Petri nets and oriented graphs in fuzzy knowledge representation for DEDS control purposes

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1 Introduction

To control complicated technical systems, especially DEDS (discrete event dynamic systems) - like flexible manufacturing systems, transport systems, communication systems, etc. - a domain oriented knowledge base (KB) is necessary. It usually expresses control task specifications (like different additional demands, restrictions, criteria, etc.) that cannot be simply expressed in analytical terms. This paper is devoted to finding a way how to represent fuzzy knowledge for the DEDS control purposes. It is supported by the author's works [1]-[5]. Two different kinds of such a rule-based representation of fuzzy knowlege are presented here. The first approach utilizes Petri nets and the second one is based on oriented graphs. The approach combining both of them is pointed out too.

2 The Petri net-based approach to the fuzzy knowledge representation

2.1 Understanding the Petri nets

Let us understand the Petri net (PN) structure to be the bipartite oriented graph with two kinds of nodes (positions and transitions) and two kinds of edges (arcs oriented from the positions to the transitions and arcs oriented in the oposite direction)

$$\langle P, T, F, G \rangle \quad ; \quad P \cap T = \emptyset \quad ; \quad F \cap G = \emptyset \tag{1}$$

$P = \{p_1, ..., p_n\}$ is a finite set of the PN positions with p_i, $i = 1, n$, being the elementary positions.

$T = \{t_1, ..., t_m\}$ is a finite set of the PN transitions with t_j, $j = 1, m$, being the elementary transitions.

$F \subseteq P \times T$ is a set of the oriented arcs entering the transitions. It can be expressed by means of the arcs incidence matrix $\mathbf{F} = \{f_{ij}\}$, $i = 1, n$; $j = 1, m$. Its element f_{ij} represents the occurrence of the arc oriented from the position p_i to its output transition t_j.

$G \subseteq T \times P$ is a set of the oriented arcs emerging from the transitions. The arcs incidence matrix is $\mathbf{G} = \{g_{ij}\}$, $i = 1, m$; $j = 1, n$. Its element g_{ij} expresses the occurrence of the arc oriented from the transition t_i to its output position p_j.

To represent the PN "dynamics" (i.e. the marking development) let us consider the quadruplet

$$\langle X, U, \delta, \mathbf{x}_0 \rangle \quad ; \quad X \cap U = \emptyset \tag{2}$$

where

$X = \{\mathbf{x}_0, ..., \mathbf{x}_N\}$ is a set of the state vectors of the PN (the states of the PN marking) with $\mathbf{x}_k = (\sigma_{p_1}^k, ..., \sigma_{p_n}^k)^T$; $k = 0, N$ being the elementary state vectors of the PN in the step k, where k is the discrete step of the PN dynamics development, and $\sigma_{p_i}^k$, $i = 1, n$ is the state of the marking of the elementary positions p_i in the step k. T symbolizes the vector or matrix transposition and N is an integer representing formally the number of different state vectors (all possible markings) during the PN dynamics development.

$U = \{u_0, ..., u_N\}$ is a set of the "control" vectors of the PN (i.e. the state vectors of the PN transitions expressing the enabling of the transitions) with $u_k = (\gamma_{t_1}^k, ..., \gamma_{t_m}^k)^T$; $k = 0, N$ being the elementary "control" vectors of the PN in the step k, where $\gamma_{t_j}^k$, $j = 1, m$ is the state of enabling the elementary transition t_j in the step k.

$\delta : X \times U \longmapsto X$ is a transition function of the PN.

x_0 is the initial state vector of the PN.

The different classes of the PN (ordinary PN (OPN), logical PN (LPN), fuzzy PN (FPN), etc.) must be distinguished as to the "dynamics" (marking development). For example in case of the safety OPN the transition function can be analytically expressed in the form of the following linear discrete system

$$x_{k+1} = x_k + B.u_k \quad , \quad k = 0, N \tag{3}$$

$$B = G^T - F \tag{4}$$

$$F.u_k \leq x_k \tag{5}$$

where

k is the disrete step of the system dynamics development.

x_k is the n-dimensional state vector of the system in the step k. Its components express the states of the OPN elementary positions. Such a component acquires its value from the set $\{0,1\}$ (0 - passivity, when no mark is present in the corresponding position; 1 - activity, when the mark is present).

u_k is the m-dimensional "control" vector of the system in the step k. Its components represent the enabling of the elementary transitions. They acquire their values from the set $\{0,1\}$ (1 - enabling, 0 - disabling).

B, F, G are respectively, $(n \times m)$, $(n \times m)$ and $(m \times n)$- dimensional structural matrices of constant elements expressing the mutual causal relations among the positions and transitions. The matrices F, G were introduced above.

$(.)^T$ 0 symbolizes the matrix or vector transposition

However, the LPN and FPN "dynamics" (defined e.g in [6]) can be uniformly expressed as follows

$$x_{k+1} = x_k \underline{or} B \underline{and} u_k \quad , \quad k = 0, N \tag{6}$$

$$B = G^T \underline{or} F \tag{7}$$
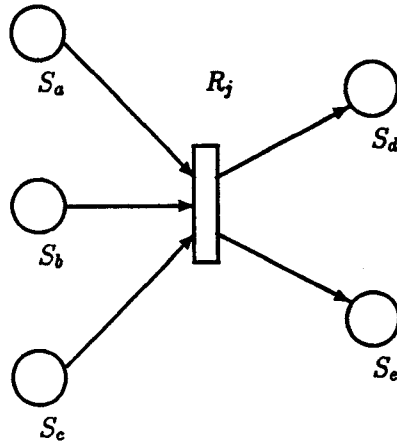
$$F \underline{and} u_k \leq x_k \tag{8}$$

where

and, *or* are, respectively, the operator of logical multiplying and additioning. In both the bivalued logic and the fuzzy one they can be uniformly defined. In scalar case *and* yields the minimum of the scalar operands and *or* yields the maximum of the scalar operands. For vector operands $a = (a_1, ..., a_n)^T$, $b = (b_1, ..., b_n)^T$ the operator *or* yields the vector $c = a \underline{or} b = ((a_1 \underline{or} b_1), ..., (a_n \underline{or} b_n))^T$ - i.e. like in scalar case, however only for corresponding components of the vector operands. The operator *and* gives as the result of two vector operands a, b the scalar d that represents an analogy with the scalar product of two vectors: $d = a^T \underline{and} b = (a_1 \underline{and} b_1) \underline{or} ... \underline{or} (a_n \underline{and} b_n)$.

2.2 The knowledge representation

To represent the KB structure - i.e. the rule-based knowledge in the whole - the analogy between the set of statements (some pieces of knowledge) S_i, $i = 1, n$ and the set of the PN positions is made as well as the analogy between the set of *IF-THEN* rules R_j, $j = 1, m$ and the set of the PN transitions. In addition to this the mutual causal interconnections among the statements and the rules can be understood to be analogical to the mutual causal interconnections among the PN positions and transitions. Consequently, the rule

$$R_j : \quad IF\ (S_a\ and\ S_b\ and\ S_c)\ THEN\ (S_d\ and\ S_e) \tag{9}$$

can be drawn like the fragment of the PN given on Fig. 1

Figure 1: The rule R_j with the input and output statements

The KB "dynamics" will be described by the system expressing uniformly both the LPN and the FPN "dynamics"

$$\mathbf{x}_{k+1} = \mathbf{x}_k \underline{or}\, \mathbf{B}\, \underline{and}\, \mathbf{u}_k \quad , \quad k = 0, N \tag{10}$$

$$\mathbf{B} = \mathbf{G}^T \underline{or}\, \mathbf{F} \tag{11}$$

$$\mathbf{F}\, \underline{and}\, \mathbf{u}_k \leq \mathbf{x}_k \tag{12}$$

where

$\mathbf{x}_k = (\sigma_{S_1}^k, ..., \sigma_{S_n}^k)^T$; $k = 0, N$ is the state vector of the KB truth propagation in the step k. $\sigma_{S_i}^k$, $i = 1, n$ is the state of the truth of the elementary statement S_i in the step k. T symbolizes the vector or matrix transposition and N is an integer representing formally the number of different state vectors occurring during the KB dynamics development.

$\mathbf{u}_k = (\gamma_{R_1}^k, ..., \gamma_{R_m}^k)^T$; $k = 0, N$ is the "control" vector of the KB (expressing the rules enabling - or better the rules evaluability, i.e. the readiness of the rules to be evaluated) in the step k. $\gamma_{R_j}^k$, $j = 1, m$ is the state of enabling the elementary rule R_j to be fired in the step k.

$\underline{and}$, $\underline{or}$ are, respectively, the operator of logical multiplying and additioning mentioned above.

The inference mechanism of the statements truth propagation can be analytically expressed as follows

$$\overline{\mathbf{x}}_k = \underline{neg}\, \mathbf{x}_k = \mathbf{1}_n - \mathbf{x}_k \tag{13}$$

$$\mathbf{v}_k = \mathbf{F}^T \underline{and}\, \overline{\mathbf{x}}_k \tag{14}$$

$$\mathbf{u}_k = \underline{neg}\, \mathbf{v}_k = \mathbf{1}_m - \mathbf{v}_k =$$
$$= \underline{neg}(\mathbf{F}^T \underline{and}\,(\underline{neg}\, \mathbf{x}_k)) \tag{15}$$

where

$\underline{neg}$ is the operator of logical negation.

$\overline{\mathbf{v}}_k$ is a m-dimensional auxiliary vector pointing out (by its nonzero elements) the rules that cannot be evaluated, because there is at least one false statement among its input statements. This declaration is qualified only in the analogy with the LPN. In the analogy with FPN any statement is always true with a fuzzy measure.

$\mathbf{u}_k$ is a m-dimensional "control" vector pointing out the rules that have all their input statements true and, consequently, they can be evaluated in the step k of the KB dynamics development. This vector is a base of the inference, because it contains information about the rules that can contribute to obtaining the new knowledge. These rules correspond to the nonzero elements of the vector $\mathbf{u}_k$. This is also qualified only in the analogy with the LPN. In the analogy with the FPN any rule is always evaluable with a fuzzy measure - i.e. it always contributes to obtaining the new knowledge.

3 The oriented graph-based approach to the fuzzy knowledge representation

Consider the ordinary oriented graph having only one type of nodes and one type of edges (the oriented arcs among the edges). In order to have a possibility to compare this approach with the previous one introduced above, suppose that the nodes correspond with the PN positions and the edges include implicitly the PN transitions (i.e. the transitions are understood to be fixed with the oriented arcs) - see Fig. 2. Hence,

$$\langle P, A \rangle \quad ; \quad P \cap A = \emptyset \tag{16}$$

where

$P = \{p_1, ..., p_n\}$ is a finite set of the PN positions with p_i, $i = 1, n$, being the elementary positions.

$A \subseteq P \times P$ is a set of the oriented arcs among the positions. It can be expressed by the arcs incidence matrix $A = \{a_{ij}\}$, a_{ij}, $i = 1, n$; $j = 1, n$. Its element a_{ij} represents formally the occurrence of the arc oriented from the position p_i to the position p_j.

Because the transitions are defined "implicitly" and $F \subseteq P \times T$ and $G \subseteq T \times P$ we can formally write

$$A \subseteq F \times G \tag{17}$$

or more precisely, the incidence matrix can be derived from the matrix

$$A = G^T . F^T \tag{18}$$

3.1 The knowledge representation

To represent the KB the analogy between the KB statements and the nodes of the oriented graph is made. The KB rules are understood to be fixed with the oriented arcs expressing the causality interconnections among the statements.

The "dynamics" of such a KB can be formally defined like the quadruplet

$$\langle X, A_k, \delta, x_0 \rangle \tag{19}$$

where

$X = \{x_0, ..., x_N\}$, $k = 0, N$ is a set of the n-dimensional state vectors $x_k = (\sigma_{S_1}^k, ..., \sigma_{S_n}^k)^T$; $k = 0, N$ Of the KB

$A_k = \{A_0, ..., A_N\}$, $k = 0, N$ is a set of the k-variant $(n \times n)$-dimensional matrices $A_k = \{a_{ij}^k\}$, $k = 0, N$. Its element $a_{ij}^k = \gamma_{R_{S_j|S_i}}^k$, $i = 1, n$; $j = 1, n$ expresses the causality amonh the statements and the rule (see Fig. 2). This element is k-variant. Its value depends on the state of enabling the corresponding rule to be fired.

$\delta : X \times A_k \longmapsto X$ is a transition function of the KB.

x_0 is the initial state vector of the KB.

There exists a relation between the arcs incidence matrix A and the matrix A_k. It consists in the fact that on the place where the former matrix has nonzero constant element a_{ij} the latter one has the functional element $a_{ij}^k = \gamma_{R_{S_j|S_i}}^k$. In such a k-variant model the states of the KB rules evaluation are "hidden" in the
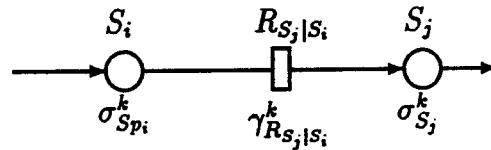

Figure 2: The causality interconnection among two statements

model parameters while the states of the KB statements in the state vector of the KB.

The KB "dynamics" (or better the transition function) can be expressed in analytical terms as follows

$$x_{k+1} = x_k \underline{or} A_k \underline{and} x_k \tag{20}$$

however, because of the mentioned implicitness of the PNs transitions in such an analogy with the KB rules, the inference mechanism (i.e. truth propagation) is also implicitly included into the model. It cannot be explicitly separated outside of the KB model itself. Hence, we have to form the k-variant model of the KB inclusive of the inference mechanism. Such a version of the KB model can be writen as follows

$$\mathbf{x}_{k+1} = \mathbf{x}_k \ \underline{or} \ \mathbf{G}_k^T \ \underline{and} \ (\underline{neg} \ (\mathbf{F}_k^T \ \underline{and} \ (\underline{neg} \ \mathbf{x}_k))) \tag{21}$$

where the sense of matrices is the following

$$\mathbf{A}_k = \mathbf{G}_k^T \ \underline{and} \ \mathbf{F}_k^T \tag{22}$$

$$\mathbf{G}_k = \{g_{rj}^k\}; \ g_{rj}^k = \gamma_{S_j|R_r}^k; \ j = 1, n; \ r = 1, m \tag{23}$$

$$\mathbf{F}_k = \{f_{ir}^k\}; \ f_{ir}^k = \gamma_{R_r|S_i}^k; \ r = 1, m; \ i = 1, n \tag{24}$$

$$\gamma_{R_{S_j|S_i}}^k = \gamma_{S_j|R_r}^k \ \underline{and} \ \gamma_{R_r|S_i}^k \tag{25}$$

4 An illustrative example

Consider a simple KB consisting of 6 rules as follows

R_1: IF $(S_1 \underline{and} S_3)$ THEN S_5 ; R_2: IF S_2 THEN S_3
R_3: IF S_2 THEN S_4 ; R_4: IF S_5 THEN S_6
R_5: IF S_4 THEN S_7 ; R_6: IF S_6 THEN S_8
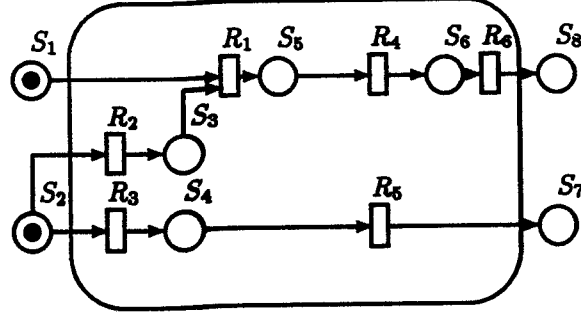
The PNs-based model of the KB is given on Fig. 3

Figure 3: The PNs-based representation of the KB

4.1 The PN-based approach

$$\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{G}_k^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Bivalued logic case: Consider the initial state vector of truth propagation given on Fig. 3.

$$\mathbf{x}_0 = (1, 1, 0, 0, 0, 0, 0, 0)^T$$
$$\underline{neg}\,\mathbf{x}_0 = (0, 0, 1, 1, 1, 1, 1, 1)^T$$
$$\mathbf{F}^T \,\underline{and}\,(\underline{neg}\,\mathbf{x}_0) = (1, 0, 0, 1, 1, 1)^T$$
$$\underline{neg}\,(\mathbf{F}^T \,\underline{and}\,(\underline{neg}\,\mathbf{x}_0)) = (0, 1, 1, 0, 0, 0)^T$$
$$\mathbf{x}_1 = (1, 1, 1, 1, 0, 0, 0, 0)^T$$
$$\underline{neg}\,\mathbf{x}_1 = (0, 0, 0, 0, 1, 1, 1, 1)^T$$
$$\mathbf{F}^T \,\underline{and}\,(\underline{neg}\,\mathbf{x}_1) = (1, 1, 1, 0, 1, 0)^T$$
$$\mathbf{x}_2 = (1, 1, 1, 1, 1, 0, 1, 0)^T$$

$$\vdots \qquad \vdots \qquad \vdots$$

Fuzzy logic case: Consider the initial state vector of truth propagation to be fuzzy.

$$\mathbf{x}_0 = (0.4, 0.7, 0, 0, 0, 0, 0, 0)^T$$
$$\underline{neg}\,\mathbf{x}_0 = (0.6, 0.3, 1, 1, 1, 1, 1, 1)^T$$
$$\mathbf{F}^T \,\underline{and}\,(\underline{neg}\,\mathbf{x}_0) = (1, 0.3, 0.3, 1, 1, 1)^T$$
$$\underline{neg}\,(\mathbf{F}_0^T \,\underline{and}\,(\underline{neg}\,\mathbf{x}_0)) = (0, 0.7, 0.7, 0, 0, 0)^T$$
$$\mathbf{x}_1 = (0.4, 0.7, 0.7, 0.7, 0, 0, 0, 0)^T$$
$$\underline{neg}\,\mathbf{x}_1 = (0.6, 0.3, 0.3, 0.3, 1, 1, 1, 1)^T$$

$$\vdots \qquad \vdots \qquad \vdots$$

4.2 The oriented graph-based approach

The corresponding matrices of parameters are as follows

$$\mathbf{A}_k = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_{R_{S_3|S_2}}^k & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_{R_{S_4|S_2}}^k & 0 & 0 & 0 & 0 & 0 & 0 \\
\gamma_{R_{S_5|S_1}}^k & 0 & \gamma_{R_{S_5|S_3}}^k & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_{R_{S_6|S_3}}^k & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_{R_{S_7|S_4}}^k & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \gamma_{R_{S_8|S_6}}^k & 0 & 0
\end{pmatrix}$$

$$\mathbf{G}_k^T = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_{S_3|R_2}^k & 0 & 0 & 0 & 0 \\
0 & 0 & \gamma_{S_4|R_3}^k & 0 & 0 & 0 \\
\gamma_{S_5|R_1}^k & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_{S_6|R_4}^k & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_{R_{S_7|R_5}}^k & 0 \\
0 & 0 & 0 & 0 & 0 & \gamma_{R_{S_8|R_6}}^k
\end{pmatrix} \ ;$$

$$\mathbf{F}_k^T = \begin{pmatrix}
\gamma_{R_1|S_1}^k & 0 & \gamma_{R_1|S_3}^k & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_{R_2|S_2}^k & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma_{R_3|S_2}^k & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_{R_4|S_5}^k & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_{R_5|S_4}^k & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \gamma_{R_6|S_6}^k & 0 & 0
\end{pmatrix}$$

Bivalued logic case: Consider the initial state vector of truth propagation given on Fig. 3. Of course, the full evaluability of rules is supposed (i.e. the values of the functions $\gamma^k_{R_{S_i|S_j}} \in \{0, 1\}$).

$$\mathbf{x}_0 = (1, 1, 0, 0, 0, 0, 0, 0)^T$$

$$neg\,\mathbf{x}_0 = (0, 0, 1, 1, 1, 1, 1, 1)^T$$

$$\mathbf{F}_0^T \underline{and}\,(\underline{neg}\,\mathbf{x}_0) = (\gamma^0_{R_1|S_3}, 0, 0, \gamma^0_{R_4|S_5}, \gamma^0_{R_5|S_4}, \gamma^0_{R_6|S_6})^T$$

$$\underline{neg}\,(\mathbf{F}_0^T \underline{and}\,(\underline{neg}\,\mathbf{x}_0)) = (\underline{neg}\,\gamma^0_{R_1|S_3}, 1, 1, \underline{neg}\,\gamma^0_{R_4|S_5}, \underline{neg}\,\gamma^0_{R_5|S_4}, \underline{neg}\,\gamma^0_{R_6|S_6})^T =$$

$$= (0, 1, 1, 0, 0, 0)^T$$

$$\mathbf{G}_0^T \underline{and}\,(\underline{neg}\,(\mathbf{F}_0^T \underline{and}\,(\underline{neg}\,\mathbf{x}_0))) = (0, 0, \gamma^0_{S_3|R_2}, \gamma^0_{S_4|R_3}, \gamma^0_{S_5|R_1} \underline{and}\,(\underline{neg}\,\gamma^0_{R_1|S_3}), \gamma^0_{S_6|R_4} \underline{and}\,(\underline{neg}\,\gamma^0_{R_4|S_5}),$$
$$\gamma^0_{S_7|R_5} \underline{and}\,(\underline{neg}\,\gamma^0_{R_5|S_4}), \gamma^0_{S_8|R_6} \underline{and}\,(\underline{neg}\,\gamma^0_{R_6|S_6}))^T$$

$$\mathbf{x}_1 = (1, 1, \gamma^0_{S_3|R_2}, \gamma^0_{S_4|R_3}, \gamma^0_{S_5|R_1} \underline{and}\,(\underline{neg}\,\gamma^0_{R_1|S_3}), \gamma^0_{S_6|R_4} \underline{and}\,(\underline{neg}\,\gamma^0_{R_4|S_5}),$$
$$\gamma^0_{S_7|R_5} \underline{and}\,(\underline{neg}\,\gamma^0_{R_5|S_4}), \gamma^0_{S_8|R_6} \underline{and}\,(\underline{neg}\,\gamma^0_{R_6|S_6}))^T =$$

$$= (1, 1, 1, 1, 0, 0, 0, 0)^T$$

$$neg\,\mathbf{x}_1 = (0, 0, 0, 0, 1, 1, 1, 1)^T$$

$$\mathbf{F}_1^T \underline{and}\,(\underline{neg}\,\mathbf{x}_1) = (0, 0, 0, \gamma^1_{R_4|S_5}, 0, \gamma^1_{R_6|S_6})^T$$

$$\underline{neg}\,(\mathbf{F}_1^T \underline{and}\,(\underline{neg}\,\mathbf{x}_1)) = (1, 1, 1, \underline{neg}\,\gamma^1_{R_4|S_5}, 1, \underline{neg}\,\gamma^1_{R_6|S_6})^T =$$

$$= (1, 1, 1, 0, 1, 0)^T$$

$$\mathbf{G}_1^T \underline{and}\,(\underline{neg}\,(\mathbf{F}_1^T \underline{and}\,(\underline{neg}\,\mathbf{x}_1))) = (0, 0, \gamma^1_{S_3|R_2}, \gamma^1_{S_4|R_3}, \gamma^1_{S_5|R_1}, \gamma^1_{S_6|R_4} \underline{and}\,(\underline{neg}\,\gamma^1_{R_4|S_5}),$$
$$\gamma^1_{S_7|R_5}, \gamma^1_{S_8|R_6} \underline{and}\,(\underline{neg}\,\gamma^1_{R_6|S_6}))^T$$

$$\mathbf{x}_2 = (1, 1, \gamma^1_{S_3|R_2}, \gamma^1_{S_4|R_3}, \gamma^1_{S_5|R_1}, \gamma^1_{S_6|R_4} \underline{and}\,(\underline{neg}\,\gamma^1_{R_4|S_5}),$$
$$\gamma^1_{S_7|R_5}, \gamma^1_{S_8|R_6} \underline{and}\,(\underline{neg}\,\gamma^1_{R_6|S_6}))^T =$$

$$= (1, 1, 1, 1, 1, 0, 1, 0)^T$$

$$\vdots \qquad \vdots \qquad \vdots$$

Fuzzy logic case: Consider the initial state vector of truth propagation to be fuzzy and the full evaluability of the rules.

$$\mathbf{x}_0 = (0.4, 0.7, 0, 0, 0, 0, 0, 0)^T$$

$$neg\,\mathbf{x}_0 = (0.6, 0.3, 1, 1, 1, 1, 1, 1)^T$$

$$\mathbf{F}_0^T \underline{and}\,(\underline{neg}\,\mathbf{x}_0) = (0.6 \underline{and}\,\gamma^0_{R_1|S_1} \underline{or}\,\gamma^0_{R_1|S_3}, 0.3 \underline{and}\,\gamma^0_{R_2|S_2}, 0.3 \underline{and}\,\gamma^0_{R_3|S_2}, \gamma^0_{R_4|S_5},$$
$$\gamma^0_{R_5|S_4}, \gamma^0_{R_6|S_6})^T$$

$$\underline{neg}\,(\mathbf{F}_0^T \underline{and}\,(\underline{neg}\,\mathbf{x}_0)) = (\underline{neg}\,(0.6 \underline{and}\,\gamma^0_{R_1|S_1} \underline{or}\,\gamma^0_{R_1|S_3}), \underline{neg}(0.3 \underline{and}\,\gamma^0_{R_2|S_2}),$$
$$\underline{neg}(0.3 \underline{and}\,\gamma^0_{R_3|S_2}), \underline{neg}\,\gamma^0_{R_4|S_5}, \underline{neg}\,\gamma^0_{R_5|S_4}, \underline{neg}\,\gamma^0_{R_6|S_6})^T =$$

$$= (0, 0.7, 0.7, 0, 0, 0)^T$$

$$\mathbf{G}_0^T \underline{and}\,(\underline{neg}\,(\mathbf{F}_0^T \underline{and}\,(\underline{neg}\,\mathbf{x}_0))) = (0, 0, \gamma^0_{S_3|R_2} \underline{and}\,(\underline{neg}(0.3 \underline{and}\,\gamma^0_{R_2|S_2})), \gamma^0_{S_4|R_3} \underline{and}\,(\underline{neg}(0.3 \underline{and}\,\gamma^0_{R_3|S_2})),$$
$$\gamma^0_{S_5|R_1} \underline{and}\,(\underline{neg}\,(0.6 \underline{and}\,\gamma^0_{R_1|S_1} \underline{or}\,\gamma^0_{R_1|S_3})),$$
$$\gamma^0_{S_6|R_4} \underline{and}\,(\underline{neg}\,\gamma^0_{R_4|S_5}), \gamma^0_{S_7|R_5} \underline{and}\,(\underline{neg}\,\gamma^0_{R_5|S_4}), \gamma^0_{S_8|R_6} \underline{and}\,(\underline{neg}\,\gamma^0_{R_6|S_6}))^T$$

$$\mathbf{x}_1 = (0.4, 0.7, \gamma^0_{S_3|R_2} \underline{and}\,(\underline{neg}(0.3 \underline{and}\,\gamma^0_{R_2|S_2})), \gamma^0_{S_4|R_3} \underline{and}\,(\underline{neg}(0.3 \underline{and}\,\gamma^0_{R_3|S_2})),$$
$$\gamma^0_{S_5|R_1} \underline{and}\,(\underline{neg}\,(0.6 \underline{and}\,\gamma^0_{R_1|S_1} \underline{or}\,\gamma^0_{R_1|S_3})), \gamma^0_{S_6|R_4} \underline{and}\,(\underline{neg}\,\gamma^0_{R_4|S_5}),$$
$$\gamma^0_{S_7|R_5} \underline{and}\,(\underline{neg}\,\gamma^0_{R_5|S_4}), \gamma^0_{S_8|R_6} \underline{and}\,(\underline{neg}\,\gamma^0_{R_6|S_6}))^T =$$

$$= (0.4, 0.7, 0.7, 0.7, 0, 0, 0, 0)^T$$

$$neg\,\mathbf{x}_1 = (0.6, 0.3, 0.3, 0.3, 1, 1, 1, 1)^T$$

$$\vdots \qquad \vdots \qquad \vdots$$

5 The combined approach to the knowledge representation

Consider the approach combining both the PN-based approach and the oriented graph-based one. In such an approach a part of the KB will be represented by means of the PN and the remainder of the KB will be represented by means of the oriented graph. Consequently, the KB structure can be formally described as follows

$$\langle P,T,\Delta,F,G \rangle \quad ; \quad P \cap T = \emptyset \quad ; \quad F \cap G = \emptyset \tag{26}$$

where

$P = \{p_1,...,p_n\}$ is a finite set of the KB statements S_i, $i = 1,n$, being the elementary statements.

$T = \{t_1,...,t_m\}$ is a finite set of the KB rules with R_j, $j = 1,m$, being the elementary rules.

$\Delta \subseteq P \times P$ is a set of the oriented arcs among the nodes of the oriented graph representing the part of the KB. It can be expressed by the arcs incidence matrix $\Delta = \{\delta_{ij}\}$, $\delta_{ij} \in \{0,1\}$, $i = 1,n$; $j = 1,n$. Its element δ_{ij} represents the absence (when 0) or presence (when 1) of the arc oriented from the statement S_i to the statement S_j.

$F \subseteq P \times T$ is a set of the oriented arcs entering the rules (it is concerning the part of the KB represented by means of the PN). It can be expressed by the arcs incidence matrix $\mathbf{F} = \{f_{ij}\}$, $f_{ij} \in \{0,1\}$, $i = 1,n$; $j = 1,m$. Its element f_{ij} represents the absence (when 0) or presence (when 1) of the arc oriented from the input statement S_i to the rule R_j.

$G \subseteq R \times S$ is a set of the oriented arcs emerging from the rules (it is concerning the part of the KB represented by means of the PN). The arcs incidence matrix $\mathbf{G} = \{g_{ij}\}$, $g_{ij} \in \{0,1\}$, $i = 1,m$; $j = 1,n$ expresses the occurrence of the arc oriented from the rule R_i to its output statement S_j.

Hence, the linear discrete dynamic model of the DES can be written as follows

$$\mathbf{x}_{k+1} = \mathbf{A} \underline{and} \mathbf{x}_k \underline{or} \mathbf{B} \underline{and} \mathbf{u}_k \quad , \quad k = 0,N \tag{27}$$

$$\mathbf{A} = \mathbf{I}_n + \Delta \tag{28}$$

$$\mathbf{B} = \mathbf{G}^T \underline{or} \mathbf{F} \tag{29}$$

$$\mathbf{F}.\mathbf{u}_k \leq \mathbf{x}_k \tag{30}$$

where

k is the disrete step of the KB dynamics development.

$\mathbf{x}_k = (\sigma_{S_1}^k,...,\sigma_{S_n}^k)^T$; $k = 0,N$ is the n-dimensional state vector of the KB statement truth propagation in the step k; $\sigma_{S_i}^k$, $i = 1,n$ is the state of the truth of the elementary statement S_i in the step k (1 - true, 0 - false).

$\mathbf{u}_k = (\gamma_{R_1}^k,...,\gamma_{R_m}^k)^T$; $k = 0,N$ is the m-dimensional "control" vector of the KB in the step k (the rules enabling - i.e. the ability of the rules to be evaluated; $\gamma_{R_j}^k$, $j = 1,m$ is the state of enabling the rule R_j in the step k (1 - enabled, 0 - disabled).

$\mathbf{A}$ is the $(n \times n)$-dimensional system matrix expressing the causal relations between the statements belonging to the corresponding part of the KB (that represented by the oriented graph). Its elements represent the states of enabling the corresponding rules.

$\mathbf{B}$ is the $(n \times m)$-dimensional structural matrix of constant elements expressing the causal relations between the statements and rules concerning the part of the KB represented by the PN. It is given by means of the matrices $\mathbf{F}$, $\mathbf{G}$ defined above.

T symbolizes the matrix or vector transposition.

Such a model of the KB utilizes advantages of both the PN-based model and the oriented graph-based one.

5.1 An example of the combined approach

The combined approach is especially suitable for cases where a spontaneous evaluation of some rules is possible. For example it is the case of two-way implication. Consider the KB as follows

R_1: IF S_1 THEN S_2 ; R_2: IF S_2 THEN S_3

R_3: IF S_3 THEN S_1 ; R_4: IF S_1 THEN S_4

R_5: IF S_5 THEN S_7 ; R_6: IF S_5 THEN S_1
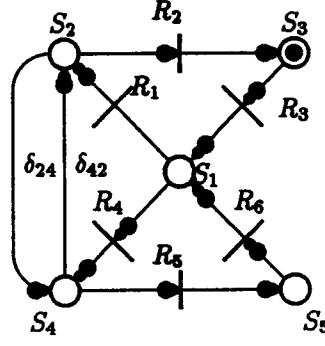
Figure 4: The combine representation of the KB

R_7: IF S_2 THEN S_4 ; R_8: IF S_4 THEN S_2

where the last rules R_7 and R_8 represent the two-way implication. The graphical expression of the KB respecting the combine approach is given on Fig. 4

The matrix A is given as follows

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The matrices F and G are simpler: $n = 5$ $m = 6$

$$F = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad G = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Bivalued logic case: Consider the initial state vector of truth propagation to be

$$
\begin{aligned}
x_0 &= (0, 0, 1, 0, 0)^T \\
\underline{neg}\, x_0 &= (1, 1, 0, 1, 1)^T \\
F^T \underline{and}\,(\underline{neg}\, x_0) &= (1, 1, 0, 1, 1, 1) \\
\underline{neg}\,(F^T \underline{and}\,(\underline{neg}\, x_0)) &= (0, 0, 1, 0, 0, 0)^T \\
x_1 &= (1, 0, 1, 0, 0)^T \\
\underline{neg}\, x_1 &= (0, 1, 0, 1, 1)^T \\
F^T \underline{and}\,(\underline{neg}\, x_1) &= (0, 1, 0, 0, 1, 1)^T \\
\underline{neg}\,(F^T \underline{and}\,(\underline{neg}\, x_1)) &= (1, 0, 1, 1, 0, 0^T \\
x_2 &= (1, 1, 1, 1, 0)^T
\end{aligned}
$$

$$\vdots \qquad \vdots \qquad \vdots$$

Fuzzy logic case: Consider the initial state vector of truth propagation to be fuzzy.

$$
\begin{aligned}
x_0 &= (0, 0, 0.7, 0, 0)^T \\
\underline{neg}\, x_0 &= (1, 1, 0.3, 1, 1)^T \\
F^T \underline{and}\,(\underline{neg}\, x_0) &= (1, 1, 0.3, 1, 1, 1)
\end{aligned}
$$

$$
\begin{aligned}
\underline{neg}\,(\mathbf{F}^T\,\underline{and}\,(\underline{neg}\,\mathbf{x}_0)) &= (0,0,,0.7,0,0,0)^T \\
\mathbf{x}_1 &= (0.7,0,0.7,0,0)^T \\
\underline{neg}\,\mathbf{x}_1 &= (0.3,1,0.3,1,1)^T \\
\mathbf{F}^T\,\underline{and}\,(\underline{neg}\,\mathbf{x}_1) &= (0.3,1,0.3,0.3,1,1)^T \\
\underline{neg}\,(\mathbf{F}^T\,\underline{and}\,(\underline{neg}\,\mathbf{x}_1)) &= (0.7,0,0.7,0.7,0,0^T \\
\mathbf{x}_2 &= (0.7,0.7,0.7,0.7,0)^T
\end{aligned}
$$

$$
\vdots \qquad \vdots \qquad \vdots
$$

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