QUALIFICATION OF IMPRECISELY AND UNCERTAINLY DEFINED OBJECTS BASED ON MULTICRITERIAL EVALUATION

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Abstract

The present paper proposes a model for the process of multicriterial evaluation, aimed at granting qualifiers and accordingly ordering the evaluated objects. The model takes into account the major characteristic of such processes, that of operating mostly with imprecise and uncertain knowledge: descriptions of objects, knowledge reflecting the experience and competence of the evaluators, types of approximate reasoning patterns.

INTRODUCTION

The process of multicriterial evaluation of objects in a class, which has as goal a graded appreciation of those objects, through the granting of adequate qualifiers, represents a slightly different alternative to the "classical" problem of multicriterial decision.

In order to outline the specificity of that process, for which we shall use the term of evaluation by competition, we just mention a few representative examples:

- competition within education systems
- competition for papers acceptance (at conferences or publications)
- competition for obtaining specific financial resources
  - differentiated financing for complex research programs (long term technology forsight, ex-ante evaluation of research programs)
  - ex-ante evaluation of research projects
  - "temporisation" strategies of firms in launching new products

Nowadays, evaluation by competition is undoubtedly an attribute of human expertise, but the modelling and simulation of the process of granting qualifiers has all chances to develop as a distinct research domain, within the large area of artificial intelligence.

The main componenets of the proposed model are presented in chap. **I, II** and **III.**

In **chap. I** are presented:

- the structure of the space where the evaluation process takes place;
- the structure of the descriptions of objects to be evaluated and how to compare them.

The distinct peculiarities of the evaluation process, induced by the proposed model, are due to the way of treating qualifiers, which become key elements, with a determinant role.

The specific aspects of this approach are presented as follows:

- **chap.II** deals with the problem of unicriterial evaluation;
- **chap.III** deals with the problem of multicriterial evaluation.

An experimentation framework for the model was embodied into an expert system implemented for the technology forsight field (short presentation in ch.IV.).

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I. The evaluation space

Def. I.1 By **evaluation space** we mean the quadruple { O, E, K, C}, where:
- O - represents the set of objects to be evaluated;
- E - represents the set of evaluators (the jury);
- K - represents the set of evaluation criteria;
- C - represents the set of qualifiers that may be granted depending on the evaluation criteria K.

I.1 Description of objects to be evaluated

Def. I.1.1 A property of an object x, namely the presence of an attribute X with a given value A, is defined by a triple of the form (x X A). A represents:

a) a precise value, if |A| = 1, respectively A = {a};

b) an imprecise value, if |A| > 1. In this situation A is a fuzzy set, defined over an universe U of possible values for X, by a membership function μ-A : U ---> [0, 1], where μ-A (u) = pos -X (u) represents the possibility for u to be a value of attribute X.

Def. I.1.2. The set of properties of object x is divided into two classes:

- the properties considered certain for object x, described through a string of triples: $((x\ X1^+\ A1^+)\ \ldots\ldots\ (x\ Xq^+\ Aq^+))$,

where it is considered that the values of attribute Xi^+ are certainly in the set Ai^+ ;

- the properties considered uncertain for object x, described through a string of triples: $(((x\ X1^-\ A1^-)\ \lambda 1)\ \ldots\ldots\ (x\ Xq^-\ Aq^-)\ \lambda q))$,

where, for an attribute Xi^- , it is considered that there is a possibility λi, that the attribute Xi^- takes values outside Ai^-.

Def. I.1.3. Consequently, the description Dx of an object x contains the set of properties:
$Dx = Dx^+ \cup Dx^-$, where $Dx^+ = \{ (x\ Xj^+\ Aj^+)\}j$, $Dx^- = \{(x\ Xq^-\ Aq^-)\ \lambda q)\}q$

Def. I.1.4. The extended description Dx' of an object x is given by:
$Dx' = Dx \cup \{(x\ Xr\ Ar)\ \lambda r\ )\ \}\ r$, where:
- the properties (x Xr Ar) result through deductions based on rules of the form (x Xp Ap) ---> (x Xr Ar), starting from properties (x Xp Ap0) $\in$ Dx

I.2 Comparison of descriptions of objects

Let the following be:
- Dx and Dy descriptions of two objects x and y
- Xx - the set of attributes of object x
- Xy - the set of attributes of object y
- Ax, Ay - the set of values for attributes Xx si Xy
- $Xxy = Xx \cap Xy = \{X1,\ldots,Xr\}$ the set of attributes common to x and y
- Xx' = Xx \ Xxy - the set of attributes specific just for obiect x
- Xy' = Xy \ Xxy - the set of attributes specific just for obiect y
- Ax', Ay' - the set of values for attributes Xx' and Xy'

Prop. I.2.1. The compatibility between the values AX-x si AX-y taken by attribute X within the descriptions Dx and Dy coincides with the degrees of mutual dependence between these values:

Pos (AX-x ^ AX-y) = Pos (AX-x I AX-y) = Pos (AX-y I AX-x)

Dem. Pos (AX-x ^ AX-y) = min (Pos (AX-x), Pos (AX-y)) =
= min ($\sup_{u \in U(X)}$ pos1-X(u), $\sup_{u \in U(X)}$ pos2-X(u)) =
= $\sup_{u \in U(X)}$ min (pos1-X(u), pos2-X(u)) =
= Pos (AX-x I AX-y) = Pos (AX-y I AX-x)

where:
- pos1-X(u) = max (μ - AX-x (u), λ1), pos2-X(u) = max (μ - AX-y (u), λ2)
- λ1 - the possibility that X takes values outside AX-x;
  λ1 = 0, if X is an attribute with certain values for object x

- $\lambda 2$ - the possibility that X takes values outside AX-y;

$\lambda 2 = 0$, if X is an attribute with certain values for object y

Prop. I.2.2. The compatibility between the values (AXi')i, taken by attributes Xi' within description Dx and the value AY' taken by the attribute Y' within description Dy coincides with the degrees of mutual dependence between these values:

Pos ((AX1'^....^ AXn') ^ AY') = Pos (AY' I (AX1'^....^ AXn')) =
= Pos ((AX1'^....^ AXn') I AY')

Dem. Pos ((AX1'^....^ AXn') ^ AY') =
= min (sup $_{v1 \in V1, ..., vn \in Vn}$ pos - X1',...Xn' (v1,.....,vn), sup $_{w \in W}$ pos -Y' (w)) =
= sup $_{v1 \in V1, ..., vn \in Vn, w \in W}$ min (pos - X1',...Xn' (v1,.....,vn), pos -Y' (w)) =
= sup $_{v1 \in V1, ..., vn \in Vn, w \in W}$ min ((pos- Xi' (vi))i, pos -Y' (w))
where pos-Y'(w) este calculated according to the projection principle.

Obs. If, for a part of the properties (x Xq Axq) specific just to x, the knowledge base contains a set of explicit rules R = (Rq)q, of the form Rq = "if X'q is AX'q then Y' is AY' ", it will be considered that the values of attribute Y' depend explicitly of the values of attributes (X'q)q. Under these circumstances the compatibility will be discussed accordingly:

Pos (Q I (^ i (Pi)) ^ (^ q (Rq)))= Pos (AY' I (^ i (AXi')) ^ (^ q (AY' I AXq))) =
= min (Pos (AY' I ^ i (Axi')), Pos (^ q (AY' I AXq))) =
= min (sup $_{v1 \in V1, ..., vn \in Vn}$ pos - X1',...Xn' (v1,.....,vn), sup $_{w \in W}$ pos -Y' (w),
sup $_{vq \in VQ, w \in W}$ min $_q$ pos-Y'I AXq (w, vq)),
where, if the Zadeh implication is used (I (a,b) = max (1 -a, min (a,b))), then
pos-Y'I AXq (w, vq) = max (1 - pos-AXq (vq), min (pos-AXq (vq), pos-Y'(w)))

Prop.I.2.3. The degree to which the description Dy becomes certain, when the description Dx is known to exist, is given by the conditional necessity:

Nec (Dy I Dx) = max (min $_{X \in Xxy}$ (Nec (AX-y I AX-x)),
min $_{Y' \in Xy', Xi' \in Xx'}$ (Nec (AY' I (^ i (AXi')) ^ (^ q Rq)))),
where Rq = " daca X'q este AX'q atunci Y' este AY' "

Dem. Nec (Dy I Dx) = 1 - Pos ($\neg$ Dy I Dx) =
= 1 - min (max $_{X \in Xxy}$ Pos ($\neg$ AX-y I AX-x),
max $_{Y' \in Xy', X' \in Xx'}$ Pos ($\neg$ AY' I (^ i (AXi')) ^ (^ q Rq))) =
= max (min $_{X \in Xxy}$ (1 - Pos ($\neg$ AX-y I AX-x)),
min $_{Y' \in Xy', X' \in Xx'}$ (1 - Pos ($\neg$ AY' I (^ i (AXi')) ^ (^ q Rq)))) =
= max (min $_{X \in Xxy}$ (Nec (AX-y I AX-x)),
min $_{Y' \in Xy', X' \in Xx'}$ (Nec (AY' I (^ i (AXi')) ^ (^ q Rq))))

II. The problem of unicriterial evaluation

Def. II.1. We consider that the problem of unicriterial evaluation resides in determining the **qualifier c** $\in$ Kc, **the most adequate for object x**, accoding to a single evaluation criterion k, given the properties specified by the description Dx of the object. The problem of unicriterial evaluation has three phases:

- **at object level:**

(i) - to compare the description of object x with the generic descriptions of the objects considered as prototypes for granting a certain qualifier and to establish, accorodingly, a preferential qualifier for x;

(ii) - to check if x fulfills the conditions for being granted the preferential qualifier (the presence of attributes and values considered of reference and/ or significant for granting that qualifier)

- **at class of objects level:**

(iii) - ordering the objects of class O, depending on the obtained qualifiers, on the basis of specific dominance relations.

Obs. Keeping in mind the so called **"principle of parcimony"** /6/ an acceptable evaluation of an object, according to a set of properties, contains a very restricted number of qualifiers, preferably a single one.

II.2 Classes of attributes and values

II.2.1 Reference and significant attributes

Def. II.2.1.1. Let c be a qualifier that may be granted according to an evaluation criterion k. The qualifier c determines over UX a fuzzy set cX, defined by the memebership function μ-cX: UX ---> [0, 1], where μ-cX (X) represents the degree of importance of attribute X in granting the qualifier c.

Def. II.2.1.2. We consider that μ-cX generates **three classes of attributes, according to their importance order in granting the qualifier c:**

- reference attributes: Xc ++ = $\{X \in UX \mid \mu\text{-cX}(X) \geq 0{,}7\}$
- significant attributes: Xc + = $\{X \in UX \mid 0{,}4 \leq \mu\text{-cX}(X) < 0{,}7\}$
- non-significant attributes: Xc0 = $\{X \in UX \mid \mu\text{-cX}(X) < 0{,}4\}$

II.2.2 Reference and significant values

Within the context of the qualifier c, the attribute X determines over the universe U of its possible values, a fuzzy set cXA, defined by the memebership function:
μ-cXA: U ---> [0, 1], where μ-cXA (u) represents the degree to which it is recommendable that an object should take the value u for the attribute X, in order to be granted the qualifier c.

Def. II.2.2.1. We consider that μ-cXA generates **three classes of possible values**, according to their role in granting the qualifier c:

- reference values: XAc ++ = $\{u \in U \mid \mu\text{-cXA}(u) \geq 0{,}7\}$
- significant values: XAc + = $\{u \in U \mid 0{,}4 \leq \mu\text{-cXA}(u) < 0{,}7\}$
- non-significant values: XAc0 = $\{u \in U \mid \mu\text{-cXA}(u) < 0{,}4\}$

Prop. II.2.2.1. The classification of the values of attribute X, within the context of the qualifier c, induces over the universe U of all possible values for X, a double structure of random set:

(i) (***F1***, m1) - ordinary random set;

(***F1***, m1) = { (A11 m11), (A12 m12), (A13 m13)}, cu

F1 = { A11 = Xac++, A12 = Xac+, A13 = Xac0} si

m1 = { m11 = m(Xac++) = $\min_{u \in Xac++}$ μ-cXA (u) = 0,7 ;
m12 = m(Xac+) = 1 - m(Xac++) = 0,3;
m13 = m(XAc0) = 1 - (m11 + m12) = 0}

(ii) (***F2***, m2) - consonant random set:

(***F2***, m2) = { (A21 m21), (A22 m22), (A23 m23)}, cu

F2 = { A21 = $cXA_{0,7}$ A22 = $cXA_{0,4}$ A23 = cXA_0}, A21 $\subset$ A22 $\subset$ A23

m2 = { m21 = a0 - a1 = 1,0 - 0,7 = 0,3
m22 = a1 - a2 = 0,7 - 0,4 = 0,3;
m23 = a2 - a3 = 0,4 - 0 = 0,4 }

Dem: Obvious in both cases: Aij $\subseteq$ U, Σ_j mij = 1, i= 1,2, j = 1,3

II.2.3 Reference and significant properties

Def. II.2.3.1 We consider that a property (x X A) of an obiect x is of **reference** for the granting of a qualifier c, if:

- X is a reference attribute, according to c: X $\in$Xxref;
- A is a reference or significant value for X: A $\in$ Axref

Def. II.2.3.2 We consider that a property (x X A) of an obiect x is of **significant** for the granting of a qualifier c, if:

- X is a significant attribute, according to c: X $\in$Xxsem;
- A is a reference or significant value for X: A $\in$ Axsem

II.3 Description of the prototype object for the granting of a given qualifier

Def. II.3.1 Let O be a class of objects and c a qualifier that may be granted according to an evaluation criterion k.

We call the **description of the prototype object (the prototype description)** for the granting of c, depending on k and O, the description pc-k (O) formed of the properties specific to class O, which are of reference or significant for the granting of c:

pc-k (O) = ((x X1 A1)...... (x Xn An)), $Xi \in Xc{+}{+} \cup Xc{+}$, $Ai \in Axref \cup Axsem$.

II.4 The horizon of a qualifier

Def. II.4.1 The horizon of a qualifier c, depending on the evaluation criterion k and the class of object O, oriz (c, k, O), is defined as the set of those descriptions pc'-k which are highly compatible with the prototype description pc-k.

Obs. If we consider that compatibility is expressed through possibility and necessity degrees higher than 0,8 , then we have, by definition:

oriz (c, k, O) = {pc'-k I Pos (pc'-k I pc-k) $\geq$ 0,8, Nec (pc'-k I pc-k) $\geq$ 0,8 }

II.5 Conditions for granting a qualifier

Def. II.5.1 Let Y be an attribute of object x.

The tendency of the property BY = (x Y B) to join the prototype description pc-k of a qualifier c, depending on the evaluation criterion k, is given by:

tend (BY, c, k) = card { (x X A) | (x X A) $\in$ pc-k ,
Nec (X | Y) $\geq$ 0,5 si Nec (A | B) $\geq$ 0,5}

Def. II.5.2 A property BY=(x Y B) is called characteristic for the granting of a qualifier c, if the tendency to join the prototype description of that qualifier is maximal, as compared to other qualifiers:

tend (BY, c, k) = $\max_i$ tend (BY, ci, k), $\forall$ ci $\neq$ c

Prop. II.5.1 A property BY=(x Y B), characteristic for the granting of a qualifier c, is joining the prototype descriptions corresponding to the horizon of c.

Dem: Let pc'-k $\in$ oriz (c, k, O). The description pc'-k becomes possible and necessary, in the presence of the prototype description pc-k of c.

Let (x X A) $\in$ pc-k, (x X' A') $\in$ pc'-k .

Since Pos/Nec (pc'-k I pc-k) $\geq$ 0,8 , and Nec (X | Y) $\geq$ 0,5 si Nec (A | B) $\geq$ 0,5 $\Rightarrow$

$\Rightarrow$ Nec (X' | Y) $\geq$ min (0,5, 0,8) = 0,5 and Nec (A' | B) $\geq$ min (0,5, 0,8) = 0,5 $\Rightarrow$

$\Rightarrow$ card { (x X' A') I (x X' A') $\in$ pc'-k , Nec (X' | Y) $\geq$ 0,5 si Nec (A' | B) $\geq$ 0,5} $\neq$ 0 $\Rightarrow$

$\Rightarrow$ tend (BY, c', k) $\neq$ 0.

Prop. II.5.2 The properties Bi specific to an object x determine a partition, according to the qualifiers for which they are characteristic:

{Bi}i = $\cup_{c \in kc}$ Bc, Bc = {BY = (x Y B) | BY - characteristic for pc-k}

Dem: Observing the definition, a property is characteristic for a given qualifier, if it mostly joins the prototype description of that qualifier (the tendency to join that description is maximal).

In the same time, two different qualifiers obviously have different prototype descriptions.

Hence, for c' $\neq$ c", tend (BY, c', k) $\neq$ tend (BY, c", k).

Corollary II.5.1. (The principle of minimum differentiation). The same object x covers up to different degrees, the prototype descriptions of two different qualifiers c1 $\neq$ c2 (for the same criterion k), respectively: Bc1 $\neq$ Bc2

Dem: Obviously, the prototype descriptions of the two different qualifiers are different: pc1-k $\neq$ pc2-k. According to the previous proposition, a property may be characteristic for a single qualifier. Hence, if BY $\in$ Bc1 then BY $\notin$ Bc2, or, respectively, if BY $\in$ Bc2 then BY $\notin$ Bc1.

Corollary II.5.2. The final qualifier granted to an object x of a class O, through evaluation depending on a single criterion k, is unique.

Dem: Results directly from the previous corollary.

II.6 The dominance relation between qualifiers granted to the same object

Def. II.2.1. The dominance relation >-DC-k between two qualifiers c' si c", granted to the same object x, through two distinct evaluation processes, depending on the same criterion k, is defined by:

c' >-DC-k c" iff Nec (pc'-k I Dx) $\geq$ Nec (pc"-k I Dx).

II.7 Preferential qualifiers granted to an object

Def. II.7.1 A qualifier c0, granted to an object x, is called preferential, as compared to a set of qualifiers C, where it belongs, if c0 dominates all the other qualifiers in C, respectively:

$\forall$ ci $\in$ C, ci $\neq$ c0, Nec (pc0 I Dx) $\geq$ Nec (pci I Dx),

where {pci}i represent the prototype descriptions for granting the qualifiers { ci}i.

Prop. II.7.1 There is only one preferential qualifier, depending on a given evaluation criterion k, that may be granted to an object x.

Dem. Let c0 $\in$ Ck be the preferential qualifier considered for the set of qualifiers Ck, that may be granted depending on the evaluation criterion k.

$\forall$ pci-k, pcj-k $\in$ PCk, i $\neq$ j, we have pci-k $\neq$ pcj-k $\Rightarrow$

$\Rightarrow$ Nec (pci-k I Dx) $\neq$ Nec (pcj-k I Dx) $\Rightarrow$ $\exists!$ c0 $\in$ Ck s.t.

for ci $\notin$ Ck, ci $\neq$ c0, Nec (pc0-k I Dx) $\geq$ Nec (pci-k I Dx)

Prop. II.7.2. The qualifiers Ak = {a1(k), ..,am(k)} granted by the evaluators {e1,...,em} are dominated by the qualifier c0 $\in$ Ck, which is preferential, as reported to set of qualifiers Ck, that may be granted depending on the evaluation criterion k.

Dem. Let c0 $\in$Ck be the preferential qualifier considered for the set of qualifiers Ck. $\Rightarrow$ $\forall$ c $\in$ Ck, c0 >-DC-k c, and $\forall$ i =1,m, ai (k) $\in$ Ck $\Rightarrow$ $\forall$ i =1,m, c0 >-DC-k ai (k)

II.8 The dominance relation between objects evaluated depending on the same evaluation criterion

Def. II.8.1 The dominance relation >-DO-k between the objects x and y, evaluated depending on the same evaluation criterion k, is defined by:

x >-DO-k y ddaca Nec (pcx-k I Dx) $\geq$ Nec (pcy-k I Dy)

III. The problem of multicriterial evaluation

Def. III.1 We consider that the problem of multicriterial evaluation of a class of objects O, depending on a set of evaluation criteria K, by a set of evaluators E, has four phases:

- **at object level**

(I) - the unicriterial evaluation of the object x, by each evaluator ei, depending on every evaluation criterion kj, taken apart, finalised with the granting of a qualifier ai (j):

ai (j) = evk (x, ei, kj) = F-i (Dx, (pcl-j)j)

Obs. The qualifier ai (j) results from the correlaltion done by the evaluator ei between the description Dx of the object and the prototype descriptions (pcl-j)l, corresponding to the qualifiers (cl)l that may be granted depending on kj.

(ii) - the multicriterial evaluation of the object x, by the evaluator ei, depending on the set K of all the criteria, taken together, finalised with the granting of a single general qualifier bi:

bi = F1 (ai (1), ..., ai (n))

Obs. The general qualifier bi results from the correlaltion done by the evaluator ei intre between the qualifiers ai (j), granted by ei depending on every criterion kj taken apart. The goal of F1 is to establish, on the basis of a dominance relation between qualifiers, the preferential qualifier granted by ei, depending on the set K.

(iii) - the multicriterial evaluation of the object x, by all the evaluators e, depending on the set K of all the evaluation criteria, taken together, finalised with the granting of a single general qualifier c:

$$c = F2 (b1, ..., bm)$$

Obs. The general qualifier c results from the correlaltion done by all the evaluators ei, between the general qualifiers bi, granted by each evaluator apart, depending on the set K of all the evaluation criteria. The goal of F2 is to establish, the general preferential qualifier, granted by all evaluators ei, depending on the set K.

- at class of objects level

(iv) - ordering the objects of class O, depending on the general qualifiers obtained, on the basis of a specific dominance relation.

Def. III. 2. We call a general qualifier a qualifier that is granted (by one or several evaluators) depending on a set containing several evaluation criteria.

III.2 Classes of evaluation criteria: reference and significant criteria

Def. III.2.1 The way a general qualifier c may be granted, by a single evaluator e, generates three classes of criteria, in accordance with their importance in granting c:

- reference criteria: $Ke\text{-}c{+}{+} = \{ k \in K \mid \mu\text{-}ce\,(k) \geq 0{,}7\}$
- significant criteria: $Ke\text{-}c{+} = \{ k \in K \mid 0{,}4 \leq \mu\text{-}ce\,(k) < 0{,}7\}$
- non-significant criteria: $Ke\text{-}c0 = \{ k \in K \mid \mu\text{-}ce\,(k) < 0{,}4\}$

III.3 The prototype description for granting general qualifiers

Def. III.3.1 The prototype description for granting a general qualifier c, depending on a whole set K of evaluation criteria, represents the union of the prototype descriptions corresponding to the unicriterial qualifiers c-k, similar to c:

$$pc = \bigcup_{c\text{-}k,\ \mu\text{-}ckX = \mu\text{-}cX} pc\text{-}k$$

III.4 The dominance relation between general qualifiers granted to the same object

Def. III.4.1. The dominance relation >-DCG between two general qualifiers c1 and c2 granted to the same object x, through two distinct evaluations, depending on the same set of evaluation criteria K, is defined by:

$c1 \succ\text{-DCG}\ c2$ ddaca $\forall k \in K,\ c'\text{-}k \succ\text{-DC-}k\ c''\text{-}k$,

where c'-k and c"-k are the qualifiers granted to x, depending on the criterion $k \in K$.

Prop. III.4.1. There is only one preferential qualifier, depending on a given set of evaluation criteria $K' \subseteq K$, that may be granted to an object x.

Dem. Let $c0 \in C = \{ c1,..., cl\}$, be the preferential qualifier considered for the set of general qualifiers C, that may be granted depending on the set of evaluation criteria $K' \subseteq K$.

$\forall$ pci, pcj $\in$ PC, $i \neq j$, we have pci $\neq$ pcj $\Rightarrow$

$\Rightarrow$ Nec (pci I Dx) $\neq$ Nec (pcj I Dx) $\Rightarrow \exists!\ c0 \in C$ s.t.

$\forall$ ci $\in$ C, ci $\neq$ c0, Nec (pc0 I Dx) $\geq$ Nec (pci I Dx)

Prop. III.4.2. The qualifiers B granted by the evaluators {e1,...,em} are dominated by the qualifier $c0 \in Ck$, which is preferential, as reported to the set of general qualifiers C = { c1,..., cl}, that may be granted depending on the set of evaluation criteria $K' \subseteq K$.

Dem. Let $c0 \in C = \{ c1,..., cl\}$, be the preferential qualifier considered for the set of general qualifiers C $\Rightarrow$

$\Rightarrow\ \forall c \in C,\ c0 \succ\text{-DCG}\ c$, and $\forall i = 1,m,\ bi \in C \Rightarrow\ \forall i = 1,m,\ c0 \succ\text{-DCG}\ bi$

III.5 Modalities for granting a qualifier

Let C^{K} = C1 x......x Cn = {(c1,....,cn) | ci qualifier granted depending on the evaluation criterion ki}, be the universe of qualifiers that may be granted depending on criteria of K, taken apart.

Let $C^{K' \subseteq K}$ = Ci1 x......x Cin' = {(ci1,....,cin') I cij qualifier granted depending on the evaluation criterion kij, 1 ≤ ij ≤ n, n' ≤ n }, be the universe of qualifiers that may be granted depending on a subset of criteria K' ⊆ K, taken apart.

Let $C^{E} = (C)^{m}$ be the universe of general qualifiers that may be granted individually, by each of the m experts, depending on the set of criteria K, taken together.

Let $C^{E' \subseteq E} = (C)^{m' < m}$, be the universe of general qualifiers that may be granted individually, by eachone of m' < m experts, depending on the set of criteria K, taken together.

III.5.1 The granting of a general qualifier by a single evaluator

Prop. III.5.1.1. The modalities that a single evaluator e uses to grant a general qualifier for an object, through an evaluation process depending on the set K of all evaluation criteria, is represented by the projection (pointwise):

F1: C^{K} ------> C, defined by F1 (c1,...,cn) = c0,

where c0 is the preferential qualifier, granted by the evaluator e, depending on the reference or significant criteria.

Dem. (Intuitive) Indeed, the evaluator will grant a general qualifier c0, if he granted this qualifier for n' (most) of the criteria he considers as important (reference or significant criteria). The index n' corresponds to a "majority" of the reference or significant criteria (elements of K-c++ ∪ K-c+).

Def. III.5.1.1. The sets of rules that derive from the modalities represented by F1 are defined by the extension (spherical):

F1': C ------> *P* ($C^{K' \subseteq K}$) ,

where F1' (c0) = {(ci1,...., cin') I n' ≅ "majority of n", 1 ≤ ij ≤ n, ∀ ij, the following are satisfied: i) kij - reference or significant criterion; ii) cij = c0}

Obs. "Majority of n" is represented by a fuzzy set: μ - major: {1,.....,n} ---> [0,1] , where, the membership degrees are as follows:

μ - major (n') ≅ ((n' < n/2; n' = n/2; n/2 < n' < n; n' = n) (0; 0,4; 0,9; 1))

III.5.2 The granting of a general qualifier by several evaluators

Prop. III.5.2.1. The modalities that m evaluators use to grant a general qualifier for an object x, through an evaluation process depending on the set K of all evaluation criteria, is represented by the projection (pointwise):

F2: C^{E} ------> C, defined by F2 (c1,...,cm) = c0,

where c0 is the general qualifier granted by the majority of the evaluators.

Dem. (Intuitive) Indeed, the object will be granted the general qualifier c0, if m' (the majority) of the evaluators E = {e1,.....,em} granted the general qualifier c0.

Def. III.5.2.1. The sets of rules that derive from the modalities represented by F2 are defined by the extension (spherical):

F2': C ------> *P* ($C^{E' \subseteq E}$) ,

where F2' (c0) = {(ci1,...., cim') I m' ≅ "majority of m", 1 ≤ ij ≤ m, and ∀ ij, cij = c0}

III.6 The dominance relation between objects evaluated depending on the same set of evaluation criteria

Def. III.6.1. The dominance relation >-DOG between the objects x and y, evaluated depending on the same set of evaluation criteria K' ⊆ K, is defined by:

x >-DOG y iff the following are satisfied:

i) ∀ k ∈ K', x >-DO-k y,   ii) Nec (pcx I Dx) ≥ Nec (pcy I Dy)

IV. An expert system for evaluation by competition in the field of technology forsight

In order to validate the proposed model for multicriterial evaluation an experiment was developed, building an incipient expert system for technology forsight.

The rule base contains:
- rules for rewriting (unicriterial/ general) qualifiers, using their prototype descriptions;
- rules expressing dependencies between values of attributes;
- rules expressing the preferences of evaluators (concerning criteria and prototype descriptions);
- rules for granting qualifiers, based on the extensions F1' and F2'.

The data used as evidence for the the expert system were taken from a comparative study concerning the evolution of future technologies in Germany and Japan /11/, based on repeated inquiries addressed to most competent experts in the technological fields had in view.

The basic inference mechanism combines the local approach through generalised modus ponens with the power of the combination/ projection principle.

The system is implemented in GC-Lisp for IBM-PC/AT/486 compatible computers.

CONCLUSIONS

The paper presents a model proposed for the multicriterial evaluation of a class of objects, described through imprecise and uncertain pieces of knowledge.

The goal of the evaluation process is the **granting of qualifiers and the final ordering** of the objects in that class. To make a distinction for that category of evaluation processes, the term of evaluation by competition was proposed.

The expert system built experimentally, for the field of technology forsight confirmed the validity of the proposed model.

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