# Fuzzy Optimization Theory and Its Application

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Abstract: This research presents, using fuzzy optimization theory, a new method of multifactorial evaluation for students' ten quarters, it really reflects every student's position in a name list in the class. This makes the administration of students scienceize, standardization, indexize and quantityize of check.

Keywords: Fuzzy optimization, fuzzy relation, multifactorial evaluation, synthetic quality.

## 1. Fuzzy optimization theory

Assume $A = (a_1, a_2, \ldots, a_n)$ is the optimum seeking set of a system S, where, $a_i$ is optimum seeking things of S, $B=(b_1, b_2, \ldots, b_m)$ is a object set of S, where, $b_j$ is object which evaluates $a_i$. Let $x_{ij}$ be characteristic value (CV, in short) of thing $a_j$ with regard to object $b_i$, then we have object characteristic value matrix of S:

$$X_{m\times n}=\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

In order to fight off the effects resulting from different dimensions of m CV's, we normalize matrix X as follows[2]:

$1^{0}$ For the object with "the more the better" , we use effect measure of the upper limit:

$$r_{ij}= x_{ij}/(x_{i1}\vee x_{i2}\vee\ldots\vee x_{in}), \qquad x_{ij}\geqslant 0 \qquad (1.1)$$

$2^{0}$ For the object with "the smaller the better" , we use effect measure of the lower limit:

$$r_{ij}=(x_{i1}\wedge x_{i2}\wedge\ldots\wedge x_{in})/x_{ij}, \qquad x_{ij}>0 \qquad (1.2)$$

$r_{ij}=1$ when $x_{ij}=0$.

$3^{0}$ For the object with "the moderater the better" , we use effect measure of the moderate:

$$r_{ij}= (x_{ij}\wedge u_{0}) / (x_{ij}\vee u_{0}) \qquad (1.3)$$

or

$$r_{ij}=u_{0}/(\left|x_{ij}-u_{0}\right|+u_{0}) \qquad (1.3')$$

where, $u_{0}$ is some moderate value assigned.

$r_{ij}$ above is optimal membership grade of thing $a_j$ under object $b_i$, and the matrix

$$R_{m\times n}=\begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

is called matrix of optimal membership grade (MOMG, in short).

Definition 1 Assume R is MOMG of system S. Then $\vec{g}=(g_1,g_2,\ldots,g_m)^T=(1,1,\ldots,1)^T$ is called top thing of S.

Definition 2 Assume R is MOMG of system S. Then $\vec{f}=(f_1,f_2,\ldots,f_m)^T=(0,0,\ldots,0)^T$ is called low thing of S.

Assume a system has n optimum seeking things, and $u_j$ is membership grade of thing $a_j$ subordinating to top thing, then $u^{c}_{j}=1-u_j$ is membership grade of thing $a_j$ subordinating to low thing.

Assume $\vec{\omega}=(\omega_1,\omega_2,\dots\omega_m)^T$ is weight vector of m objects, where $\omega_i$ is weight number of object $b_i$, $\sum_{i=1}^{m}\omega_i=1$. In order to get optimal value of membership grade $u_j$ of thing $a_j$, we want some majorizing criterion. For this we have

Definition 3 Suppose R is MOMG of system S, and $\vec{r}_j=(r_{1j},r_{2j},\dots,r_{mj})^T$ is a vector of optimal membership grade of thing $a_j$. Then

$$D(\vec{r}_j,\vec{g})=u_j[\sum_{i=1}^{m}(\omega_i|r_{ij}-1|)^p]^{1/p}$$

is called weight distance from top of thing $a_j$, and

$$D(\vec{r}_j,\vec{f})=u^c{}_j[\sum_{i=1}^{m}(\omega_i|r_{ij}-0|)^p]^{1/p}$$

is called weight distance from low of thing $a_j$, where, P is distance parameter. p=1 is called Hamming's distance, and p=2 is called Euclid's distance.

In order to get optimal value of membership grade $u_j$ of thing $a_j$, we present a objective function as follows:

$$\min(F(u_j)=u_j^2[\sum_{i=1}^{m}(\omega_i|r_{ij}-1|)^p]^{2/p}+(1-u_j)^2[\sum_{i=1}^{m}(\omega_i r_{ij})^p]^{2/p}),$$

that is, it makes the sum smallest of square for weight distance from top and for weight distance from low of thing $a_j$.

Let $dF(u_j)/du_j=0$, then it follows that formula computing the optimal value of $u_j$:

$$u_j=1/(1+[\sum_{i=1}^{m}(\omega_i|r_{ij}-1|)^p/\sum_{i=1}^{m}(\omega_i r_{ij})^p]^{2/p}),$$

$j=1,2,\dots,n$ (1.4)

Let U and V be two finite universes of a multi-objective system S. For object i of S, all element pair $(u, v)\in U\times V$ constitutes Dartesian product set with regard to object i. Then we get the following fuzzy relational matrix of object i with regard to excellent:

$$iR_{a\times c}=\begin{bmatrix} ir_{11} & ir_{12} & \cdots & ir_{1c} \\ ir_{21} & ir_{22} & \cdots & ir_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ ir_{a1} & ir_{a2} & \cdots & ir_{ac} \end{bmatrix}=(ir_{kh})_{a\times c}$$

where, a is the total of elements in universe U, and c is that in V, and $ir_{kh}$ is membership grade of (u, v) with regard to object i to excellent.

Definition 4 Suppose that $iR_{a\times c}$ is fuzzy relational matrix of object i with regard to excellent. If

$$iG_{a\times c}=(g_{kh})_{a\times c}=\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots \\ 1 & 1 & \cdots & 1 \end{bmatrix},$$

then $iG_{a\times c}$ is called top relational matrix.

Definition 5 Assume $iR_{a\times c}$ is fuzzy relational matrix of object i with regard to excellent. If

$$iF_{a\times c}=(f_{kh})_{a\times c}=\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 \end{bmatrix},$$

then $iF_{a\times c}$ is called low relational matrix.

Similarly, we can get the following model computing the optimal value of membership grade $u_{kh}$:

$$u_{kh}=1/\left(1+\left[\sum_{i=1}^{m}\left(\left|\ ir_{kh}-1\ \right|\right)^{p}/\sum_{i=1}^{m}(\omega_{i}\cdot ir_{kh})^{p}\right]^{2/p}\right) \qquad (1.5)$$

Suppose that system S has multi-objective fuzzy relational matrix $iR_{a\times c}$, i=1,2, . . . , m. To synthesize m fuzzy relation matrixes $iR_{a\times c}$ by model (1.5) , we get a fuzzy relational synthetical matrix $U_{k\times h}=(u_{kh})_{a\times c}$ of a multi-objective system in $U\times V$.

2. Basic Models

(1) First, we set up a list of objects and weights and optimal membership grade (OMG, in short).

| | | | | |
|---|---|---|---|---|
| second layer | subsystem | 1 | 2 ... | m |
| | class of subsystems | * | * * | * |
| | weight | $\omega_1$ | $\omega_2$ ... | $\omega_n$ |
| first layer | order | $1,2,\ldots,k_1$ | $1,2,\ldots,k_2$ | $1,2,\ldots,k_m$ |
| | object | * | * | * |
| | weight | $\vec{\omega}_{1k1}$ | $\vec{\omega}_{2k2}$ ... | $\vec{\omega}_{mkm}$ |
| OMG | 1<br>2<br>...<br>n | $R^T_{k1\times n}$ | $R^T_{k2\times n}$ ... | $R^T_{km\times n}$ |

(2) We begin with the first layer, for m fuzzy relational matrixes $R_{ki\times m}$, $i=1,2,\ldots,m$, in above list, comput $u_{1j},u_{2j},\ldots,u_{mj}$ by formula(1.4) with taking p=2, i.e., the following formula:

$$u_j=1/\left(1+\left[\sum_{i=1}^{m}(\omega_i\left|r_{ij}-1\right|)^2/\sum_{i=1}^{m}(\omega_i r_{ij})^2\right]\right),\ j=1,2,\ldots,n \qquad (1.4')$$

(3) Let $u_{ij}=r_{ij}$, $i=1,2,\ldots,m$; $j=1,2,\ldots n$, using formula (1.4′) we can get $u_j$, then arrange order of n things awaiting evaluation according to the principle of the bigger membership grade is, the excellenter.

3. Fuzzy optimization for 33 students' synthetic quality

In this section, using above basic model,33 students in Class 1, Grade 92, Department of Mathematics,Liaocheng Teacher's College, are multifactorially evaluated,the relust is as follows.

Table of objects,weights and OMG's

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| second | subsy-stemx | 1 | | | 2 | | | | 3 | | |
| layer | class of subsyx-stems | intelligential quality | | | moral character | | | | health quality | | |
| | weight | 0.4 | | | 0.3 | | | | 0.3 | | |
| first layer | order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | object | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ | $B_6$ | $B_7$ | $B_8$ | $B_9$ | $B_{10}$ |
| | weight | 0.2 | 0.2 | 0.6 | 0.25 | 0.25 | 0.25 | 0.25 | 0.4 | 0.3 | 0.3 |
| OMG | 1 | 0.698 | 0.989 | 1.000 | 0.990 | 0.978 | 0.979 | 0.958 | 0.918 | 0.973 | 0.987 |
| | 2 | 0.640 | 0.800 | 0.902 | 0.885 | 0.843 | 0.845 | 0.948 | 0.845 | 0.893 | 0.890 |
| | 3 | 0.780 | 0.871 | 0.934 | 0.900 | 0.819 | 0.969 | 0.927 | 0.979 | 0.918 | 0.925 |
| | 4 | 0.965 | 0.817 | 0.862 | 0.865 | 0.994 | 0.897 | 0.927 | 0.753 | 0.841 | 0.876 |
| | 5 | 0.872 | 0.839 | 0.915 | 0.844 | 0.712 | 0.990 | 0.917 | 0.814 | 0.879 | 0.880 |
| | 6 | 0.814 | 0.860 | 0.896 | 0.938 | 0.785 | 0.990 | 0.750 | 0.959 | 0.850 | 0.883 |
| | 7 | 0.759 | 0.800 | 0.896 | 0.885 | 0.725 | 0.887 | 0.708 | 0.938 | 0.902 | 0.871 |
| | 8 | 0.884 | 0.688 | 0.899 | 0.927 | 0.724 | 1.000 | 0.958 | 0.918 | 0.875 | 0.888 |
| | 9 | 0.698 | 0.774 | 0.825 | 0.750 | 0.700 | 0.918 | 0.760 | 0.784 | 0.788 | 0.798 |
| | 10 | 0.674 | 0.806 | 0.830 | 0.844 | 0.764 | 0.918 | 0.792 | 0.742 | 0.875 | 0.833 |
| | 11 | 0.581 | 0.860 | 0.909 | 1.000 | 0.983 | 0.845 | 0.969 | 0.959 | 0.950 | 0.959 |
| | 12 | 0.884 | 0.796 | 0.891 | 0.719 | 0.785 | 0.866 | 0.979 | 0.959 | 0.866 | 0.868 |
| | 13 | 0.814 | 0.870 | 0.946 | 0.833 | 0.799 | 0.948 | 0.969 | 0.773 | 0.918 | 0.902 |
| | 14 | 0.605 | 1.000 | 0.974 | 0.958 | 0.908 | 0.969 | 0.979 | 1.000 | 0.995 | 0.998 |
| | 15 | 0.581 | 0.882 | 0.870 | 0.948 | 0.941 | 0.979 | 0.750 | 0.907 | 0.870 | 0.904 |
| | 16 | 0.733 | 0.935 | 0.937 | 0.844 | 0.750 | 0.990 | 0.885 | 0.918 | 0.961 | 0.935 |
| | 17 | 0.774 | 0.919 | 0.902 | 0.948 | 0.768 | 0.969 | 0.802 | 0.918 | 0.902 | 0.910 |
| | 18 | 0.977 | 0.903 | 0.944 | 0.990 | 1.000 | 0.907 | 0.906 | 0.845 | 0.964 | 0.958 |
| | 19 | 0.581 | 0.849 | 0.775 | 0.698 | 0.846 | 0.887 | 0.844 | 0.784 | 0.743 | 0.799 |
| | 20 | 0.674 | 0.801 | 0.821 | 0.896 | 0.719 | 0.866 | 0.927 | 0.804 | 0.743 | 0.810 |
| | 21 | 0.988 | 0.914 | 0.829 | 0.427 | 0.638 | 0.876 | 0.823 | 0.742 | 0.806 | 0.780 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 22 | 1.000 | 0.849 | 0.928 | 0.969 | 0.868 | 0.887 | 0.990 | 0.948 | 0.916 | 0.923 |
| | 23 | 0.756 | 0.914 | 0.909 | 0.927 | 0.881 | 0.928 | 0.833 | 0.856 | 0.843 | 0.892 |
| | 24 | 0.663 | 0.828 | 0.869 | 0.781 | 0.699 | 0.938 | 0.958 | 0.897 | 0.738 | 0.816 |
| OMG | 25 | 0.605 | 0.849 | 0.865 | 0.635 | 0.809 | 0.887 | 0.875 | 0.814 | 0.806 | 0.826 |
| | 26 | 0.814 | 0.935 | 0.980 | 0.979 | 0.983 | 0.918 | 1.000 | 1.000 | 1.000 | 1.000 |
| | 27 | 0.640 | 0.785 | 0.821 | 0.979 | 0.817 | 0.990 | 0.729 | 0.845 | 0.893 | 0.889 |
| | 28 | 0.779 | 0.753 | 0.862 | 0.938 | 0.921 | 0.959 | 0.896 | 0.835 | 0.788 | 0.857 |
| | 29 | 0.791 | 0.742 | 0.867 | 0.854 | 0.755 | 0.928 | 0.750 | 0.887 | 0.843 | 0.844 |
| | 30 | 0.628 | 0.925 | 0.865 | 0.927 | 0.594 | 0.969 | 0.885 | 0.660 | 0.743 | 0.803 |
| | 31 | 0.663 | 0.800 | 0.861 | 0.844 | 0.757 | 0.928 | 0.719 | 0.990 | 0.793 | 0.833 |
| | 32 | 0.744 | 0.731 | 0.915 | 0.979 | 0.889 | 0.938 | 0.958 | 0.907 | 0.891 | 0.913 |
| | 33 | 0.733 | 0.941 | 0.874 | 0.854 | 0.669 | 0.990 | 0.865 | 0.887 | 0.861 | 0.880 |

In above table,object $B_1$—$B_{10}$ are as follows:

$B_1$----foreign language,

$B_2$----politics,

$B_3$----speciality,

$B_4$----political thought,

$B_5$----observe discipline,

$B_6$----specialized thought ,

$B_7$----labour hygiene,

$B_8$----physical result,

$B_9$----come up to standard ,

$B_{10}$----result of the game .

From these data in above table, we can get, by formula (1.4') , $\vec{u}_{1j}$, $\vec{u}_{2j}$, $\vec{u}_{3j}$, as follows:

$\vec{u}_{1j}$= (0.991,0.970,0.986,0.976,0.988,0.983,0.900,0.977,0.945,0.948,

0.969,0.981,0.992,0.984,0.939,0.988,0.982,0.996,0.908,0.943, 0.936, 0.993, 0.984, 0.964, 0.971, 0.996, 0.996, 0.965, 0.945, 0.963, 0.959, 0.977, 0.942)

$\overrightarrow{u_{2j}}$=(0.999, 0.980, 0.985, 0.990, 0.960, 0.964, 0.934, 0.978, 0.919, 0.950, 0.993, 0.952, 0.978, 0.997, 0.980, 0.968, 0.969, 0.995, 0.947, 0.963, 0.801, 0.991, 0.984, 0.953, 0.929, 0.996, 0.967, 0.993, 0.948, 0.941, 0.941, 0.995, 0.951)

$\overrightarrow{u_{3j}}$= (0.996, 0.969, 0.996, 0.908, 0.968, 0.988, 0.990, 0.987, 0.933, 0.938, 0.998, 0.987, 0.962, 1.000, 0.987, 0.995, 0.991, 0.986, 0.923, 0.933, 0.916, 0.945, 0.975, 0.956, 0.951, 1.000, 0.977, 0.958, 0.975, 0.864, 0.977, 0.989, 0.981)

Let component $u_{ij}=r_{ij}$ in above vectors, from formula(1.4′), we get:

$u_j$= (0.9999, 0.99917, 0.99984, 0.99730, 0.99920, 0.99957, 0.99730, 0.99956, 0.99510, 0.99670, 0.99951, 0.99913, 0.99940, 0.99987, 0.99795, 0.99964, 0.99955, 0.99996, 0.99460, 0.99610, 0.98350, 0.99800, 0.99980, 0.99850, 0.99750, 0.99998, 0.99912, 0.99888, 0.99747, 0.99261, 0.99799, 0.99970, 0.99746).

The order of 33 students are arranged according to OMG′s from high to low:

1, 26, 18, 14, 3, 23, 32, 16 , 6, 8, 17, 11, 13, 5, 2, 12, 27, 28, 24, 22, 31, 15, 25, 29, 33, 4, 10, 20, 9, 19, 7, 30, 21.

It is in keeping with reality that order arranged by the fuzzy optimization model.

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