

## MEMBERSHIP FUNCTIONS IN MEDICAL DIAGNOSIS

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*Abstract:*

Usually are presented two models membership functions. The easiest way is to ask some subjects directly for membership values. Often a certain concept can be considered as a context - specific version of a more general feature. Then the degree of membership can evaluate as the comparison between of an object with a standard (ideal) which results in some distance. In this paper we present another model for membership function based on probability method.

**Key Words:** medical diagnosis, fuzzy set theory, membership functions.

**Introduction**

Measurements means assigning numbers to object, such that certain relations between numbers reflect analogous relations between objects. If it is possible to prove that there is a homomorphic mapping  $f: A \rightarrow N$  from an empirical relational structure

$$\langle A, R_1, \dots, R_n \rangle$$

with a set of object  $A$  and an  $n$ -tuple of relations  $R_i$  into a numerical relational structure

$$\langle N, P_1, P_2, \dots, P_n \rangle$$

with a set of numbers  $N$  and relations  $P_i$ , then a scale  $\langle A, N, f \rangle$  exists.

By specifying the admissible transformations the grade of uniqueness is determined. Therefore measurements starts by formulating the properties of empirical structure, but this has not been possible for the concept of membership. Membership has a clear cut formal definition. However, explicit requirements for its empirical measurement are still missing, but for the purpose of empirical research it may be

tolerable to use plausible techniques. Usually are present two models for membership functions.

The purpose of the first model is to empirically investigate aggregation operators. It is sufficient to determine degrees of membership for a predefined set of object rather than continuous membership functions. One way to obtain data is to ask some expert directly for membership values.

Often a certain concept can be considered as a context/specific version of a more general feature. We assume that the observer has some idea about general feature. The valuation of membership can be regarded as the comparison of some object with a standard which results in some distance. distance shall be infinity. Then we assume, that degree of membership is a function of this distance. This way yields the logistic S-shaped function.

The use of objective measurements for diagnostic purposes is only possible to a certain degree. The assignment of laboratory test results to the ranges "normal" or "pathological" is arbitrary in borderline cases and many observation are very subjective. The relationship between symptoms and diseases are generally far from crisp and unique.

The second model for membership function is based on characteristic function which allows various degrees of membership for the elements or objects of a given fuzzy set. For our purposes elements or objects are patients and fuzzy set is set all patients with their diagnoses.

The task is to determine degrees of membership  $g_k$  with the patient has the diagnose  $k$ , based on his results of medical tests. Sumall these membership is equal to 1. Hence

$$0 \leq g_k \leq 1 \quad \sum g_k = 1 \quad (1)$$

where  $K$  is number of diagnoses. We apologise that results of medical tests are discrete . In other words we apologise that for every test exists finite number of results.

The method is based on basic collection of patient with clearly given diagnose. The membership of every patient is equal 1, and we can write

$$g_k=1 \text{ for } i=k \text{ and}$$

$$g_k=0 \text{ for } i \neq k. \quad i=1,2,\dots, K$$

Hence they are given on the basis unformalised (intuitive) method.

From basis collection of patients we determine probability ( $\lambda_{kix}(x)$ ). This is probability that patient with k-th diagnose has in j-th test result x. These probability can be given direct by medical experts, too, or can be obtained from the following expression

$$\lambda_{kix}(x) = \frac{N_{kix}}{N_k}$$

where  $N_{kix}$  is number of patients with k-th diagnose and has in j-th test result x.

$N_k$  is number of patient with diagnose k. On the basis of these probabilities can be computed new values  $g_k$  for the patients from basic collection of patient. These values are degrees of membership computed formal mathematical procedure shortly presents in the next. From definition of probability  $\lambda_{kix}(x)$  we know, that

$$\sum \lambda_{kix} = 1, \lambda_{kix} \geq 0 \quad (2)$$

Probabilities  $\lambda_{kix}(x)$  makes matrix  $\lambda$ , type  $K \times J \times X$

where  $K$  is number of diagnoses,  $J$  number of tests and  $X$  is number of results in the test. Our task is to determine from the results of tests

$$x=(x_1, x_2, \dots, x_j),$$

for the patient the following vector

$$g=(g_1, g_2, \dots, g_K)$$

For this purpose we have basic collection of patient with vectors

$$x^{(1)}, x^{(2)}, \dots, x^{(I)},$$

and given

$$g^{(1)}, g^{(2)}, \dots, g^{(I)},$$

where I is number patients of basic collection.

We use likelihood function X to determine grades of memberships.

$$X = (g_1 \lambda_{11x_1} + g_2 \lambda_{21x_1} + \dots + g_K \lambda_{K1x_1}) \cdot (g_1 \lambda_{12x_2} + g_2 \lambda_{22x_2} + \dots + g_K \lambda_{K2x_2}) \dots (g_1 \lambda_{1Jx_J} + g_2 \lambda_{2Jx_J} + \dots + g_K \lambda_{KJx_J})$$

If the matrix  $\lambda$  is known then g can be obtained from

$$X \rightarrow \max \quad (3)$$

and condition (1) holds.

To solve equation (3) we used method of gradient and condition (1) we changed the following condition, [1]

$$\|g\| = \sqrt{g_1^2 + g_2^2 + \dots + g_K^2} = 1, \quad g_k \geq 0 \quad k = 1, 2, \dots, K$$

On the basis briefly given method a program has been done. The program is a part of interactive classification system[2]. This system was used in various areas of medicine for example in cardiology, ophthalmology or psychology, [3],[4].

## CONCLUSION

Fuzzy set theory has developed as a very powerful modelling language, that can cope with a large fraction of uncertainties of real-life situations. It can be well adapted to different circumstances and contexts. The most publications contain contributions or monographs that focus on fuzzy set theory. This paper deals with

basis element fuzzy set theory, about membership function. Model for membership function based on probability approach is given. The program made on this base was used in interactive classification system and applied on various areas of medicine.

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