. Otherwise, if $C'=R_{(\mu,\upsilon)}(C_{1R}^{(\mu,\upsilon)},C_{2R}^{(\mu,\upsilon)})$ there exist $C=R_{(\mu,\upsilon)}(C_1,C_2)$ which makes $C_R^{(\mu,\upsilon)}=C'$.

Proof. Use Definition 9 and Definition 13.

Theorem 6 $S_{PR}^{(\mu,\upsilon)}$ is (μ,υ)—identically false iff $S_R^{(\mu,\upsilon)}$ is identically false.

Proof. Let $\mu\geqslant 0.5,\upsilon\leqslant 0.5$ might as well, using Definition 6, Definition 12 and Definition 13.

Theorem 7 For $(\mu,\upsilon)\in L$ if a clause set of S is (μ,υ)—identically false, there exist (μ,υ)—resolution deduction which can be deduce (μ,υ)—empty clause from S and an arbitrary literal $(\mu^*,\upsilon^*)P$ in empty clause will be satisfied with $\upsilon\leqslant\mu^*\leqslant\mu$ or $\upsilon\leqslant\upsilon^*\leqslant\mu$.

Proof. Use Theorem 4, Theorem 5 and Theorem 6.

One of the properties of IOFL is $(\mu_1,\upsilon_1)(\mu_2,\upsilon_2)\ P\neq((\mu_1,\upsilon_1)\text{o}(\mu_1,\upsilon_2))\ P$ (we can make example to proof it). So the literal $(\mu_1,\upsilon_1)\cdots(\mu_n,\upsilon_n)\ P$ can not be simplified to $(\mu^*,\upsilon^*)\ P$. In order to introduce (μ,υ)—resolution, the concept of (μ,υ)—complementary literals and (μ,υ)—similar literals will be extended.

Definition 14 $(\mu_{11},\upsilon_{11})\cdots(\mu_{1n},\upsilon_{1n})P$ and $(\mu_{21},\upsilon_{21})\cdots(\mu_{2n},\upsilon_{2n})P$ are called (μ,υ)—complementary literals if they satisfy following:

(1)If $\mu \geqslant 0.5, \upsilon \leqslant 0.5$,

when P is appointed T by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} > \mu \\ \upsilon_{11}\cdots\upsilon_{1n} < \upsilon\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} < \upsilon \\ \upsilon_{21}\cdots\upsilon_{2n} > \mu\end{cases}$;

when P is appointed F by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} < \upsilon \\ \upsilon_{11}\cdots\upsilon_{1n} > \mu\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} > \mu \\ \upsilon_{21}\cdots\upsilon_{2n} < \upsilon\end{cases}$;

(2)If $\mu < 0.5, \upsilon > 0.5$

when P is appointed T by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} > \upsilon \\ \upsilon_{11}\cdots\upsilon_{1n} < \mu\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} < \mu \\ \upsilon_{21}\cdots\upsilon_{2n} > \upsilon\end{cases}$;

when P is appointed F by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} < \mu \\ \upsilon_{11}\cdots\upsilon_{1n} > \upsilon\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} > \upsilon \\ \upsilon_{21}\cdots\upsilon_{2n} < \mu\end{cases}$.

Definition 15 $(\mu_{11},\upsilon_{11})\cdots(\mu_{1n},\upsilon_{1n})P$ and $(\mu_{21},\upsilon_{21})\cdots(\mu_{2n},\upsilon_{2n})P$ are called (μ,υ)—similar literals if they satisfy following:

(1)If $\mu \geqslant 0.5, \upsilon \leqslant 0.5$,

when P is appointed T by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} > \mu \\ \upsilon_{11}\cdots\upsilon_{1n} < \upsilon\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} > \mu \\ \upsilon_{21}\cdots\upsilon_{2n} < \upsilon\end{cases}$;

when P is appointed F by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} < \upsilon \\ \upsilon_{11}\cdots\upsilon_{1n} > \mu\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} < \upsilon \\ \upsilon_{21}\cdots\upsilon_{2n} > \mu\end{cases}$;

(2)If $\mu < 0.5, \upsilon > 0.5$,

when P is appointed T by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} > \upsilon \\ \upsilon_{11}\cdots\upsilon_{1n} < \mu\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} > \upsilon \\ \upsilon_{21}\cdots\upsilon_{2n} < \mu\end{cases}$;

when P is appointed F by I, $\begin{cases}\mu_{11}\cdots\mu_{1n} < \mu \\ \upsilon_{11}\cdots\upsilon_{1n} > \upsilon\end{cases}$, $\begin{cases}\mu_{21}\cdots\mu_{2n} < \mu \\ \upsilon_{21}\cdots\upsilon_{2n} > \upsilon\end{cases}$.

The concepts of two varibles (μ,υ)—resolution form, (μ,υ)—factor and (μ,υ)—resolution form are the same as aforesaid definitions. Thus the (μ,υ)—resolution method of this general clause set can be discussed similar.

Note that, when $\mu+\upsilon=1$, lattice L and operator lattice on [0,1] are isomorphism. The result of this paper is showed no difference from article[3]. Thus the method of this paper is a extension of article[3].

references

[1]K. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy sets and system, Vol 20(1986), pp87—96.

[2]K. Atanassov, New Variants of Model Operators in intuitionistic Fuzzy Model Logic, BUSEFAL, No. 54(1993), pp79—83.

[3]Liu XuHua, Automatic Reasoning Based On Resolution Method, Science Publishing House, 1994, pp347—360. (in Chinese).

[4]Wang Yuanyuan, Logic in Computer Science, Science Publishing house, 1989, pp130—131. (in Chinese).