Research into the Forming Models of Fuzzy (strong, weak) Including Relations

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Abstract: In this paper, I have researched into the fuzzy (strong, weak) including relations, fuzzy (strong, weak) similar relations and some of their forming models on $\mathscr{F}(X)$. Also I have discussed their internal relations.

Keywords: Fuzzy (strong, weak) including relation, Fuzzy (strong, weak)similar relation, Fuzzy measure and Probability measure.

1. Introduction

Let X be the basic field, $\mathscr{F}(X)$ be all the fuzzy sets on X, $\mathscr{P}(X)$ be all the classical sets on X.

We call the fuzzy subset $R\in\mathscr{F}(X\times X)$ fuzzy relation on X. Because every fuzzy relaton corresponds to a membership function $R: X\times X\to[0, 1]$, for convenience' sake, from now on I will not distnguish them. I call $R: X\times X\to[0, 1]$fuzzy relation on X and call value $R(x_1, x_2)$ of membership function the relation degree of x_1 to x_2 about fuzzy relation R. So the key to researching into fuzzy relation is to form their membership function. But this is not easy.

In this paper, I defined fuzzy (strong, weak) including relation and fuzzy (strong, weak) similar relation on the basis of the classical including relation. I used the measure methods to form their membership functions. Also, I researched into the models of forming fuzzy similar relations with fuzzy including relations.

In this paper, let T be T—norm, namely, T be the opration with two variables

$T:[0,1]\times[0,1]\to[0,1]$ that satisfies the following conditions:

(1) $T(a,1)=a$;

(2) $T(a,b)=T(b,a)$;

(3) $T(T(a,b),c)=T(a,T(b,c))$;

(4) $T(a,b) \leqslant T(c,d)$ (when $a \leqslant c$ and $b \leqslant d$).

If we change condition (1) into (1') $T(a,\ 0)=a$ and change symbol T into S, then the operations is called S—norm. If T and S satisfies

$T(1-a,1-b)=1-S(a,b)$

we say S and T are dual norms.

Let $A,B \in \mathscr{F}(X)$, the definitions of operatons $\bigcap_S$, $\bigcap_T$ and complement as follows,

$$(A \bigcup_S B)(x)=S(A(x),B(x));$$
$$(A \bigcap_T B)(x)=T(A(x),B(x));$$
$$A^C(x)=1-A(x);$$

If mapping $M:\mathscr{F}(X) \to [0,1]$ satisfies

(1) $M(\Phi)=0$ and $M(X)=1$

(2) $M(A) \leqslant M(B)$ when $A \subset B$

then we call M fuzzy measure.

If M is a fuzzy measure and satisfies

$$M(A \bigcup B)=M(A)+M(B)-M(A \bigcap B)$$

then we call M probability measure.

If a fuzzy measure M satisfies

$$M(A \bigcap B)=M(A) \vee M(B)$$

then we call M possibility measure.

If a fuzzy measure M satisfies

$$M(A \bigcap B)=M(A) \wedge M(B)$$

then we call M inevitablity measure.

2. Fuzzy(strong, weak)including relation and their forming models

First, we can use the following mapping to define the classical including relation.

If mapping $R:\ \mathscr{F}(X) \times \mathscr{F}(X) \to \{0,1\}$ satisfies condition

$R(B,A)=\begin{cases} 1, A \subset B \\ 0, A \not\subset B \end{cases}$, where $A,B \in \mathscr{F}(X)$, then we call R the classical including relation on $\mathscr{F}(X)$. It has the self—self character, anti—symmetric character and the transmit character.

I defined the fuzzy (strong, weak) including relation as follows:

Definition 1 If mapping $D:\mathscr{F}(X) \times \mathscr{F}(X) \to [0,1]$ satisfies

(1) $D(B,A)=1$ when $A,B \in \mathscr{P}(X)$ and $A \subset B$.

(2) $D(A,C) \leqslant D(A,B) \wedge D(B,C)$ when $A,B,C \in \mathscr{F}(X)$ and A

$\subset B\subset C$, then we call D fuzzy including relation on $\mathscr{F}(X)$ and call value $D(A,B)$ including degree of A to B.

Definition 2 If change condition (1) in definition 1 into

(1′) $D(B,A)=1$ when $A,B\in\mathscr{F}(X)$ and $A\subset B$

then we call D fuzzy strong including relation on $\mathscr{P}(X)$ and call value $D(A,B)$ strong including degree of A to B.

Definition 3 If chang condition (2) in definition 1 into

(2′) $D(A,C)\leqslant D(A,B)\vee D(B,C)$ when $A,B,C\in\mathscr{F}(X)$ and $A\subset B\subset C$

then we call D fuzzy weak including relation and call value $D(A,B)$ weak including degree of A to B.

Property 1 If D is a fuzzy strong including relation, then D is a fuzzy including relation certainly. If D is a fuzzy including relaion, then D is a fuzzy weak including relation surely. The convers is not right.

Theorem 1 let M be a fuzzy measure, and

$$D_1(A,B)=M(A\cap B)/M(B)$$

then D_1 is a fuzzy strong including relation.

Proof For any $A,B\in\mathscr{F}(X)$, $D_1\in[0,1]$ is easy to see. If $A,B\in\mathscr{F}(X)$ and $A\subset B$ then $A\cap B=A$, thus $D_1(B,A)=1$, If $A,B,C\in\mathscr{F}(X)$ and $A\subset B\subset C$, we have $M(A)\leqslant M(B)\leqslant M(C)$ and

$$D_1(A,C)=M(A\cap C)/M(C)=M(A)/M(C),$$
$$D_1(A,B)=M(A\cap B)/M(B)=M(A)/M(B),$$
$$D_1(B,C)=M(B\cap C)/M(C)=M(B)/M(C),$$

thus $D_1(A,C)\leqslant D_1(A,B)\wedge D_1(B,C)$

So D_1 is a fuzzy strong including relation according to definition 1.

Corollary 1. 1 If M is probability measure P, then we have

$$D_1(A,B)=P(A\cap B)/P(B)=P(A|B)$$

Corollary 1. 2 If M is fuzzy measure $M(A)=\sum A(x)P(x)$, where P is a probability distribution on X, then we have

$$D_1(A,B)=\sum(A(x)\wedge B(x))P(x)/\sum B(x)P(x)$$

Specially, If P is a well—distribution, then

$$D_1(A,B)=\sum(A(X)\wedge B(X))/\sum B(X)$$

Corollary 1. 3 If M is a inevitablity measure, then we have

$$D_1(A,B)=(M(A)\wedge M(B)/M(B))$$

Theorem 2 Let M be a fuzzy measure, and

$$D_2(A,B)=M(A^c\cap B^c)/M(A^c)$$

then D_2 is a fuzzy strong including relation.

Proof $D_2(A,B)\in[0,1]$ for any $A,B\in\mathscr{F}(X)$ obviously.

when $A,B \in \mathscr{F}(X)$ and $A \subset B$, we have $B^C \subset A^C, A^C \cap B^C = B^C$,

$$D_2(B,A) = M(B^C \cap A^C)/M(B^C) = 1.$$

When $A,B,C \in \mathscr{F}(X)$ and $A \subset B \subset C$, we have $C^C \subset B^C \subset A^C, D_2(A,C) = M(A^C \cap C^C)/M(A^C)$. $D_2(B,C) = M(C^C)/M(B^C)$, thus $D_2(A,C) \leqslant D_2(A,B) \wedge D_2(B,C)$

So D_2 is a fuzzy strong including relation.

Corollary 2. 1 If M is a probability measure P, then

$$D_2(A,B) = P(A^C \cap B^C)/P(A^C) = P(B^C | A^C)$$

Corollary 2. 2 If M is a inevitability measure, then

$$D_2(A,B) = (M(A^C) \wedge M(B^C))/M(A^C)$$

Corollary 2. 3 If M is fuzzy measure $M(A) = \sum A(x)\ P(x)$ and probability distribution $P(x)$ is a well—distribution, then

$$D_2(A,B) = \sum (A^C(x) \wedge B^C(x)) / \sum A^C(x)$$
$$= \sum ((1-A(x)) \wedge (1-B(x)) / \sum (1-A(x))$$

Theorem 3 Let P be a probability distribution and $P(B|A) = P(AB)/P(A)$, MYCIN determinary factor be

$$CF(B/A) = \begin{cases} [P(B/A) - P(B)]/[1-P(B)], & P(B/A) \geqslant P(B); \\ [P(B/A) - P(B)]/P(B), & P(B/A) \leqslant P(B). \end{cases}$$

If we take $D_3(A,B) = (1/2)[CF(B/A)+1]$, then D_3 is a fuzzy strong including relation.

Proof For any $A,B \in \mathscr{F}(X)$, we have $|CF(B/A)| \leqslant 1, D_3(A,B) \in [0,1]$.

When $A \subset B$, we have

$$P(B/A) = 1, CF(B/A) = 1, \text{ then } D_3(B,A) = 1.$$

when $A \subset B \subset C$ we have

$$P(A) \leqslant P(B) \leqslant P(C), P(A/B) = P(A)/P(B) \geqslant P(A)$$
$$P(A/C) = P(A)/P(C) \geqslant P(A), P(B/C) \geqslant P(B), \text{ then}$$
$$CF(A/C) = [P(A)/P(C) - P(A)]/[1-P(A)]$$
$$\leqslant [P(A)/P(B) - P(A)]/[1-P(A)]$$
$$\leqslant CF(A/B) \text{ and}$$
$$CF(B/C) = [P(B)/P(C) - P(B)]/[1-P(B)]$$
$$= P(B)[1-P(C)]/P(C)[1-P(B)]$$

thus $D_3(A,C) \leqslant D_3(A,B) \wedge D_3(B,C)$. So D_3 is a fuzzy strong including relation.

Theorem 4 If M is a fuzzy measure, $D_4(A,B) = M(A \cup_S B^C)$, then D_4 is a fuzzy including relation.

Proof $D_4(A,B) \in [0,1]$, obviously.

When $A,B \in \mathscr{P}(X)$ and $A \subset B$, we have $A \cup_S A^C = X, (A \cup_S A^C) \subset (B$

$\bigcup_S A^c$), then $M(B\bigcup_S A^c)=1$ *i.e.* $D_4(B,A)=1$, however this is not sure for $A,B\in\mathscr{F}(X)$.

When $A,B,C\in\mathscr{F}(X)$ and $A\subset B\subset C$, we have $C^c\subset B^c\subset A^c$,

$D_4(A,C)=M(A\bigcup_S C^c)\leqslant M(A\bigcup_S B^c)=D_4(A,B)$,

$D_4(A,C)\leqslant M(B\bigcup_S C^c)=D_4(B,C)$, thus

$D_4(A,C)\leqslant D_4(A,B)\wedge D_4(B,C)$

So D_4 is a fuzzy including relation.

Theorem 5 If M is a fuzzy measure, $D_5(A,B)=1-M(A^c\bigcap B)/M(B)$, then D_5 is a fuzzy weak inluding relation.

proof For any $A,B\in\mathscr{F}(X)$, $0\leqslant M(A^c\bigcap B)\leqslant M(B)$, $D_5(A,B)\in[0,1]$.

When $A,B\in\mathscr{P}(X)$ and $A\subset B$, we have $A\bigcap B^c=\Phi$, $M(B^c\bigcap A)=0$, $D_5(B,A)=1$.

When $A,B,C\in\mathscr{F}(X)$ and $A\subset B\subset C$, we have

$D_5(A,C)=1-M(A^c\bigcap C)/M(C)$

$D_5(A,B)=1-M(A^c\bigcap B)/M(B)$

$D_5(B,C)=1-M(B^c\bigcap C)/M(C)$

From $A\subset B$ get $B^c\subset A^c$, $M(A^c\bigcap C)\geqslant M(B^c\bigcap C)$, $D_5(A,C)\leqslant D_5(B,C)$. However, $D_5(A,C)\leqslant D_5(A,B)$ is not sure.

Thus $D_5(A,C)\leqslant D_5(A,B)\vee D_5(B,C)$.

So D_5 is a fuzzy weak including relation.

3. Fuzzy(strong, weak) similar relations and their forming models

Definition 4 If mapping $S:\mathscr{F}(X)\times\mathscr{F}(X)\to[0,1]$ satisfies conditions as follows

(1) $S(A,B)=S(B,A)$ for any $A,B\in\mathscr{F}(X)$

(2) $S(A,A)=1$ for any $A\in\mathscr{F}(X)$

(3) $S(A,C)\leqslant S(A,B)\wedge S(B,C)$ for $A,B,C\in\mathscr{P}(X)$ and $A\subset B\subset C$,

then we call mapping S a fuzzy similar relation on $\mathscr{F}(X)$ and call value $S(A,B)$ similar degree of A and B.

Definition 5 If conditions(2), (3) above are changed so that they can be sastisfied for fuzzy sets, then we call S a fuzzy strong similar relation and call value $S(A,B)$ strong similar degree of A and B.

Definition 6 If condition (3) in definition 4 is changed into

(3′) $S(A,C)\leqslant S(A,B)\vee S(B,C)$ when $A,B,C\in\mathscr{P}(X)$ and $A\subset B\subset C$

then S is called a fuzzy weak similar relation and value $S(A,B)$ is called weak similar degree of A and B.

Property 2 If S is a fuzzy strong similar relation, then it is a fuzzy similar relation surely. If S is a fuzzy similar relation then it is a fuzzy weak similar relation surely. The converse is not right.

Theorem 6 Suppose T is the T—norm and

$$S_1(A,B)=T(D(A,B),D(B,A))$$

$$S_2(A,B)=D(A\cap B,A\cup B)$$

We have

1° If D is a fuzzy including relation, then S_1 and S_2 are both fuzzy similar relations.

2° If D is a fuzzy strong including relation, then S_1 and S_2 are both fuzzy strong similar relations.

3° If D is a fuzzy weak including relation, then S_1 and S_2 are both fuzzy weak similar relations.

Proof 1° $0\leqslant S_1(A,B)\leqslant 1$, $0\leqslant S_2(A,B)\leqslant 1$ and $S_1(A,B)=S_1(B,A)$, $S_2(A,B)=S_2(B,A)$ for any $A,B\in\mathscr{F}(X)$ obviously.

We have $S_1(A,A)=S_2(A,A)=D(A,A)=1$ when $A\in\mathscr{P}(X)$ according as D is a fuzzy including relation.

When $A,B,C\in\mathscr{P}(X)$ and $A\subset B\subset C$, we have $D(C,A)=D(B,A)=D(C,B)=1$ and

$$S_1(A,C)=T(D(A,C),D(C,A))=T(D(A,C),1)=D(A,C)$$

$$S_1(A,B)=T(D(A,B),D(B,A))=T(D(A,B),1)=D(A,B)$$

$$S_1(B,C)=T(D(B,C),D(C,B))=T(D(B,C),1)=D(B,C)$$

We get $S_1(A,C)\leqslant S_1(A,B)\wedge S_1(B,C)$ according to $D(A,C)\leqslant D(A,B)\wedge D(B,C)$. So S_1 is a fuzzy similar relation. About S_2, we have

$$S_2(A,C)=D(A\cap C,\ A\cup C)=D(A,C)$$

$$S_2(A,B)=D(A\cap B,A\cup B)=D(A,B)$$

$$S_2(B,C)=D(B\cap C,B\cup C)=D(B,C)$$

So $S_2(A,C)\leqslant S_2(A,B)\wedge S_2(B,C)$. As a result, S_2 is a fuzzy similar relation also.

We can prove 2°, 3° similarly.

Corollary 1 We can form some fuzzy strong similar relations respectively according to theorem 1, 2, 3 and 6. Some fuzzy similar relations can be formed according to theorem 4 and 6. Some fuzzy weak similar relations can be formed according to theorem 5and 6.

Corollary 2 Let M be a fuzzy measure, then the fuzzy strong similar relations

$S_{11}(A,B)=T(M(A\cap B)/M(B),M(B\cap A)/M(A))$ and

$S_{21}(A,B)=M(A\cap B)/M(A\cup B)$ are formed according to theorem 1 and 6.

Example 1 If M is a probability measure, then

$$S_{11}(A,B)=T(P(A/B),P(B/A)),$$

$$S_{21}(A,B)=P(A\cap B|A\cup B).$$

Example 2 Let M be a well—distribution, then we get

$$S_{11}(A,B)=T(\sum(A(x)\wedge B(x)/\sum B(x),\sum B(x)\wedge A(x))/\sum A(x))$$

$$S_{21}(A,B)=\sum(A(x)\wedge B(x))/\sum A(x)\vee B(x))$$

Specially, if T—norm $T(a,b)=a\cdot b$, then

$$S_{11}(A,B)=[\sum A(x)\wedge B(x)]^2/(\sum A(x))(\sum B(x)),$$

if $T(a,b)=a\wedge b$, then

$$S_{11}(A,B)=\sum(A(x)\wedge B(x))/(\sum A(x))\vee(\sum B(x))$$

Corollary 3 Let M be a fuzzy measure, then the fuzzy similar relations formed according to theorem 4 and 6 are

$$S_{14}(A,B)=T(M(A\cup_s B^c),M(B\cup_s A^c))$$

$$S_{24}(A,B)=M((A\cap B)\cup_s(A\cup B)^c)=M((A\cap B)\cup_s(A^c\cap B^c))$$

Specially, when M is probability measure P, $\cup_s$ is $\cup$, T—norm $T(a,b)=a\wedge b$, then we get

$$S_{14}(A,B)=P(A\cup B^c)\wedge P(B\cup A^c)$$

$$S_{24}(A,B)=P((A\cap B)\cup(A^c\cap B^c))$$

Corollary 4 If M is a fuzzy measure, then the fuzzy weak similar relations formed according to theorem 5 and 6 are

$$S_{15}(A,B)=T(1-M(A^c\cap B)/M(B),1-M(B^c\cap A)/M(A))$$

$$S_{25}(A,B)=1-M((A^c\cup B^c)\cap(A\cup B))/M(A\cup B)$$

Specially, when M is probability measure P and T—norm $T(a,b)=a\cdot b$. We get

$$S_{15}(A,B)=[1-P(A^c/B)][1-P(B^c/A)]$$

$$S_{25}(A,B)=1-P(A^c\cup B^c/A\cup B)$$

4. Fuzzy including relation, fuzzy similar relation and their forming models on space $\mathscr{F}$

Definition 7 Let $X_i(i=1,2,\cdots,m)$ be the basic fields, $\mathscr{F}(X_i)$ be all fuzzy sets on X_i, $\mathscr{P}(X_i)$ be all classical sets on $X_i(i=1,\cdots,m)$. $A=(A_1,$

$\cdots, A_m)$ be m—dimension fuzzy set vector, where $A_i \in \mathscr{F}(X_i)(i=1,\cdots, m)$.

We call $\mathscr{F}=\prod_{i=1}^{m}\mathscr{F}(X_i)$ the space of m—dimension fuzzy set vectors, call $\mathscr{P}=\prod_{i=1}^{m}\mathscr{P}(X_i)$ the space of m—dimension classial set vectors. For short, they are called space $\mathscr{F}$ and space $\mathscr{P}$ separately.

Definition 8 Suppose $A=(A_1, A_2, \cdots, Am), B=(B_1, \cdots, B_m)$ $A_i, B_i \in \mathscr{F}(X_i)(i=1,\cdots,m)$. we call $A \subset B$ if and only if $A_i \subset B_i$ are true for any $i=1,2,\cdots,m$.

Definition 9 If change $\mathscr{F}(X)$ and $\mathscr{P}(X)$ in definition 1(2,3) into $\mathscr{F}=\prod \mathscr{F}(X_i)$ and $\mathscr{P}=\prod \mathscr{P}(X_i)$ separately, then we call mapping $D: \mathscr{F} \times \mathscr{F} \rightarrow [0, 1]$ the fuzzy (strong, weak) including relation on space $\mathscr{F}$.

Definition 10 If change $\mathscr{F}(X)$ and $\mathscr{P}(X)$ in the definition 4(5,6) into $\mathscr{F}(X)$ and $\mathscr{P}(X)$ separately, then we call mapping $S: \mathscr{F} \times \mathscr{F} \rightarrow [0, 1]$ the fuzzy (strong, weak) similar relation on space $\mathscr{F}$.

Theorem 7 If $A_i, B_i \in \mathscr{F}(X_i)$ and D_i is a fuzzy (strong, weak) including relation on $\mathscr{F}(X_i)(i=1,2,\cdots,m)$. $A=(A_1,\cdots,A_m), B=(B_1,\cdots,B_m)$, then

$$D_P(A,B)=\sum_{i=1}^{m}P_iD_i(A_i,B_i) \text{ (where } P_i\in[0,1], \sum_{i=1}^{m}P_i=1)$$
$$D_N(A,B)=\min_{1\leqslant i\leqslant m}\{D_i(A_i,B_i)\}$$
$$D_M(A,B)=\max_{1\leqslant i\leqslant m}\{D_i(A_i,B_i)\}$$

are all fuzzy (strong, weak) including relations on space $\mathscr{F}$.

Proof

(1) $0\leqslant D_P(A,B)\leqslant \sum_{i=1}^{m}P_i=1$, $D_M(A,B)\in[0,1], D_N(A,B)\in[0,1]$ for any $A,B\in\mathscr{F}$ are obvious.

(2) when $A,B\in\mathscr{P}$ and $A\subset B$, We get $A_i\subset B_i, D_i(B_i,A_i)=1$ $(i=1,\cdots,m)$, thus $D_P(B,A)=\sum_{i=1}^{m}P_i=1, D_N(B,A)=1, D_M(B,A)=1$.

(3) When $A,B,C\in\mathscr{F}$ and $A\subset B\subset C$, we have

$$\begin{aligned} D_P(A,C) &= \sum_{i=1}^{m}P_iD_i(A_i,C_i) \\ &\leqslant \sum P_i[D_i(A_i,B_i)\wedge D_i(B_i,C_i)] \\ &= [\sum P_iD_i(A_i,B_i)]\wedge[\sum P_iD_i(B_i,C_i)] \end{aligned}$$

$$=D_P(A,B)\wedge D_P(B,C)\ .$$

$$D_N(A,C)=\min_{1\leqslant i\leqslant m}\{D_i(A_i,C_i)\}$$
$$\leqslant[\min\{D_i(A_i,B_i)\}]\wedge[\min\{D_i(B_i,C_i)\}]$$
$$=D_N(A,B)\wedge D_N(B,C)$$

Similarly, we can get $D_M(A,C)\leqslant D_M(A,B)\wedge D_M((B,C)$. As a result D_P, D_N and D_M are all fuzzy including relations on space $\mathscr{F}$.

The situation about fuzzy strong and weak including relations can be proved similaly.

Theorem 8 If $A_i,B_i\in\mathscr{F}(X_i)$, S_i is a fuzzy (strong, weak) similar relation on $\mathscr{F}(X_i)(i=1,2,\cdots,m)$, $A=(A_1,\cdots,A_m)$, $B=(B_1,\cdots,B_m)$ then

$$S_P(A,B)=\sum_{i=1}^{m}P_iS_i(A_i,B_i)\ \text{(where } P_i\in[0,1],\ \sum P_i=1)$$
$$S_N(A,B)=\min_{1\leqslant i\leqslant m}\{S_i(A_i,B_i)\}$$
$$S_M(A,B)=\max_{1\leqslant i\leqslant m}\{S_i(A_i,B_i)\}$$

are all fuzzy (strong, weak) similar relations on space $\mathscr{F}$.

This theorem can be proved as theorem 7.

Definition 11 Suppose $A,B\in\mathscr{F}$, we definited

$$A\cap B=(A_1\cap B_1,\cdots,A_m\cap B_m)$$
$$A\cup B=(A_1\cup B_1,\cdots,A_m\cup B_m)$$
$$A^C=(A_1^C,\cdots,A_m^C)$$

Theorem 9 The theorem 6 is correct on space $\mathscr{F}$ also.

According to theorems 6, 7, 8 and 9, we can get two means of forming fuzzy similar relations on space $\mathscr{F}$ with fuzzy including relations D_i on $\mathscr{F}(X_i)(i=1,\cdots,m)$.

Mean 1° $D_i\xrightarrow{\text{(theorem 6)}}S_i\xrightarrow{\text{(theorem 8)}}S_{68}$

Where S_i *is the fuzzy similar relation on* $\mathscr{F}(X_i)(i=1,\cdots,m)$ and that is formed by D_i according to theorem 6. S_{68} is the fuzzy similar relation on space $\mathscr{F}$ and it is formed by $S_i(i=1,\cdots,m)$ according to theorem 8.

Mean 2° $D_i(i=1,\cdots,m)\xrightarrow{\text{(theorem 7)}}D\xrightarrow{\text{(theorem 9)}}S_{79}$.

Where D is a fuzzy including relation on $\mathscr{F}$ and that is formed by $D_i(i=1,\cdots,m)$ according to theorem 7, S_{79} is the fuzzy similar relation on $\mathscr{F}$ and that is formed by D according to thorem 9

Generally , the results obtained from the same $D_i(i=1,\cdots,m)$ by two means above are not same, i. e. $S_{67}\neq S_{79}$

Example 3 Let D_i be a fuzzy including relation on $\mathscr{F}(X_i)(i=1,\cdots,m)$, $A=(A_1,\cdots,A_m)$, $B=(B_1,\cdots,B_m)$, $A_i,B_i\in\mathscr{F}(X_i)(i=1,\cdots,m)$.

1° According to S_1 in theorem 6 from D_i we first get the fuzzy similar relations on $\mathscr{F}(X_i)$ as follows:

$$S_{1i}(A_i,B_i)=T(D_i(A_i,B_i),D_i(B_i,A_i)),(i=1,\cdots,m)$$

then from $S_{1i}(A_i,B_i)$ according to S_P in theorem 8 we get the fuzzy similay relation on $\mathscr{F}$

$$S_{1P}(A,B)=\sum_{i=1}^{m}P_iS_{1i}(A_i,B_i)$$

$$=\sum P_iT(D_i(A_i,B_i),D_i(B_i,A_i))$$

2° From $D_i(i=1,\cdots,m)$ according to D_P in theorem 7 we first get the fuzzy including relatoin on $\mathscr{F}$

$$D_P(A,B)=\sum_{i=1}^{m}P_iD_i(A_i,B_i)$$

then according to S_1 in theorem 9 we get the fuzzy similar relation on $\mathscr{F}$

$$S_{P1}(A,B)=T(D_P(A,B),D_P(B,A))$$

$$=T(\sum_{i=1}^{m}P_iD_i(A_i,B_i),\sum_{i=1}^{m}P_iD_i(B_i,A_i))$$

We can see $S_{1P}\neq S_{P1}$

The definitions and forming modles of fuzzy including relation and fuzzy similar relation are very important for fuzzy expert systems.

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