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Fuzzy Modelling of Mortgage Loans

Danuše Bauerová

Abstract

This paper shows possibilities of creating **fuzzy analogies in area of financial mathematics**. Attention is given to annuities as a system of fuzzy cash flows, and in particular to calculation of **amount of periodical mortgage payment at vagueness.**

Key words:

Fuzzy numbers, fuzzy present value of annuity, fuzzy periodic payment.

1. Introduction

Mathematical methods have become at present a **powerful tool** in the process of acquiring knowledge due to highly effective computers. However, in general we very often encounter the problem in **mathematization** of reality which is not as precise as mathematical methods themselves, but is only **vague** without possibility to be described with absolute precision. The above-mentioned imprecision could, to a certain extent, be eliminated by a fuzzy approach to solution. Fuzzy approach to uncertainty can be an alternative of probabilistic models that have been wide-spread in the world business community but have not been received without criticism.

Fuzzy set theory is considered to be a very valuable contribution to existing knowledge. Today, its application in areas of **fuzzy control, multi-criteria analysis, descriptive decision theory** as well as **expert systems** is entirely common. However, in **economics** applications of the fuzzy set theory are still very **rare.** Though it is obvious that economics and especially microeconomics study the world in which **uncertainty** and **vagueness** play an important role. It is necessary to emphasize that subject of study of economics are not only economical objects (goods), but also economical **subjects** (agents) (see [4], pg. 1). This fact unavoidably causes a contradiction. The contradiction in the sense, that a theory (for instance a theory of consumer), which requires a strict unification of subject's behaviour, does not correspond with his real behaviour. This approach allows only a certain type of rationality, which forbids differences. Such economic subjects are then deprived of their basic freedom: they could not have behaviour different of their neighbours. A question arises: what remains of a theory if consumers themselves (agents) are introduced into analyses without respecting usual standards of behaviour?

It is obvious from the above-mentioned that the observed reality is not precise. The reason is that the observed objects or subjects (or both) **simply do not behave in such a unified manner**. However, important fact which affects unavoidably the process of knowledge acquisition is the fact that the description of subject's behaviour and the language used in the description is imprecise.

In addition to the above-mentioned consumer theory there are other areas of economics in which **disproportions** occur between **inexactness of input data and their subsequent precise processing**. We are faced with the problem of application of "precise methods on imprecise data" in each step. The endeavour to manage them leads to emergence of works which **are not aimed at development of fuzzy set theory**, but problems in them, for instance economic ones, enter into a model as a **predominant factor** and **fuzzy set theory is only a tool for improvement of economic models and theory**.

One of such areas seems to be **the mathematics of finance** where it is possible to create **fuzzy analogues** of the elementary compound interest problems as an **alternative** to the use exact **amounts for interest rates, cash amounts and time periods**.

For example such statement as "approximately between 14 000 Kč and 18 000 Kč" has to be turned into exact amount, for instance "16 000 Kč" before it can be used in any financial formula. To case with the vagueness, we may use fuzzy numbers. For example, it is possible to determine the **fuzzy** amount in an account, if **around** A Kč is invested for about n periods at **approximately** r percent interest per period, or it is possible to create the **fuzzy** future value and **fuzzy** present value. (see [1]).

2. Topicality of model

In the Czech Republic the institution of **mortgage loans** is only being prepared at present time. Discussions among experts are abundant, but they all have one thing in common: **uncertainty** and **vagueness** of estimates of values entering into theoretical considerations or even into exact calculations.

There is for instance a prerequisite that banks should create cheap financial resource of long-term crediting for this purpose. But how cheap such money can be, what will be a concrete number characterising "long-term" to guarantee, at the same time, the acceptable return for lenders (creditors)? What will be the size of state´s contribution in interest payment of bond certificates? And finally, how much can the person interested in obtaining a mortgage loan afford to borrow with respect to his/her family budget? At present (when a lot on both sides of future financial transactions is already being decided), all such questions can be answered **only with uncertainty and vagueness**.

This uncertainty **will continue** even in the period following the start-up of these loans because a great number of subjective, changeable with time, influences will still be in effect on either the borrower´s or lender´s side (for instance it is certain that a possibility to change interest rates during amortisation of debt will be included in contract). It is important for the borrower to estimate well his/her possibilities to assure a certain security in the future. On the other hand it is important for a creditor to enable only such attractiveness of credits, which for themthelves would guarantee the effectiveness sufficient for financing of mortgage loans by registered mortgage bonds, but which will ensure their successful sale.

There are so many variables entering into play that **required foundations for future decision making cannot be in any case secured without exact calculations**. The above-mentioned fuzzy approach is a suitable tool which enables us to join vagueness of input characteristics with exactness of calculations based on them.

3. Classical approach

Let us first pay attention to the necessary parts of classical financial mathematics and secondly to their fuzzy analogies.

Mortgage loan is to be classified as
real annuity
with payments at the **end** of payment period where
payment period **complies** with interest period (see [6], p. 43).

Therefore, the mortgage payments can be viewed as a **system of financial flows** the regularity of which enables us to arrive at simple **explicit formulas** for current or final value of annuity. These formulas then **can be used** with advantage in various financial decisions concerning these flows (for instance determination of their size, number, selection of optimal system of instalments etc.) both for requirements of mortgage lender and borrower.

Let us consider mortgage payments to be a sure annuity, however, the specific nature of this annuity - its considerable uncertainty - cannot be neglected. In general, this uncertainty is caused especially by the **long-term nature** of financial flows at changing conditions of economic reality. In the conditions our national economy is presently in, the uncertainty is yet multiplied by vagueness of every **new project**, because in our country a model of mortgage loans is still only being developed.

Let us recall **classical explicit formulas** in the area of financial mathematics:

Suppose the present value of a regular annuity A_n **is the present value of one adequate to this cash flow**, i. e.

- A_n is the **present value** of this future cash flow.
  - If
    - r is the **interest rate** per period in decimal notation, and $v = \frac{1}{1+r}$,
    - P is **equal to periodic payment** made at the **end** of each payment period for **n** periods,
  - then

$$A_n = P \cdot v + P \cdot v^2 + \ldots + P \cdot v^n = P \cdot \frac{1 - v^n}{r} = P \cdot \gamma \text{, i. e.}$$

$$A_n = P \cdot \gamma \qquad (1)$$

where the **actuarial function** γ is

$$\gamma\ (n, r) = \frac{1 - v^n}{r} = \frac{1 - (1+r)^{-n}}{r} = \frac{(1+r)^n - 1}{(1+r)^n \cdot r}, \text{ (note)}^1 \qquad (2)$$

[1] Current value of A_n:

Our considerations will be now limited to the analysis of mortgage loans **in view of the amount of monthly payments**.

From the formula (1) the value of equivalent periodical payment P can be expressed as the quotient

$$P = \frac{A_n}{\gamma}. \tag{3}$$

Let us denote $\frac{1}{\gamma} = \delta\,(n, r)$, then

$$P = A_n \,.\, \delta, \qquad \text{where} \qquad \delta = \frac{(1+r)^n \,.\, r}{(1+r)^n - 1}. \tag{4}$$

4. Fuzzy approach

Assume the **fuzzy amount of equal periodic payments $\underline{P}_n$** , i.e. the amount of payments is determined in the fuzzy environment.

Symbols applied in review:

Mathematics of finance		Fuzzy mathematics of finance	
Symbol		Fuzzy set	Membership function
A	Cash amount in the present	$\underline{A}$	$\mu(x \mid \underline{A})$
P	Equal periodic payments	$\underline{P}$	$\mu(x \mid \underline{P})$
r	Interest rate per period	$\underline{r}$	$\mu(x \mid \underline{r})$
n	Number of interest periods	$\underline{n}$	$\mu(x \mid \underline{n})$

The fuzzy sets **$\underline{A}$, $\underline{P}$, $\underline{r}$** are usually **fuzzy numbers**, but **$\underline{n}$** is a **discrete positive fuzzy subset** of the **real** numbers (see 7. Appendix).

4.1 Number of interest periods determined exactly

Now a fuzzy present value **$\underline{A}_n$** is presumed. **$\underline{A}_n$** is defined as current value of the corresponding system of n payments related to first interest period. The payments **$\underline{P}_n$** are always made at the end of the interest period during n (real number) periods at fuzzy interest rate **$\underline{r}$**. The payment **$\underline{P}_n$** is equal to

$$\underline{P}_n = \underline{A}_n \otimes \left\{ (1 \oplus \underline{r})^{-1} \oplus (1 \oplus \underline{r})^{-2} \oplus K \oplus (1 \oplus \underline{r})^{-n} \right\}^{-1}. \tag{6}$$

System of payments a_1, a_2, ...a_n can be considered as a **geometric progression** with the first member $a_1 = P\,.v$ and the coefficient $q = v$. The current value of A_n can then be determined as **the sum of n members of the progression.**

The membership function $\mu(x \mid \underline{P}_n)$ for $\underline{P}_n$ is determined by

$$f_{ni}(y \mid \underline{P}_n) = f_i(y \mid \underline{A}_n)\,\delta(n; f_i(y \mid \underline{r}), \tag{7}$$

for $i = 1, 2$ and where

$$p_{n1} = f_{n1}(0 \mid \underline{P}_n), \quad p_{n2} = f_{n1}(1 \mid \underline{P}_n),$$
$$p_{n3} = f_{n2}(1 \mid \underline{P}_n), \quad p_{n4} = f_{n2}(0 \mid \underline{P}_n).$$

If $\delta(n, r)$) is an increasing function r, it can be verified that
$f_{n1}(y \mid \underline{P}_n)$ is increasing function,
$f_{n2}(y \mid \underline{P}_n)$ is decreasing function and
$p_{n2} \leq p_{n3}$.

Therefore it can be asserted that $\underline{P}_n$ is a fuzzy number.

4. 2. Number of interest periods determined vaguely

If the number of interest periods $\underline{n}$ is fuzzy, then the size of payments must be modelled using the **extension principle** (see [7], pg. 37) in equation (4):

Let $\underline{P}$ be the amount deposited at the end of each interest period with the membership function $\mu(x \mid \underline{P})$. Then

$$\mu(x \mid \underline{P}) = \sup_{\{(u,v,w) \mid u\delta(w,v) = x\}} \min\{\mu(u \mid \underline{A}), \mu(v \mid \underline{r}), \mu(w \mid \underline{n})\}. \tag{8}$$

It can be proven that $\mu(x \mid \underline{P}) = \mu(x \mid \underline{P}_n)$ holds for $\underline{n} = \mathbf{n}$. This assertion implies that

$$\mu(x \mid \underline{P}) = \max_{1 \leq i \leq K}\left\{\min\left(\mu\left(x \mid \underline{P}_{\mathbf{n}_i}\right), \lambda_i\right)\right\}. \tag{9}$$

Thus, according to the previous formulas, $\mu(x \mid \underline{P})$ can be simply found from equation (9). Equation (9) means that, at first, $\underline{P}_{ni}$ will be found from equation (6), **cut at the level** of λ_i and then **maximum** of these fuzzy sets will be taken. The $\underline{P}$ is not necessarily fuzzy number (see[1], pg. 261). Finding of $\underline{P}$ is illustrated by the following example.

5. Examples of determination of fuzzy value of periodical payment

The situation will be presented in **two examples** (5.1., 5.2.) based on

- equal **present values** of mortgage loan $\underline{A}_n$ (about million Kč),
- paid in **monthly** payments during the same amortisation period $\underline{n}$ (of about 18 years) but
- for **different** annual interest **rates** $\underline{r}^*$ (about 6 % p.a., and then about 9 % p.a.) to which their **twelfths** correspond to **monthly** interest rates $\underline{r}$.

Then the value of **fuzzy periodic payment** $\underline{P}$ will be determined.

5.1 Example 1

Let the membership functions of individual fuzzy numbers be given as:

- Present value of the mortgage loan, therefore the amount which the debtor shall obtain is $\underline{A}$ = (950000 / 1000000, 1000000 / 1100000)
- at interest rate p.a.
  $\underline{r}^*$ = (0.055 / 0.06, 0.065 / 0.07) p.a.
- for the period of about 18 years, i.e. for the number of periods (months)
  $\underline{n}$ = (180/216,216/240) with membership function μ (n_i | $\underline{n}$) given in the table

n_i	180 (15 years)	216 (18 years)	240 (20 years)
μ (n_i | $\underline{n}$)	0.6	1	0.8

Using formula (7) for n_i = 180, 216, 240, μ (x | $\underline{P}_{ni}$) is the following:

n_i	$\underline{P}_{ni}$
180	(7762 / 8439, 8711 / 9887)
216	(6938 / 7582, 7866 / 8971)
240	(6535 / 7164, 7456 / 8528)

These results can be applied to formulate μ(x | $\underline{P}$) based on equation (9). All is represented graphically (see **fig. a**).

5.2 Example 2

Input values from example 1 **differ** only in p.a. interest rate which is in this case is
$\underline{r}^*$ = (0.085 / 0.09, 0.095 / 0.10) p.a.
The obtained values are substantially different in comparison with the final values from example 1:

n_i	$\underline{P}_{ni}$
180	(9355 / 10143, 10442 / 11821)
216	(8602 / 9364, 9679 / 10998)
240	(8244 / 8997, 9321 / 10615)

This is illustrated graphically (see **fig. b**).

Comparison:

- While the value of the monthly payment at interest rate of about 6% was **about 8000 Kč**, at interest rate of about 9% it reached **about 10000 Kč**.
- We might be puzzled by the **considerable fuzziness** of result (for instance in example 1 from 6535 Kč up to 9887 Kč). It is determined by the immense uncertainty of the amortisation period of mortgage loan.
  - Whole project of mortgage loan modelling would greatly benefit from clients´ greater certainty in the length of mortgage loan amortisation period.

- Let us illustrate this and other considerations in the following model situations.

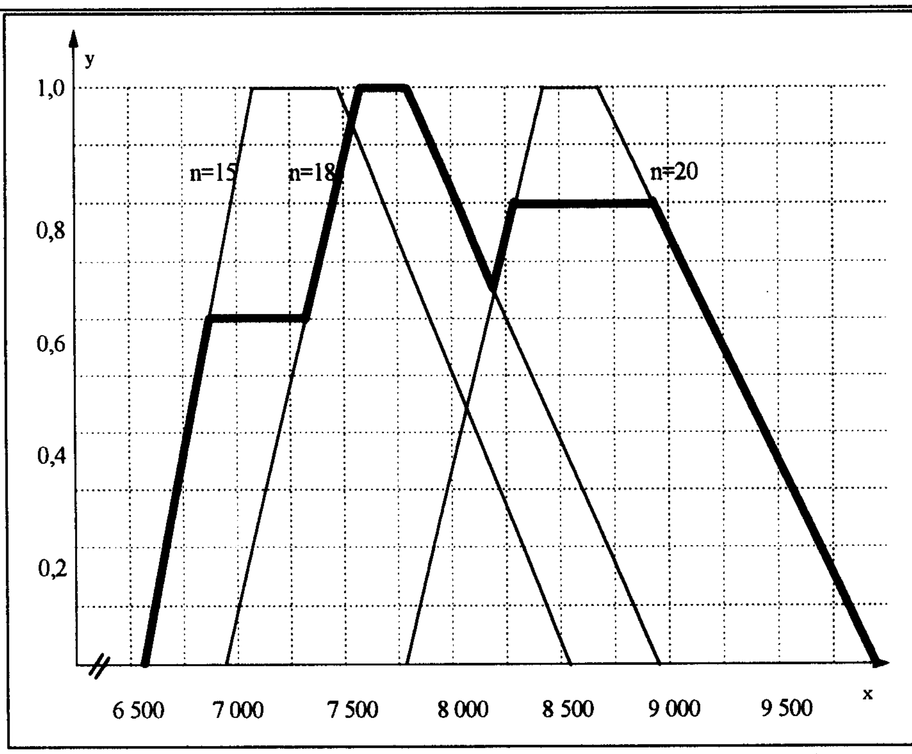

Fig. A: Membership function of fuzzy payment **P** for interest rate "**approximately 6%**" p.a.

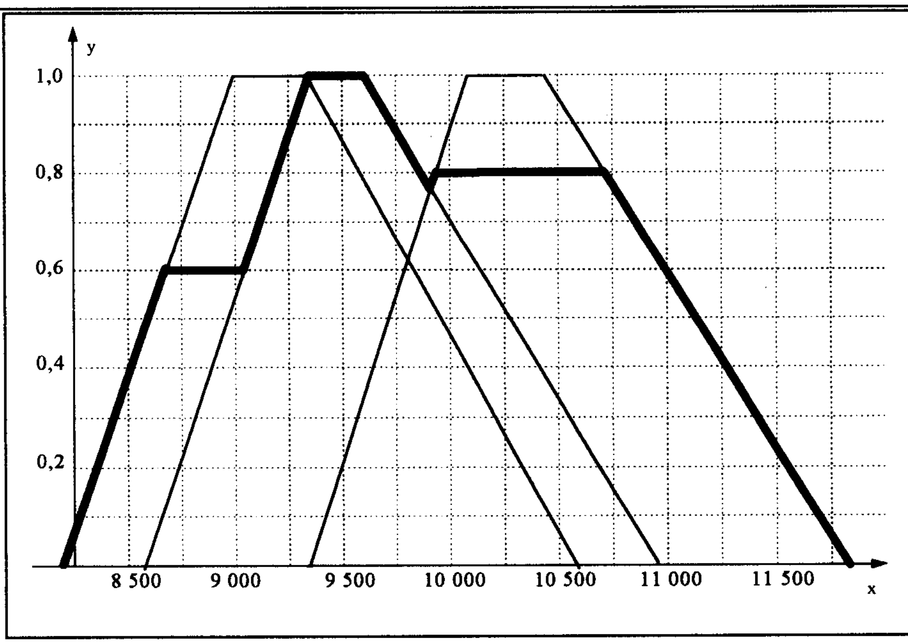

Fig. B: Membership function of payment **P** for interest rate "**approximately 9%**" p.a.

6. Solutions of selected problems of mortgage loans based on fuzzy approach

6.1 Reduction of fuzziness of amortisation period

As already the creditors will weigh carefully what values of individual inputs could guarantee stressed them a required effectiveness of the granted loans so that a possibility of their financing by registered mortgage bonds could by secured. One of the items necessary to be determined in this process is the amortisation period. It can be proven that a change of the membership function μ (n_i | **n**) **by ability of banks to increase the certainty** and thus to reduce fuzziness when determining the number of payment periods in which they allow their clients to repay loans, results in **the reduction of fuzziness** of the resulting membership function of payment **P** to acceptable level.

All input data will remain the same as in example 1, except for the **fuzziness of determination of the number of interest periods** which **will be reduced**, i.e. the statement of "up to twenty years" will be defined more sharply:

n_i	216 (18 years)	228 (19 years)	240 (20 years)
μ (n_i | **n**)	0.6	1.0	0.8

Now the fuzzy payment **P**, which corresponds to situation with smaller fuzziness in amortisation period, will be determined according to formulas (7), (9):

n_j	P_{nj}
216	(6938 / 7582, 7866 / 8971)
228	(6725 / 7361, 7649 / 8736)
240	(6535 / 7164, 7456 / 8528)

The results are represented graphically (see **fig. c**).

Comparison:

- In example 1 the fuzziness of amortisation period was 5 years and fuzziness of the result was very considerable **3352** Kč.
- Now the fuzziness of amortisation period was reduced to 2 years and also the fuzziness of the periodic payment was reduced to acceptable level, i.e. to **2436** Kč.
- Comparison might be interesting not only in absolute amounts, but also in relative ones.

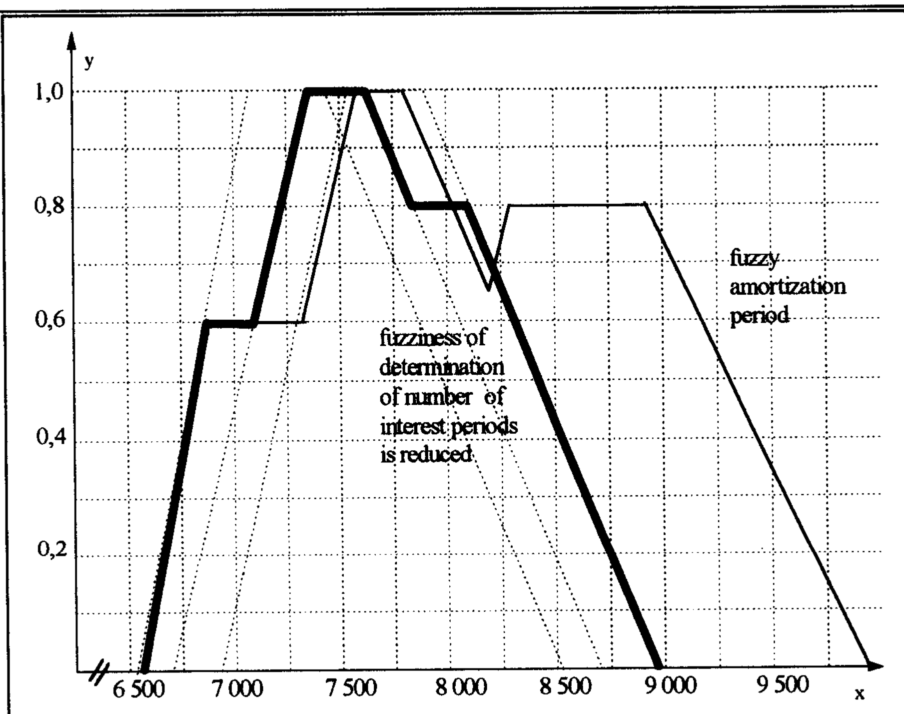

Fig. C: At sharper determination of amortisation period "up to 20 years" the fuzziness of resulting periodic payment will be reduced considerably.

6.2 Modelling of payments for different lengths of amortisation periods

- Let the amortisation period be determined exactly, i.e.
  a) **15** years, b) **20** years.

- Let the same fuzzy interest rate of about **6%** and an identical fuzzy present value of loan of about **1 million** Kč remain in both cases.

- A **fuzzy value of the periodical payment** will be sought.

According to formulas (6), (7), the fuzzy payment $\underline{\mathbf{P}}_n$ can be determined which is directly the fuzzy payment $\underline{\mathbf{P}}$. It will be determined separately for each case a) and b). The results are summarised in the following table and they are represented graphically (see **fig. d**).

n	$\underline{\mathbf{P}}_n$
180 (15 years)	(7762 / 8439, 8711 / 9887)
240 (20 years)	(6535 / 7164, 7456 / 8528)

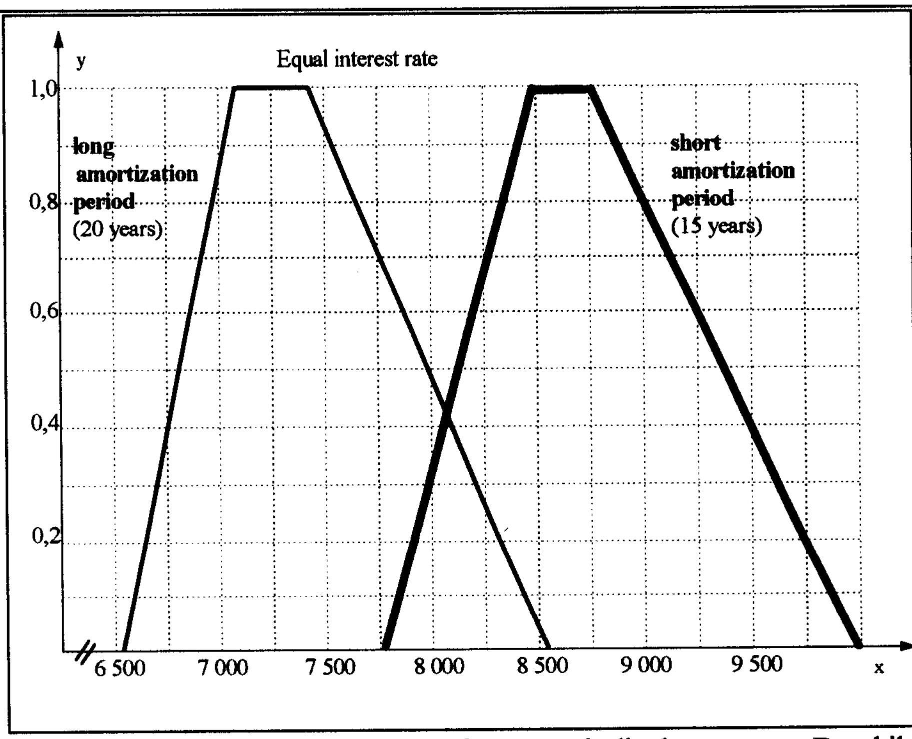

Fig. D: Extension of amortisation period will reduce fuzzy periodical payment $\underline{\mathbf{P}}$ while fuzziness remains.

6.3 Modelling of payments for different interest rates

Too many factors play a role in determining the interest rates and so, it is not possible to eliminate the uncertainty. How cheap will be money the banks can obtain? How great will be the state contribution which is being prepared in the form of interest rate reduction? What will be the effect of inflation in determining new rates during the duration of the loan? Next example will demonstrate substantial effect which the level of the interest rate has on the level of payment even in the fuzzy environment.

Let us determine the size of payments for the interest rates which are at the present time stated as extreme alternatives of future mortgage loan model after the state dotation is granted. Let us model for

a) 5 % p.a., b) 9 % p.a.

The other input values are the same as in example 1.

The calculated values are summarised in the following tables and represented in a figure (see fig. e).

a) 5% p.a.

n_i	P_{ni}
180	(7513 / 7908, 7908 / 8699)
216	(6679 / 7030, 7030 / 7733)
240	(6970 / 6600, 6600 / 7260)

b) 9% p.a.

n_i	P_{ni}
180	(9636 / 10143, 10143 / 11157)
216	(8896 / 9364, 9364 / 10301)
240	(8547 / 8997, 8997 / 9897)

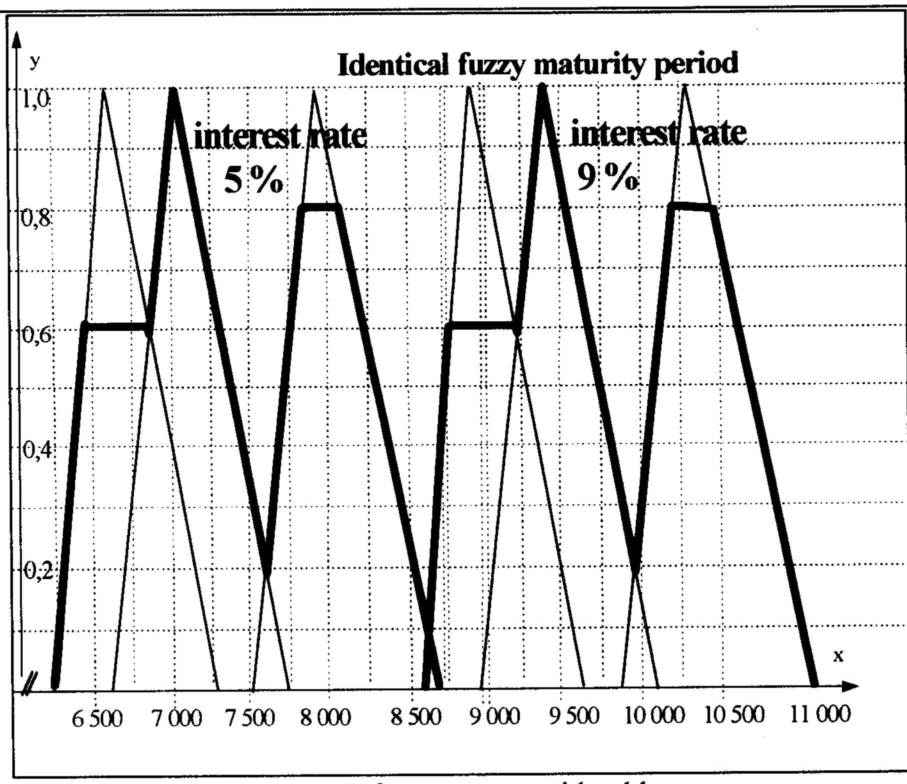

Fig. E: Level of interest rate will affect the amount of payment considerably.

6.4 Compensation of increase of interest rate

If the **family budget** of the debtor is **limited** to the extent that he can afford only a certain level of payments, while the interest rates are set so high that the corresponding payments for the given amortisation period would exceed the debtor´s payment possibilities, then requirements of the highest possible periodical payments can be met **by extention of the amortisation period**. The extension of the amortisation period thus could compensate an increase in monthly payments so that monthly payments will not be increased but amortisation period of the loan will be extended. However, the question is whether the bank would be able to accommodate the client´s request.

Let us assume that the level of a **periodic monthly payment** $\underline{P}$ which will secure the payment of the loan $\underline{A}$ within a period of **15 years** at an interest rate $\underline{r}_1$ is the greatest possible for the given client. The determined size of payment $\underline{P}$ will be taken as an input value into the second part of example and the amortisation period will be extend up to **20 years**. Let us determine a new interest rate $\underline{r}_2$ which will also create "reserve" for client with limited financial potential for the eventual rise of the interest rate by the bank.[2]

[2] It would be also meaningful to make **analoguous calculation of new length of amortization period** for a certain **changed** level of the interest rate at unchanged monthly payment. Such considerations would by actual, for instance, at a renewal of contract during amortization period, if the bank would require a higher interest rate and at the same time to agree to an extension of amortization period of the loan due to the client´s inability

Let the actual membership functions which characterise values entering into the example be the following:

- The present value of loan A_n = (950000 / 1000000, 1000000 / 1100000)
- The interest rate $\underline{r}^*_1$= (0.055 / 0.06, 0.065 / 0.07).
- The amortisation period $\underline{n}^*$= 15 years, i. e. number of period is 180.

- **The change of amortisation period from 15 to 20 years.**
- Newly found (designed) interest rate $\underline{r}^*_2$= 0.076/0.082/0.087/0.09).

Again the results are summarised and depicted on Fig. 6.

$\mathbf{n_i}$	$\underline{\mathbf{P}}_{ni}$
180	(7762 / 8439, 8711 / 9887)
216	(7711 / 8489, 8805 / 9897) (see footnote[3])

Comparison:

Extention of amortisation period from 15 to 20 years provides us, while the monthly family financial budget stays the same, with a room for a change in interest rates from about 6.25 % up to 8.45 %. So called **monthly payments would not have to be increased**, if the increase in interest rates will be compensated by the extention of the amortisation period.

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to increase his/her periodical payments. However, in such case other than original (present) value of the loan would enter into play, because its part has already been paid.

[3] The values were found out by interpolation and therefore the payments $\underline{P}_{180,i}$ are equal to payments $\underline{P}_{216,i}$ only approximately.

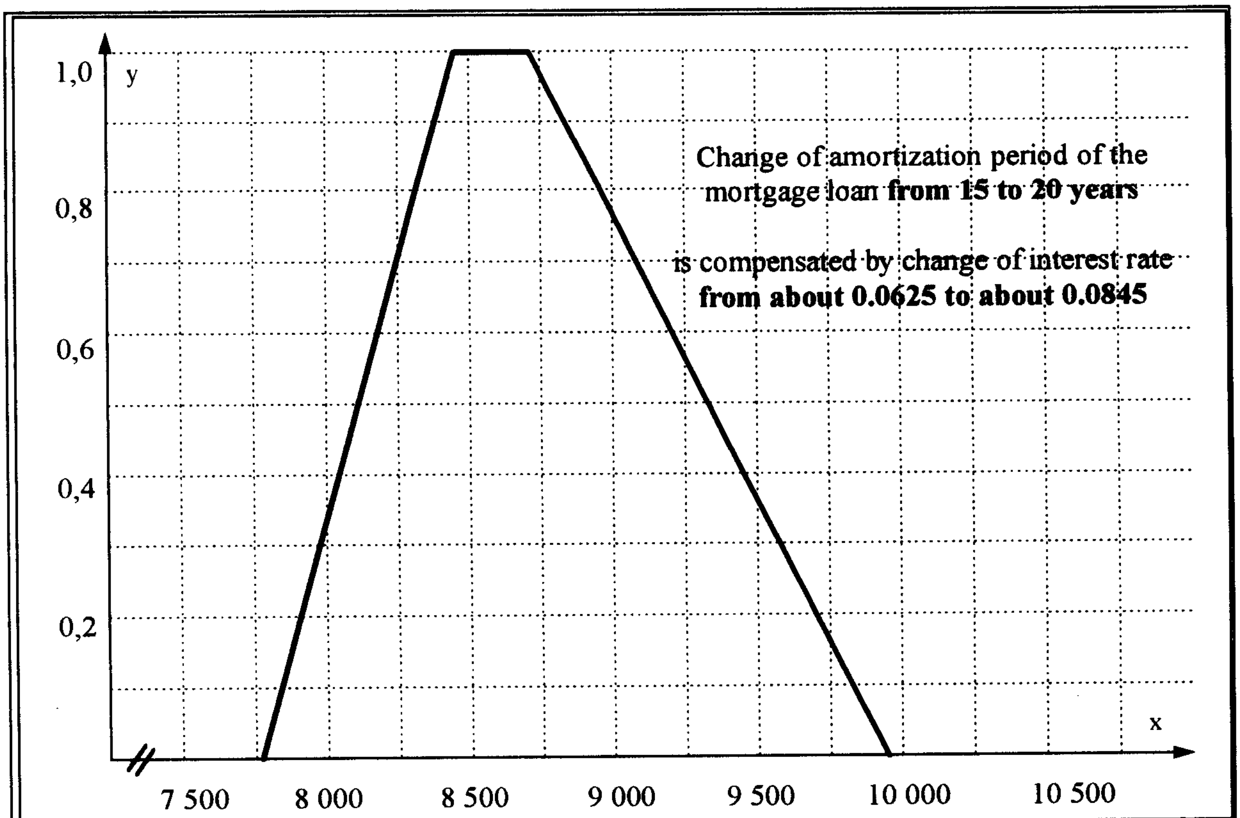


Fig. F: Change in length of amortisation period of mortgage loan and change in interest rate compensate each other, so there is no change in the size of periodic payment.

7. Appendix

<u>Fuzzy number</u>

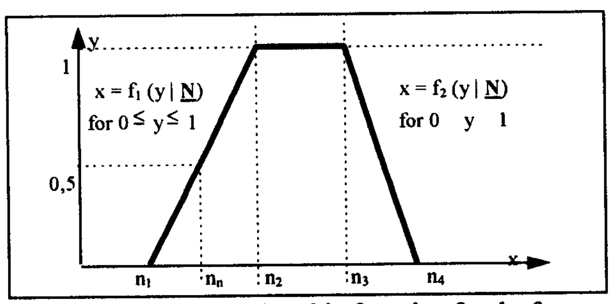
A fuzzy number $\underline{N}$ is a special fuzzy **subset of the real** numbers. Its **membership function** is defined by

$\mu(x \mid \underline{N}) = (n_1,\ f_1(y \mid \underline{N}) / n_2,\ n_3 / f_2(y \mid \underline{N}),\ n_4)$, where

$n_1 < n_2 \leq n_3 < n_4$,

$f_1(y \mid \underline{N})$ is a continuous monotone **increasing** function of y for $0 \leq y \leq 1$
with $f_1(0 \mid \underline{N}) = n_1$ and $f_1(1 \mid \underline{N}) = n_2$, and

$f_2(y \mid \underline{N})$ is a continuous monotone **decreasing** function of y for $0 \leq y \leq 1$
with $f_2(0 \mid \underline{N}) = n_4$ and $f_2(1 \mid \underline{N}) = n_3$,



Graph of $\mu(x \mid \underline{N})$, the **membership function** for the fuzzy number $\underline{N}$ (see [1], pg. 258).

It follows from this picture that, for example, the value n_2 belongs to **N** with the degree 1 (i. e. it is in 100 % true that n_1 belongs to **N**), the value n_1 does not belong to **N** (its membership degree is 0), while it is only half true that n_n belongs to **N** (its membership degree is 0.5).

Footnotes:

- In this paper the standard **arithmetics** of fuzzy numbers will be used (see [3]). The **addition** and **multiplication** of fuzzy numbers is $\oplus$ and $\otimes$, respectively.
- The functions f_1, f_2, may not be **straight line** in each case. If f_1, f_2 are straight lines, then we denote μ (x | **N**) **simply as** $(n_1 / n_2, n_3 / n_4)$.
- If $n_2 = n_3$ then fuzzy number **N** is called **triangular** fuzzy number. If $n_2 \neq n_3$ then fuzzy number **N** is called **flat** fuzzy number.
- The **real** number N is a special fuzzy number, where μ (x | **N**) = 1 if and only if x = N and it is zero otherwise.
- The fuzzy number **N** is **positive** if $n_1 \geq 0$ and **negative** if $n_4 \leq 0$.

Fuzzy number n

The fuzzy number n is a **discrete** positive fuzzy subset of the real numbers.

The membership function μ (x | **n**) is defined on a **collection** of positive **integers** n_i, $1 \leq i \leq K$, where

- $\mu (n_i \mid \mathbf{n}) = \lambda_i, \quad 0 < \lambda_i \leq 1$ for $1 \leq i \leq K$ and
- $\mu (x \mid \mathbf{n}) = 0$ otherwise.

The value of λ_i can be interpreted as the **possibility** that the number of interest periods is n_i (see [1], pg. 259).

For instance, the fuzzy time period **n** is characterised by the membership function $\mu (n_i \mid \mathbf{n})$ which is defined on the following table:

n_i	15	18	20
$\mu (n_i \mid \mathbf{n})$	0,6	1,0	0,8

8. Conclusion

Financial mathematics uses **explicit formulas for calculation of exact numbers** for the interest rate, loan amount, number of interest periods and others. However, the data entering into these calculations are not crisp. Very frequently, the future interest rates, future periodic payments, future cash flows as well as the duration of investments **are estimated**. Statements like "about 500 000 Kč", approximately between 8% and 10% have to **be first transformed** into crisp values and only then they can be used in any of explicit formulas.

Fuzzy approach is presented here as a possible alternative in solving of the above-mentioned contradiction, where the vagueness, inaccuracy and **uncertainty are not overlooked but enter into exact model**. This is possible by using **fuzzy numbers** for interest rates, cash amounts or positive discrete fuzzy sets for the duration of the investment.

Besides the possibility to create fuzzy current value of the cash amount to be repaid, its fuzzy future value, fuzzy pure current value and besides potential of fuzzification of the other concepts in the area of financial mathematics, there is a possibility to pay attention **to modelling of the annuities at uncertainty, namely of mortgage loan at uncertainty**.

At present there is too much uncertainty and vagueness when determining conditions for the mortgage loans in the Czech Republic. Respite this, concrete numbers are necessary to determine the credit. Such numbers can be obtained only by application of explicit formulas from the area of financial mathematics, but the question is how to focus, "crisp" values of variables in the way that enable us to enter them into formulas and, at the same time, to preserve the exact nature of calculations. But every approach requires sharpening **at the classical beginning** of calculation, which necessarily results in **multiplication of errors**. A possible alternative is the fuzzy approach to solution of the whole problem in which **vagueness is not transformed in the beginning but itself enters into exact calculations** resulting in presence of uncertainty even in final solution.

In this article, attention is also put to methodical procedure at fuzzy modelling of the mortgage loans as a **system of cash flows at uncertainty** including examples illustrating this procedure. In the following part, possible solutions of some of the many questions, which presently (during the period, when mortgage loan conditions are being established) arise, are being presented in the way which will give us vital, optimal model expectable to side of the lender and also the borrower.

It would be suitable to elaborate further the indicated considerations. Interesting questions could be asked for instance in connection with **inconvexity** of results in part 6.3. This could be avoided by selecting a smaller fuzziness of input data, by finer definition of membership function $\mu\ (x \mid \underset{\sim}{\mathbf{n}}\ )$ but also by shortening of periods theoretically up to the conversion to **continuous** interest rating. However, in connection with this, it is necessary to point out the fact that in similar considerations we must not neglect, that financial area is decisive one in this modelling, and a formal solution by means of fuzzy sets is only a tool in solving of the problems, which have to be real ones.

The presentation of **fuzzy sets in the area of finance** calls attention to the existence of areas in economics which can be broadened by fuzzy approach in solving of their problems.

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Authors′ address:

Danuše Bauerová, Department of Mathematical Methods in Economics, Faculty of Economics, VŠB-TU, Sokolská 33, 701 21 Ostrava 1, Czech Republic, e-mail: danuse.bauerova@vsb.cz