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EXPLOITATION OF FUZZY SET THEORY IN CERTAIN ECOLOGICAL PROBLEMS

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ABSTRACT. In 1991 in Łódź (a big city in Poland) studies of the concentration of SO_2 and the level of noise were carried out. These results are analysed by means of the non-probabilistic entropy method of A. de Luca and S. Termini. In the case when the phenomenon being examined bears a random character and fuzzy one at the same time, the analysis of it has respect to the indefiniteness following from both the characters.

Another aspect of connecting fuzzy set theory with a random event is the calculation of fuzzy probability by example of the states of weather.

1. Introduction

The majority of phenomena occurring in nature bear imprints of some indefiniteness. This indefiniteness may derive from the random character of those phenomena or from an inaccurate description of them. Undoubtedly, the indefiniteness connected with an experiment lying in the observation of the sex of first person encountered in the street differs from that consisting in the observation of the species of a first bird encountered in the wood. The above indefinitenesses are the outcome of the random nature of these experiments. It is important in practice to be able to evaluate such an indefiniteness in number in order to make it possible to compare the experiments and, in consequence, to make proper decisions.

A rational way of measuring indefiniteness of random experiments was proposed in 1948 by C.E. Shannon who introduced the notion of entropy and, in consequence, the notion of information. Another aspect of analysis random phenomena is probability theory with parameters it operates with in numerical problems.

Key words and phrases. Random experiment, probability measure, entropy, energy, fuzzy set, fuzziness measures of a fuzzy set

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In the present paper we wish to pay attention to some other of non--uniqueness in phenomena of natural type, to the non-uniqueness connected with the non-precision of the very description of the phenomenon, for we can examine the "onerousness of a small" on the gathering ground of a sewer, the water pollution in some water reservoir, or the "intensity of atmospheric fall" on a chosen area. The notions mentioned here are non-unique indeed: after all, it is difficult to indicate a clear boundary between the small which is "onerous" and the one which is not onerous any longer (or is not onerous yet). Such notions really bear a fuzzy character [9], they can be described precisely by using the membership function of a fuzzy set and analysed in a suitable manner.

PROBLEM 1

Let, for instance, the table given below ilustrate the mean monthly concentrations of SO_2 in Łódź (a big city in Poland) in 1991, hanging over on the northern outskirts of the urban centre (according to the measurement made by DEPŁ [3]):

month	I	II	III	IV	V	VI
$SO_2[mg/m^3]$	120	180	100	50	30	20

VII	$VIII$	IX	X	XI	XII
10	10	15	40	70	200

Treating the December observation as a really high concentration of SO_2, we can model the set of "high concentrations" of SO_2 in Łódź by means of a fuzzy set A with the membership function μ_A given in the table:

month x	I	II	III	IV	V	VI
$\mu_A(x)$	0.60	0.90	0.50	0.25	0.15	0.10

VII	$VIII$	IX	X	XI	XII
0.05	0.05	0.08	0.20	0.35	1

In virtue of the table, e.g. the April observation states that in this month there is a "high concentration" of SO_2 to the degree 0.25, while in February the concentration is "high" up to the degree 0.90.

The indefiniteness resulting from the fuzzy complexion of phenomena can be analysed mathematically. One of the possibilities is given by the conception of A. De Luca and S. Termini [5] of the so-called non-probabilistic entropy of such a set. Following on the lines of the properties of Shannon's entropy, they imposed the conditions below on the non-probabilistic entropy $D(A)$ of the fuzzy set A with the membership function μ_A:

d1) $D(A) = 0 \Longleftrightarrow \mu_A(x) \in \{0,1\}$,

d2) $D(A)$ is maximum $\Longleftrightarrow \mu_A(x) \equiv \frac{1}{2}$,

d3) $D(A) > D(A^*)$ where A^* is a sharper version of the set A, that is, $\mu_{A^*}(x) > \mu_A(x) > \frac{1}{2}$ or $\mu_{A^*}(x) \leq \mu_A(x) \leq \frac{1}{2}$.

The effect of the papers devoted to the entropy of a fuzzy set is the definition of the quantity of this entropy to be

$$D(A) = -K \cdot \sum_{i=1}^{n} [\mu_A(x_i) \cdot \log_a \mu_A(x_i) + (1 - \mu_A(x_i)) \cdot \log_a(1 - \mu_A(x_i))]$$

where $K > 0$ is an arbitrary constant, and A is a fuzzy set in the finite space of considerations $\{x_1, x_2, ..., x_n\}$. By using the Shannon function

$$S(x) = \begin{cases} -x \cdot \log_a x - (1-x) \cdot \log_a(1-x) & \text{for } x \in (0,1), \\ 0 & \text{for } x \in \{0,1\}, \end{cases}$$

the value of $D(A)$ can be written down in the shorter form:

$$D(A) = K \cdot \sum_{i=1}^{n} S(\mu_A(x_i)).$$

In contradistinction (in a way) to the notion of entropy as a fuzziness measure of a fuzzy set, in 1977 D. Dumitrescu introduced a sharpness measure, the so-called energy of a fuzzy set [1]. The starting point of this conception is not Shannon's entropy but Onicescu's informational energy [8]. D. Dumitrescu imposed the following conditions on the energy $E(A)$ of the fuzzy set A:

e1) $E(A)$ attains its minimum $\Longleftrightarrow \mu_A(x) \equiv \frac{1}{2}$, e2) $E(A)$ attains its maximum $\Longleftrightarrow \mu_A(x) \in \{0,1\}$,

e3) $E(A^*) > E(A)$ where A^* is as in (d3). The effect is the formula [1]

$$E(A) = \sum_{i=1}^{n} [\mu_A^2(x_i) + (1 - \mu_A(x_i))^2].$$

More information on the measuring of the fuzziness and sharpness of fuzzy sets, as well as on the investigations of the algebra and other aspects of fuzzy set theory, can be found, for instance, in papers [1], [4], [5], [6].

Let us consider, for example, a fuzzy set B of "high-level noise" defined similarly, as earlier, the set of "concentrations of SO_2 in Łódź" on the basis of the investigations of noise carried out on Karolewska Street in Łódź in 1991 during the course of a workday (on the basis of the data from [3]):

hour	6	7	8	9	10	11
*	0.8	0.9	1	0.7	0.1	0.5

12	13	14	15	16
0.4	0.6	0.7	0.8	0.9

17	18	19	20	21
0.5	0.6	0.4	0.3	0.2

* — the degree μ_B of the pitch of noise

Then the entropy $D(B)$ of the fuzzy set thus defined is (for $K = 1$) equal to $D(A) = S(0.8) + S(0.9) + ... + S(0.2) \approx 3.64$ decimal units. This number may be interpreted as the total amount of information carried by the set B. Similarly, for the set B, after calculations we have $E(B) = 10.28$ energy units.

In the case where the phenomenon under examination bears the imprints of randomness and fuzziness at the same time, its indefiniteness is the sum of random indefiniteness (entropy) and fuzzy one (non-probabilistic entropy). Appropriate examples can be found in [2] and [7].

PROBLEM 2

Another aspect of the use of fuzzy set theory in the analysing of natural phenomena can be the notion of a probability of fuzzy events. Since most observations of nature wear a discrete complexion, therefore, reducing the situation to the set Ω of a finite number of elementary observations, i.e. $\Omega = \{x_1, x_2, ..., x_n\}$, with the introduced probability distribution $p_i = P(\{x_i\})$ (for $i = 1, 2, ..., n$ and $\sum_{i=1}^{n} p_i = 1$), we calculate the probability of the fuzzy event A according to L.A. Zadeh's formula to be

$$\widetilde{P}(A) = \sum_{i=1}^{n} \mu_A(x_i) \cdot p_i.$$

To illustrate this question, let us consider a weather observation in a certain locality. We distinguish six conventional weather states $x_1, x_2, ..., x_6$, their annual frequencies being presented by the table

state	x_1	x_2	x_3	x_4	x_5	x_6
frequency	10%	20%	5%	5%	40%	20%

which can be interpreted probabilistically as

state	x_1	x_2	x_3	x_4	x_5	x_6
probability p_i	0.1	0.2	0.05	0.05	0.4	0.2

To each of the states x_i let there correspond a subjective definition C of "fine weather" on the level

state	x_1	x_2	x_3	x_4	x_5	x_6
level($\mu_C(x_i)$)	0.8	0.1	0	1	0.3	0.5.

Now, we want to compute the probability $\widetilde{P}(C)$ of the occurrence of a day with "fine weather". In conformity with the formula for the probability of fuzzy events, we have

$$\widetilde{P}(C) = 0.8 \cdot 0.1 + 0.1 \cdot 0.2 + 0 \cdot 0.05 + 1 \cdot 0.05 + 0.3 \cdot 0.4 + 0.5 \cdot 0.2 = 0.37.$$

Let us notice here that classical approach to this event would give (after accepting the states x_1, x_4 and, perhaps, x_6 to be "fine weather" the result

$$P(C) = 0.1 + 0.05 + 0.2 = 0.35.$$

The difference between the results issues from the fact that the taking account of fuzziness also gives "fine weather" with the states x_2 and x_5 (although "fine" to a small degree), whereas the classical approach absolutely rejects these states.

Conclusion

The apparatus of fuzzy set theory constitutes, as one could be convinced of, a very simple but simultaneously elegant and elastic theory providing facilities for the mathematical formalism of non-sharp phenomena unprecisely defined, phenomena we very often have to do with in natural observations. It adds its owen tools to those of classical mathematics, and thereby increases the reliability of the results being obtained. It finds wide applications in the newest domains of life and also bravely enters into biology and ecology.

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