LINGUISTICALLY VALUED OPERATOR LOGIC

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ABSTRACT

Zadeh presented linguistically valued logic in 1975. Liu Xuhua presented the concept of operator logic in 1984. In this paper, a system of Linguistically Valued Operator Logic (LVOL) is suggested. In this system, truth values may be expressed clearly by operator. $\underset{\sim}{\lambda}$ — resolution and completeness theorem are introduced at last.

Keywords: linguistically valued operator, $\underset{\sim}{\lambda}$—identically true, $\underset{\sim}{\lambda}$—identically false, $\underset{\sim}{\lambda}$—resolution.

1 Linguistically valued operator lattice

Definition 1.1[1] Let $(L, \leqslant)$ be a completely complemented lattice of distribution and let L be a operator set. Operator $\cdot$ is a binary operation on L. If $\forall\ a, b, c \in L$, satisfies the following conditions:

(1)$a \cdot (b * c) = (a \cdot b) * (a \cdot c)$; (2)$a \cdot (b \oplus c) = (a \cdot b) \oplus (a \cdot c)$; (3)$(a \cdot b)' = a' \cdot b'$, then $(L, \leqslant)$ is called a operator lattice, where $*$, $\oplus$ and$'$ are operation of infimum, supremum and complement respectively.

Theorem 1.1 Let $\tilde{L}_1$ be a set of fuzzy numbers[2] on $[0,1]$. $\forall\ \underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c} \in \tilde{L}_1$, if we define $'$, $*$, $\oplus$ and $\cdot$ as following,

$$\underset{\sim}{a}' = \bigcup_{\lambda \in (0,1]} \lambda[1-a_\lambda^+, 1-a_\lambda^-],$$

$$\underset{\sim}{a} * \underset{\sim}{b} = \underset{\sim}{a} \wedge \underset{\sim}{b} = \bigcup_{\lambda \in (0,1]} \lambda[a_\lambda^- \wedge b_\lambda^-, a_\lambda^+ \wedge b_\lambda^+],$$

$$\underset{\sim}{a} \oplus \underset{\sim}{b} = \underset{\sim}{a} \vee \underset{\sim}{b} = \bigcup_{\lambda \in (0,1]} \lambda[a_\lambda^- \vee b_\lambda^-, a_\lambda^+ \vee b_\lambda^+],$$

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = \bigcup_{\lambda \in (0,1]} \lambda\left[\frac{a_\lambda^- + b_\lambda^-}{2}, \frac{a_\lambda^+ + b_\lambda^+}{2}\right],$$

then $\tilde{L}_1$ is a operator lattice (we call linguistically valued operator lattice too).

2 Basic concept

Definition 2.1 Let P be a atomic symbol, $\underset{\sim}{\lambda} \in \tilde{L}_1$. Then call $\underset{\sim}{\lambda} P$ fuzzy atom.

Definition 2.2 Formulas of LVOL are defined recursively:

(1) Fuzzy atom is a formula;

(2) If G is formula, then $\underset{\sim}{\lambda}G$, $\sim G$ are formula, where $\underset{\sim}{\lambda}\in L_1$;

(3) If G, H are formula, then $G\wedge H, G\wedge H, G\to H$ and $G\to H$ are formula;

(4) If G is formula, x is a free variable in G, then $(\forall\ x)\ G(x)$, $(\exists\ x)\ G(x)$, $(\underset{\sim}{\lambda}\forall\ x)\ G(x)$ and $(\underset{\sim}{\lambda}\exists\ x)\ G(x)$ are formula;

(5) All of formulas are generated by using (1)~(4).

$\lambda_1(\lambda_2(\cdots\cdots(\lambda_n P)\cdots\cdots))$ is called literal. It is expressed simply by $\lambda_1\lambda_2\cdots\cdots\lambda_n P$, specially, fuzzy atom is literal.

Definiiton 2.3 Solution I of G is made of non-empty universe and the following rules:

(1) Assign a member of D to every variable symbol;

(2) Assign a mapping from D^n to D to every n-tuple function symbol;

(3) Assign a mapping from D^n to $\{F, T\}$ to every predicate symbol.

Definition 2.4 Truth values $T_I(G)$ is defined by the following rules:

(1) If $\underset{\sim}{\lambda}P$ is fuzzy atom, then $T_I(\lambda_j P)=\lambda_j$. iff P is defined T by I; $T_I(\underset{\sim}{\lambda}P)=\underset{\sim}{\lambda}'$ iff P is defined F by I.

(2) If G and H are formulas, then

$$T_I(\underset{\sim}{\lambda}G)=\underset{\sim}{\lambda}\cdot T_I(G);$$
$$T_I(\sim G)=(T_I(G))';$$
$$T_I(G\vee H)=T_I(G)\oplus T_I(H);$$
$$T_I(G\wedge H)=T_I(G)*T_I(H);$$
$$T_I(G\to H)=T_I(\sim G\vee H);$$
$$T_I(G\to H)=T_I((G\to H)\wedge(H\to G));$$
$$T_I((\forall\ x)G(x)))=\prod_{x\in D}(T_I(G(x)));$$
$$T_I((\exists\ x)G(x))=\sum_{x\in D}(T_I(G(x)));$$
$$T_I((\underset{\sim}{\lambda}\forall\ x)G(x))=T_I(\underset{\sim}{\lambda}((\forall\ x)G(x)));$$
$$T_I((\underset{\sim}{\lambda}\exists\ x)G(x))=T_I(\underset{\sim}{\lambda}((\exists\ x)G(x))),$$

where $x_1\oplus x_2\oplus\cdots\cdots$ is denoted simply by $\sum_i x_i$, $x_1 * x_2 * \cdots\cdots$ is denoted simply by $\prod_i x_i$.

3 Normal form of formulas in LVOL

Theorem 3.1 Arbitrary formula in LVOL is equivalent to a former constraint normal form.

Definition 3.1 Let G be a formula, $\underset{\sim}{\lambda}\in L_1$. If $\forall\ I, T_I(G)\leqslant\underset{\sim}{\lambda}$, then G is $\underset{\sim}{\lambda}$-identically false; if $\forall\ I, T_I(G)\geqslant\underset{\sim}{\lambda}$, then G is $\underset{\sim}{\lambda}$-identically true.

Theorem 3.2 Formula G is $\underset{\sim}{\lambda}$-identically false iff Skolem's normal form of G is $\underset{\sim}{\lambda}$-identically false.

4 $\underset{\sim}{\lambda}$—resolution

Because $\tilde{L}_1$ is a partial ordered set, importing $\underset{\sim}{\lambda}$—resolution is difficult. We may take a completely ordered subset of $\tilde{L}_1$ as our discussing field. The truth values of "unknowing to be true or false" is represented by U_n, where $\mu_{U_n}(x)\equiv 1$. We take a completely ordered subset including U_n from $\tilde{L}_1$, it is denoted by $\tilde{N}$. From now on, truth values of formulas are taken from $\tilde{N}$.

In order to discuss conventiently, we only think about clauses being made of fuzzy atom. Its conclusions are easy to extend to normal clauses.

Definition 4. 1 Let $\underset{\sim}{\lambda}_1 L$ and $\underset{\sim}{\lambda}_2 L$ be two literals, $\underset{\sim}{\lambda}\in\tilde{N}$. If $\underset{\sim}{\lambda}\geqslant U_n$, $\underset{\sim}{\lambda}_1>\underset{\sim}{\lambda}$ and $\lambda_2<\underset{\sim}{\lambda}'$; or $\underset{\sim}{\lambda}_1<\underset{\sim}{\lambda}'$ and $\underset{\sim}{\lambda}_2>\underset{\sim}{\lambda}$ (if $\underset{\sim}{\lambda}<U_n$, then just contrary), then $\underset{\sim}{\lambda}_1 L$ and $\underset{\sim}{\lambda}_2 L$ are $\underset{\sim}{\lambda}$—complementative literal.

Definition 4. 2 Let $\underset{\sim}{\lambda}_1 L$ and $\underset{\sim}{\lambda}_2 L$ be two literals, $\underset{\sim}{\lambda}\in\tilde{N}$. If $\underset{\sim}{\lambda}\geqslant U_n$, $\underset{\sim}{\lambda}_1>\underset{\sim}{\lambda}$ and $\underset{\sim}{\lambda}_2>\underset{\sim}{\lambda}$; or $\underset{\sim}{\lambda}_1<\underset{\sim}{\lambda}'$ and $\lambda_2<\underset{\sim}{\lambda}'$ (if $\underset{\sim}{\lambda}<U_n$, then iust contrary), then $\underset{\sim}{\lambda}_1 L$ and $\underset{\sim}{\lambda}_2 L$ are $\underset{\sim}{\lambda}$—samiliar literal.

Definition 4. 3 Let C_1 and C_2 be clauses without same variables, let $\underset{\sim}{\lambda}_1 L_1$ and $\underset{\sim}{\lambda}_2 L_2$ be respectively literal of C_1 and C_2. If L_1 and L_2 have MGUσ, $\underset{\sim}{\lambda}_1 L_1^\sigma$ and $\underset{\sim}{\lambda}_2 L_2^\sigma$ are $\underset{\sim}{\lambda}$—complementative literal, then $(C_1^\sigma-S_1)\vee(C_2^\sigma-S_2)$ is called binary $\underset{\sim}{\lambda}$—resolution formula about C_1 and C_2, denoted by $R_{\underset{\sim}{\lambda}}(C_1,C_2)$, where

$S_1=\{\underset{\sim}{\lambda}^* L^\sigma\mid(\underset{\sim}{\lambda}^* L^\sigma\in C_1^\sigma)\wedge(\underset{\sim}{\lambda}^* L^\sigma$ and $\underset{\sim}{\lambda}_1 L_1^\sigma$ are $\underset{\sim}{\lambda}$—samiliar literal$)\}$,

$S_2=\{\underset{\sim}{\lambda}^* L^\sigma\mid(\underset{\sim}{\lambda}^* L^\sigma\in C_2^\sigma)\wedge(\underset{\sim}{\lambda}^* L^\sigma$ and $\underset{\sim}{\lambda}_2 L_2^\sigma$ are $\underset{\sim}{\lambda}$—samiliar literal$)\}$.

Definition 4. 4 Let S be clause set. Treat disjunct $\underset{\sim}{\lambda}^* L$ in C by the following rules, where $C\in S$,

(1) if $\underset{\sim}{\lambda}\geqslant Un$, $\underset{\sim}{\lambda}'\leqslant\underset{\sim}{\lambda}^*\leqslant\underset{\sim}{\lambda}$, then cross off $\underset{\sim}{\lambda}^* L$ from C;

(2) if $\underset{\sim}{\lambda}<Un$, $\underset{\sim}{\lambda}\leqslant\underset{\sim}{\lambda}^*\leqslant\underset{\sim}{\lambda}'$, then cross off $\underset{\sim}{\lambda}^* L$ from C.

The remained clause is called $\underset{\sim}{\lambda}$—primary reductive clause, denoted by $C^{\underset{\sim}{\lambda}}_{PR}$. The remained clause set is called $\underset{\sim}{\lambda}$—primary reductive clause set, denoted by $S^{\underset{\sim}{\lambda}}_{PR}$.

Theorem 4. 1 Let S be a clause set, $\underset{\sim}{\lambda}\geqslant Un$, then S is $\underset{\sim}{\lambda}$—identically false iff $S^{\underset{\sim}{\lambda}}_{PR}$ is $\underset{\sim}{\lambda}'$—identically false.

Definition 4. 5 Let $S^{\underset{\sim}{\lambda}}_{PR}$ be $\underset{\sim}{\lambda}$—primary reductive clause set of S. Treat the disjunct $\underset{\sim}{\lambda}^* L$ in $C^{\underset{\sim}{\lambda}}_{PR}$ by the following rules; where $C^{\underset{\sim}{\lambda}}_{PR}\in S^{\underset{\sim}{\lambda}}_{PR}$,

(1) when $\underset{\sim}{\lambda}\geqslant Un$, $\underset{\sim}{\lambda}^*>\underset{\sim}{\lambda}$, then $\underset{\sim}{\lambda}^* L$ is substituted by L,

(2) when $\underset{\sim}{\lambda}\geqslant Un$, $\underset{\sim}{\lambda}^*<\underset{\sim}{\lambda}'$, then $\underset{\sim}{\lambda}^* L$ is substituted by $\sim L$,

(3) when $\underset{\sim}{\lambda}<Un$, $\underset{\sim}{\lambda}^*>\underset{\sim}{\lambda}'$, then $\underset{\sim}{\lambda}^* L$ is substituted by L,

(4) when $\underset{\sim}{\lambda}<Un$, $\underset{\sim}{\lambda}^*<\underset{\sim}{\lambda}$, then $\underset{\sim}{\lambda}^* L$ is substituted by $\sim L$.

By this way, the attained clause is called $\underset{\sim}{\lambda}$—reductive clause, denoted by $C^{\underset{\sim}{\lambda}}_R$; the attained clause set is called $\underset{\sim}{\lambda}$—reductive clause set, denoted by $S^{\underset{\sim}{\lambda}}_R$.

Theorem 4. 2 $S^{\underset{\sim}{\lambda}}_{PR}$ is $\underset{\sim}{\lambda}$—identically false iff $S^{\underset{\sim}{\lambda}}_R$ is $\underset{\sim}{\lambda}$—identically false.

Theorem 4.3 Let C_1 and C_2 be two clauses, let C_{1R}^{λ} and C_{2R}^{λ} be respectively λ—reductive clause of C_1 and C_2. If $C=R_{\lambda}(C_1,C_2)$, then there exists $C'=R(C_{1R}^{\lambda},C_{2R}^{\lambda})$ such that $C'=C_R^{\lambda}$ is satisfied; In the contrary, if $C'=R(C_{1R}^{\lambda}, C_{2R}^{\lambda})$, then there exists $C=R_{\lambda}(C_1,C_2)$ such that $C_R^{\lambda}=C'$ is satisfied.

Theorem 4.4 Let $\lambda \geqslant Un$. If clause set S is λ—identically false, then there exists λ—resolution such that λ—empty clause can be deduced by the resolution (each literal $\lambda^* L$ in λ—empty clause satisfies $\lambda' \leqslant \lambda^* \leqslant \lambda$, λ—empty clause is denoted by $\lambda-\square$).

5 λ—weak implication and λ—strong implication

The following results are similar to [3].

Definition 5.1 Let G and H be formula, $\lambda \in \tilde{N}$. If $(G \to H)$ is λ—identically true, then we call G λ—weak implication H, denoted by $G \Rightarrow H$.

Theorem 5.1 $(G \to H)$ is λ—identically true iff $\forall I$ if $T_I(G) > \lambda'$, then $T_I(H) \geqslant \lambda$.

Definition 5.2 Let G and H be formulas, $\lambda \in \tilde{N}$. If $\forall I, T_I(G) \geqslant \lambda$, then $T_I(H) \geqslant \lambda$, then we call G λ—strong implication H, denoted by $G \stackrel{+}{\Rightarrow} H$.

Proposition 5.1 If $G \Rightarrow H, \lambda > Un$, then $G \stackrel{+}{\Rightarrow} H$; if $G \stackrel{+}{\Rightarrow} H, \lambda = Un$, then $G \Rightarrow H$.

Proposition 5.2 Let G be formula, then (1) when $\lambda \leqslant Un, A \Rightarrow A$, (2) $A \stackrel{+}{\Rightarrow} A$.

Proposition 5.3 Let A, B and C be formulas, then

(1) when $\lambda > Un$, if $A \Rightarrow B$ and $B \Rightarrow C$, then $A \Rightarrow C$; (2) if $A \stackrel{+}{\Rightarrow} B$ and $B \stackrel{+}{\Rightarrow} C$, then $A \stackrel{+}{\Rightarrow} C$.

Proposition 5.4 Let A, B and C be formulas, then

(1) if $A \Rightarrow B, A \Rightarrow C$, then $A \Rightarrow (B \wedge C)$; (2) if $A \stackrel{+}{\Rightarrow} B, A \stackrel{+}{\Rightarrow} C$, then $A \stackrel{+}{\Rightarrow} (B \wedge C)$.

Theorem 5.2 Let $\lambda \geqslant Un$. If there exists λ—resolution such that λ—empty clause can be deduced from clause set S by the resolution, then S is λ—identically false.

Theorem 5.3 Let $\lambda \geqslant Un$, let S be a clause set. Then S is λ—identically false iff there exists λ—resolution such that λ—empty clanse can be deduced from clause set S by the resolution.

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